Synchronous Machine pu Model

A q-axis Model - per-unit

$$\frac{d\psi_G}{dt} = \frac{1}{T'_q}(-\psi_G + \psi_q)$$

$$\frac{d\psi_K}{dt} = \frac{1}{T''_q}(-\psi_K + \psi_q)$$

$$\psi_q = x''_q i_q + \frac{(x'_q - x''_q)}{x'_q}\psi_K + \frac{(x_q - x'_q)}{x_q}\frac{x''_q}{x'_q}\psi_G$$

$$\frac{d\psi_q}{dt} = \omega\psi_d - \omega_B R_a i_q - \omega_B v_q$$

A d-axis Model - in pu (assuming $T''_{dc} = T''_d$)

$$\frac{d\psi_H}{dt} = \frac{1}{T''_d}(-\psi_H + \psi_d)$$

$$\frac{d\psi_F}{dt} = \frac{1}{T'_d}(-\psi_F + \psi_d + \frac{x'_d}{(x_d - x'_d)}E_{fd})$$

$$\psi_d = x''_d i_d + \frac{(x'_d - x''_d)}{x'_d}\psi_H + \frac{(x_d - x'_d)}{x_d}\frac{x''_d}{x'_d}\psi_F$$

$$\frac{d\psi_d}{dt} = -\omega\psi_q - \omega_B R_a i_d - \omega_B v_d$$

$$E_{fd} = \frac{\omega_B M_{df}}{R_f} \frac{v_f}{V_{BASE}}$$

Zero Sequence Equations in pu

(not required for balanced situations)

$$\frac{d\psi_o}{dt} = \omega_B R_a i_o - \omega_B v_o$$

Torque Equation in per-unit

$$T_e = -(\psi_d i_q - \psi_q i_d)$$

ω is the electrical angular speed in rad/s.

ω_B is the base electrical angular speed in rad/s.

Model of Line

$$\begin{bmatrix} L_s & L_m & L_m \\ L_s & L_s & L_m \\ L_m & L_m & L_s \end{bmatrix} \frac{d}{dt} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} = -R \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix}$$

$$\left(\begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} - \begin{bmatrix} E_a \\ E_b \\ E_c \end{bmatrix} - \begin{bmatrix} V_{Ca} \\ V_{Cb} \\ V_{Cc} \end{bmatrix} \right)$$

Model of interconnection (d-q pu form)

$$\frac{d}{dt}\begin{bmatrix} i_d \\ i_q \end{bmatrix} = \begin{bmatrix} -\frac{R\omega_B}{x} & -\omega \\ \omega & -\frac{R\omega_B}{x} \end{bmatrix} \begin{bmatrix} i_d \\ i_q \end{bmatrix}$$

$$+\frac{\omega_B}{x}\left(\begin{bmatrix} v_d \\ v_q \end{bmatrix} - \begin{bmatrix} E_d \\ E_q \end{bmatrix} - \begin{bmatrix} V_{Cd} \\ V_{Cq} \end{bmatrix} \right)$$

$$x = \frac{\omega_B(L_s - L_m)}{Z_{base}}$$

Infinite Bus

$$E_d = -E \sin \delta \quad E_q = E \cos \delta$$

$$\frac{d\delta}{dt} = \omega - \omega_o$$

$$E = 1, \omega_o = \omega_B$$

If resistances are small ....

$$\frac{d}{dt}(\psi_d + xi_d) = -\omega(\psi_q + xi_q) - \omega_B E_d - \omega_B V_{Cq}$$

$$\frac{d}{dt}(\psi_q + xi_q) = \omega(\psi_d + xi_d) - \omega_B E_q - \omega_B V_{Cq}$$

If rotor fluxes are assumed to be constant and $x_d'' = x_q'' = x''$

$$\frac{d}{dt}(x+x'')i_d = -\omega(x+x'')i_q + \omega E_1 - \omega_B E_d - \omega_B V_{Cd}$$

$$\frac{d}{dt}(x+x'')i_q = \omega(x+x'')i_d + \omega E_2 - \omega_B E_q - \omega_B V_{Cq}$$

E_1 and E_2 are dependent on rotor fluxes.

Using the Transformation

$$(f_Q + jf_D) = (f_q + jf_d)e^{j\delta}$$

$$\frac{d}{dt}(x + x'')i_D = -\omega_B(x + x'')i_D + \omega E_1' \\ -\omega_B E_D - \omega_B V_{CD}$$

$$\frac{d}{dt}(x + x'')i_Q = \omega_B(x + x'')i_Q + \omega E_2' \\ -\omega_B E_Q - \omega_B V_{CQ}$$

Capacitor Equations

$$\begin{bmatrix} C & 0 & 0 \\ 0 & C & 0 \\ 0 & 0 & C \end{bmatrix} \frac{d}{dt} \begin{bmatrix} V_{Ca} \\ V_{Cb} \\ V_{Cc} \end{bmatrix} = \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix}$$

Capacitor Equations

$$\frac{d}{dt}\begin{bmatrix} V_{CD} \\ V_{CQ} \end{bmatrix} = \begin{bmatrix} 0 & -\omega_B \\ \omega_B & 0 \end{bmatrix}\begin{bmatrix} V_{CD} \\ V_{CQ} \end{bmatrix} + \frac{\omega_B}{b_c}\begin{bmatrix} i_D \\ i_Q \end{bmatrix}$$