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Lec-15
8/2/10

SYNCHRONOUS MACHINE EQUATIONS

STATOR FLUX EQNS.

$$\boxed{K_d = K_q = \sqrt{\frac{2}{3}},\ K_0 = \frac{1}{\sqrt{3}}}$$

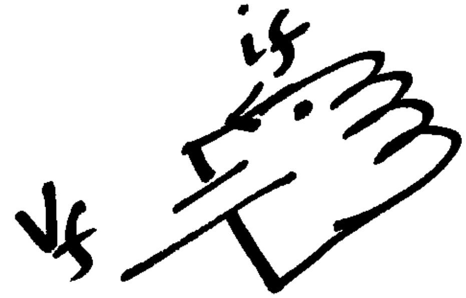
(a) $-\frac{d\psi_d}{dt} - \omega\psi_q - R_a i_d = V_d$

$\omega = \frac{d\theta}{dt}$

(b) $-\frac{d\psi_q}{dt} + \omega\psi_d - R_a i_q = V_q$

(c) $-\frac{d\psi_0}{dt} - R_a i_0 = 0$

ROTOR FLUX EQNS



(a) $$\frac{d\psi_f}{dt} + R_f i_f = V_f$$

(b) $$\frac{d\psi_h}{dt} + R_h i_h = 0$$

direct axis

(c) $$\frac{d\psi_g}{dt} + R_g i_g = 0$$

(d) $$\frac{d\psi_K}{dt} + R_K i_K = 0$$

quadrature axis.

$$
\begin{bmatrix} \psi_d \\ \psi_f \\ \psi_h \\ \psi_q \\ \psi_g \\ \psi_k \\ \psi_0 \end{bmatrix}
=
\begin{bmatrix}
L_d & M_{df} & M_{dh} & 0 & 0 & 0 & 0 \\
M_{df} & L_{ff} & L_{fh} & 0 & 0 & 0 & 0 \\
M_{dh} & L_{fh} & L_{hh} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & L_q & M_{qg} & M_{qk} & 0 \\
0 & 0 & 0 & M_{qg} & L_{gg} & L_{gk} & 0 \\
0 & 0 & 0 & M_{qk} & L_{gk} & L_{kk} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & L_0
\end{bmatrix}
\begin{bmatrix} i_d \\ i_f \\ i_h \\ i_q \\ i_g \\ i_k \\ i_0 \end{bmatrix}
$$

$$\frac{2}{p} J \frac{d\omega}{dt} = T_m - \frac{p}{2}\left(\psi_d\, \underset{\substack{\uparrow \\ T_e'}}{i_q} - \psi_q i_d\right)$$

$$\omega = \frac{d\theta}{dt}$$

$$\downarrow$$

$$\frac{2H}{\omega_B} \frac{d\omega}{dt} = \overset{pu}{T_m} - \frac{(\psi_d i_q - \psi_q i_d)}{(MVA_{base}/\omega_B)}$$

$$= \frac{MVA_{base}}{\omega_m} H = \frac{1}{2} J \omega_{mB}^2 / MVA_{base}.$$

$$\begin{bmatrix} f_d \\ f_q \\ f_0 \end{bmatrix} = \begin{bmatrix} K_1 \cos\theta & K_1 \cos\left(\theta - \frac{2\pi}{3}\right) & K_1 \cos\left(\theta + \frac{2\pi}{3}\right) \\ K_2 \sin\theta & K_2 \sin\left(\theta - \frac{2\pi}{3}\right) & K_2 \sin\left(\theta + \frac{2\pi}{3}\right) \\ K_3 & K_3 & K_3 \end{bmatrix} \begin{bmatrix} f_a \\ f_b \\ f_c \end{bmatrix}$$

$C_p^T = C_p^{-1}$

$K_1 = \frac{2}{3K_d}$ $\quad K_2 = \frac{2}{3K_q}$ $\quad K_3 = \frac{1}{3K_0}$

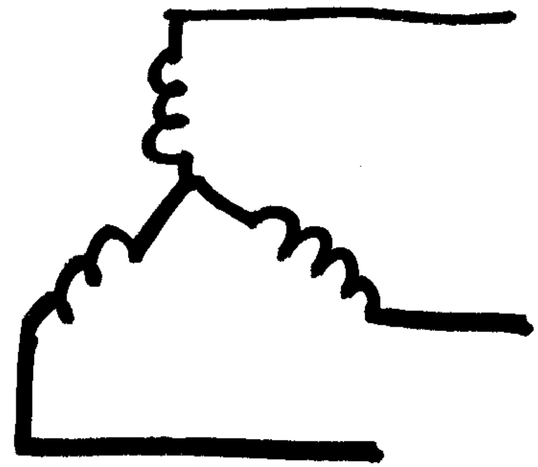
$$\begin{bmatrix} f_a \\ f_b \\ f_c \end{bmatrix} = \begin{bmatrix} K_d \overset{\cos}{\cancel{\sin}}\,\theta & K_q \sin\theta & K_0 \\ K_d \cos\left(\theta - \frac{2\pi}{3}\right) & K_q \sin\left(\theta - \frac{2\pi}{3}\right) & K_0 \\ K_d \cos\left(\theta + \frac{2\pi}{3}\right) & K_q \sin\left(\theta + \frac{2\pi}{3}\right) & K_0 \end{bmatrix} \begin{bmatrix} f_d \\ f_q \\ f_0 \end{bmatrix}$$

$\uparrow C_p$

$$V_q = \sqrt{\frac{2}{3}}\left[V_a \sin\theta + V_b \sin\left(\theta - \frac{2\pi}{3}\right) + V_c \sin\left(\theta + \frac{2\pi}{3}\right)\right]$$

$$V_o = \frac{1}{\sqrt{3}}\left[V_a + V_b + V_c\right]$$

if $\theta = \omega_0 t$



$$V_a = V_m \sin(\omega_0 t)$$

$$V_b = V_m \sin\left(\omega_0 t - \frac{2\pi}{3}\right)$$

$$V_c = V_m \sin\left(\omega_0 t - \frac{4\pi}{3}\right)$$

$$V_d \quad , \quad V_q \quad , \quad V_0$$

$$V_d = \sqrt{\frac{2}{3}}\left[V_a(t)\cos\theta + V_b(t)\cdot\cos\left(\theta - \frac{2\pi}{3}\right) + V_c\cos\left(\theta + \frac{2\pi}{3}\right)\right]$$

$$V_0 = 0$$

$$V_q = V_m\sqrt{\frac{2}{3}}\left[\sin^2\theta + \sin^2\left(\theta\ \frac{2\pi}{3}\right) + \sin^2\left(\theta + \frac{2\pi}{3}\right)\right]$$

$$= V_m \cdot \sqrt{\frac{3}{2}} = V_{LL\,rms}$$

$$V_d = 0$$

$$\frac{d\psi_f}{dt} + R_f i_f = v_f$$

$$\frac{d\psi_f}{dt} \to 0$$

$$\boxed{v_f / R_f = i_f}.$$

Open circuit

$$i_d = 0 = i_q.$$

$$\boxed{\begin{aligned} -\omega\psi_q - R_a i_d &= V_d \\ \omega\psi_d - R_a i_q &= V_q \end{aligned}}$$

$$-R_a i_0 = 0 \Rightarrow i_0 = 0$$

$$i_g = i_h = i_k = 0$$

$$\frac{d\psi_h}{dt} + R i_h = 0$$

$$\begin{bmatrix} \psi_d \\ \psi_q \\ \psi_o \\ \hline \psi_f \\ \psi_h \\ \psi_g \\ \psi_k \end{bmatrix} = \left[\begin{array}{ccc|cccc} L_d & 0 & 0 & M_{df} & M_{dh} & 0 & 0 \\ 0 & L_q & 0 & 0 & 0 & M_{qg} & M_{qk} \\ 0 & 0 & L_o & 0 & 0 & 0 & 0 \\ \hline M_{df} & 0 & 0 & L_{ff} & L_{fh} & 0 & 0 \\ M_{dh} & 0 & 0 & L_{hf} & L_{hh} & 0 & 0 \\ 0 & M_{qg} & 0 & 0 & 0 & L_{gg} & L_{gk} \\ 0 & M_{gk} & 0 & 0 & 0 & L_{gk} & L_{kk} \end{array}\right] \begin{bmatrix} i_d \\ i_q \\ i_o \\ \hline i_f \\ i_h \\ i_g \\ i_k \end{bmatrix}$$

$$L_{aa2}$$

$$M_{df} = \underline{M_{af}} / K_d \qquad L_d = L_{aa0} - L_{ab0} + \frac{3}{2} L_{a}$$

$$M_{dh} = M_{ah} / K_d \qquad L_q = L_{aa0} - L_{ab0} - \frac{3}{2} L_{aa2}$$

$$M_{qg} = M_{ag} / K_q \qquad L_0 = L_{aa0} + 2 L_{aa2}$$

$$M_{qk} = M_{ak} / K_q$$

$$v_a = \sqrt{\frac{2}{3}} \cdot 0 \cdot \cos\theta + \sqrt{\frac{2}{3}} \cdot \frac{M_{df}}{R_f} \cdot v_f \sin\theta + 0$$

$$v_a = \sqrt{\frac{2}{3}} \frac{M_{df}}{R_f} \cdot v_f \sin\theta$$

$$\theta = \omega_0 t$$

$$\left.\begin{aligned} -\omega \psi_q &= V_d \\ \omega \psi_d &= V_q \\ V_f / R_f &= i_f \\ \psi_d &= M_{df}\, i_f \\ \psi_q &= 0 \end{aligned}\right\}$$

$V_d = 0$

$V_q = \dfrac{M_{df}}{R_f} V_f \cdot \omega_0$

$V_0 = 0$

$\theta = \omega_0 t$

$$V_a = \omega_0 \sqrt{\frac{2}{3}} \frac{M_{af}}{\sqrt{2/3}} \cdot \left(\frac{V_f}{R_f}\right) \cdot \sin \omega_0 t$$

$$= \omega_0 M_{af}\, i_f \sin \omega_0 t$$

$$\boxed{\sqrt{\tfrac{3}{2}}\, \omega_0 M_{af} \cdot i_f}$$

O.C.

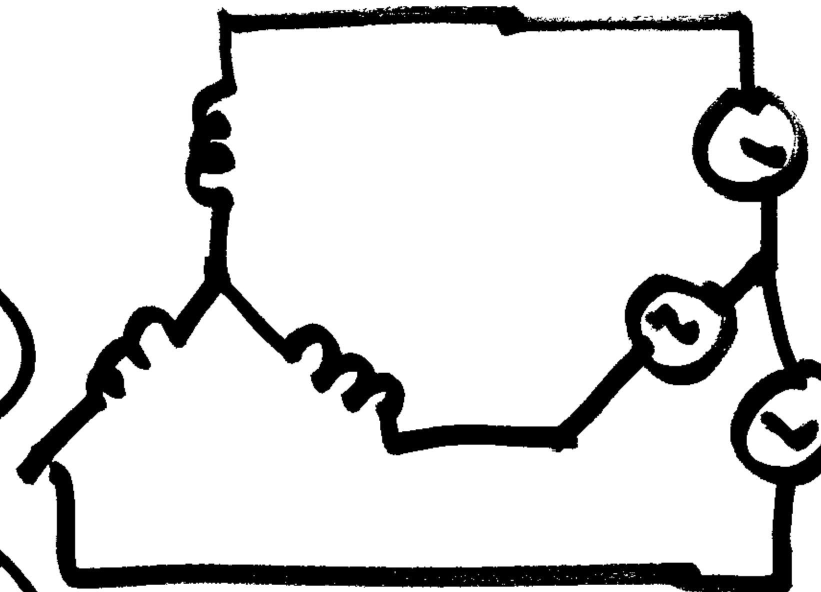
$\underline{\theta = \omega_0 t}$

$$V_b = \sqrt{\frac{2}{3}}\, V_f \frac{M_{af}}{R_f} \sin\left(\theta - \frac{2\pi}{3}\right) \cdot \omega_0$$

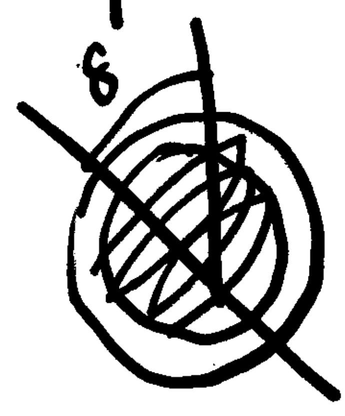
$$V_a = V_m \sin \omega_0 t$$

$$V_b = V_m \sin\left(\omega_0 t - \frac{2\pi}{3}\right)$$

$$V_c = V_m \sin\left(\omega_0 t + \frac{2\pi}{3}\right)$$



$$\theta = \omega_0 t + \delta .$$



$$V_d = ?$$

$$V_q = ?$$

$$V_0 = ?$$

$$V_a = \sqrt{\frac{2}{3}}\left[V_d \cos\theta + V_q \sin\theta + \underset{\substack{\downarrow \\ 0}}{V_0}\right]$$

$$V_m \sin\omega_0 t = \sqrt{\frac{2}{3}}\left[V_d \cos(\omega_0 t + \delta) + V_q \sin(\omega_0 t + \delta)\right]$$

$$\cos(A+B) = \cos A \cos B - \sin A \sin B$$

$$\sin(A+B) = \cos A \sin B + \cos B \sin A$$

$$\begin{cases} V_d = \underbrace{\sqrt{\frac{3}{2}}\, V_m}\,(-\sin\delta) \\ V_q = \underbrace{\sqrt{\frac{3}{2}}\, V_m}\, \cos\delta. \\ V_0 = 0 \end{cases}$$

Set $\frac{d}{dt}$'s $= 0$ for all flux

$$i_g = i_h = i_k = 0$$

$$\underline{v_0 = 0} \qquad \frac{d\psi_0}{dt} = 0 \qquad i_0 = 0$$

$$\boxed{v_f / R_f = i_f .}$$

$$-\omega_0 \psi_q = V_d$$
$$\omega_0 \psi_d = V_q$$

$$\theta = \omega_0 t + \delta$$
$$\frac{d\theta}{dt} = \omega_0 .$$

ψ_d , ψ_q

$$\psi_d = L_d i_d + M_{af} i_f$$
$$\psi_q = L_q i_q$$

$$T_e' = \psi_d i_q - \psi_q i_d$$

$$\frac{P}{2} T_e' = T_e$$

$$T_e' = \frac{V_q}{\omega_0} \cdot \frac{\psi_q}{L_q} - \left(\frac{-V_d}{\omega_0}\right) \cdot i_d$$

$$= \frac{V_q}{\omega_0} \cdot \frac{\psi_q}{L_q} + \frac{V_d}{\omega_0} \cdot \left[\frac{\psi_d - M_{df}\, i_f}{L_d}\right]$$

$$T_e' = \cancel{V_q \, [V_d - M}$$

$$= \frac{V_q \cdot \left(-\frac{V_d}{\omega_0}\right)}{\omega_0 L_q} + \frac{V_d}{\omega_0}\left[\frac{\frac{V_q}{\omega_0} - M_{df} i_f}{L_d}\right]$$

$$T_e' = \frac{-V_q V_d}{\omega_0^2 L_q} + \frac{V_d V_q}{\omega_0^2 L_d}$$

$$- \frac{V_d}{\omega_0 L_d} \cdot M_{df}\, i_f .$$

$$V_d = -\sqrt{\frac{3}{2}}\, V_m \sin\delta \qquad V_q = \sqrt{\frac{3}{2}}\, V_m \cos\delta$$

$$T_e' = \frac{3}{2} \cdot \frac{1}{X_q} \cdot \frac{V_m^2 \sin\delta \cos\delta}{\omega_0}$$

$$- \frac{3}{2} \cdot \frac{1}{X_d} \cdot \frac{V_m^2 \sin\delta \cos\delta}{\omega_0}$$

$$+ \sqrt{\frac{3}{2}} \frac{V_m \sin\delta}{X_d} \frac{X_{df}}{\omega_0} \cdot i_f$$

$$\underbrace{\frac{P}{2} T_e' \cdot \omega_0} = \frac{P}{2} V_{LLrms}^2 \cdot \frac{\sin 2\delta}{2} \times \left[\frac{1}{X_q} - \frac{1}{X_d} \right]$$

$$\boxed{T_m = T_e}$$

$$+ \frac{P}{2} \frac{V_{LLrms} \cdot (X_{df}\, i_f)}{X_d} \cdot \sin\delta$$

$$\frac{P}{2} T_e' = T_e$$

open circuit voltage
LL rms.