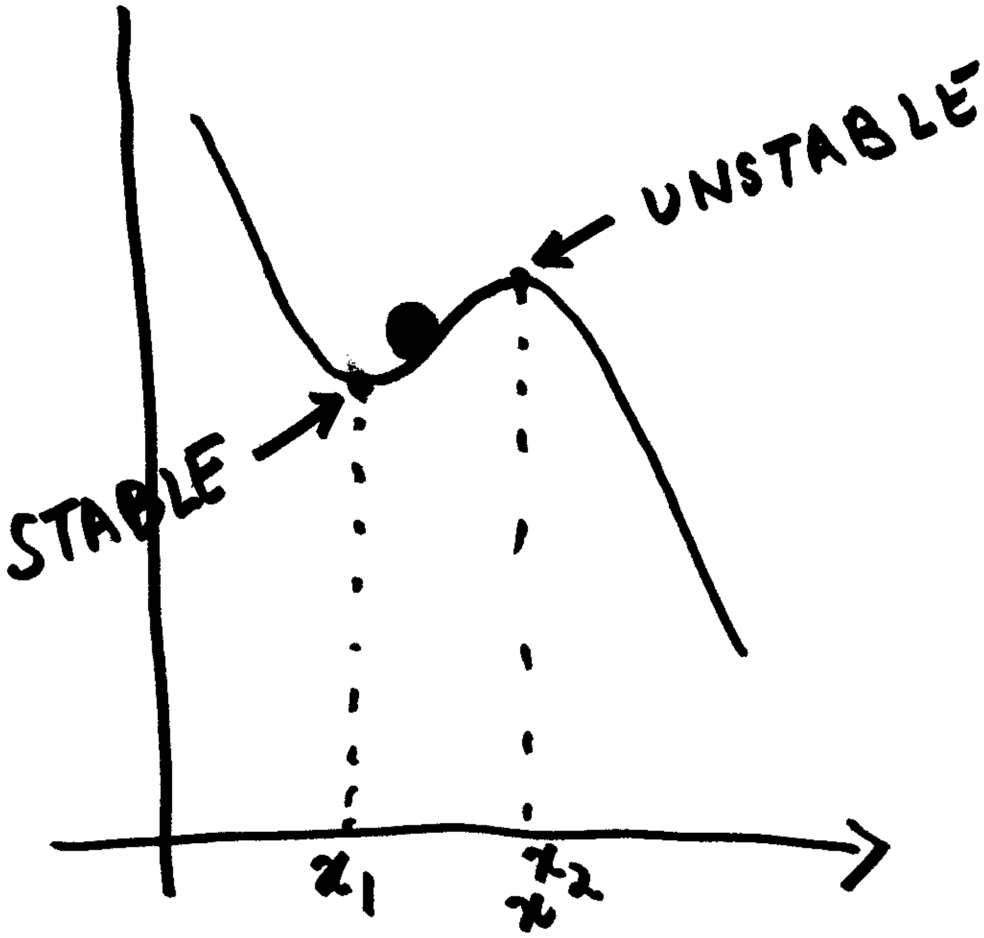
Prof. A. M. Kulkarni
Lec - 3
9/10/09



I

If $x = x_1$
AND
$\dot{x} = \frac{dx}{dt} = 0$

II

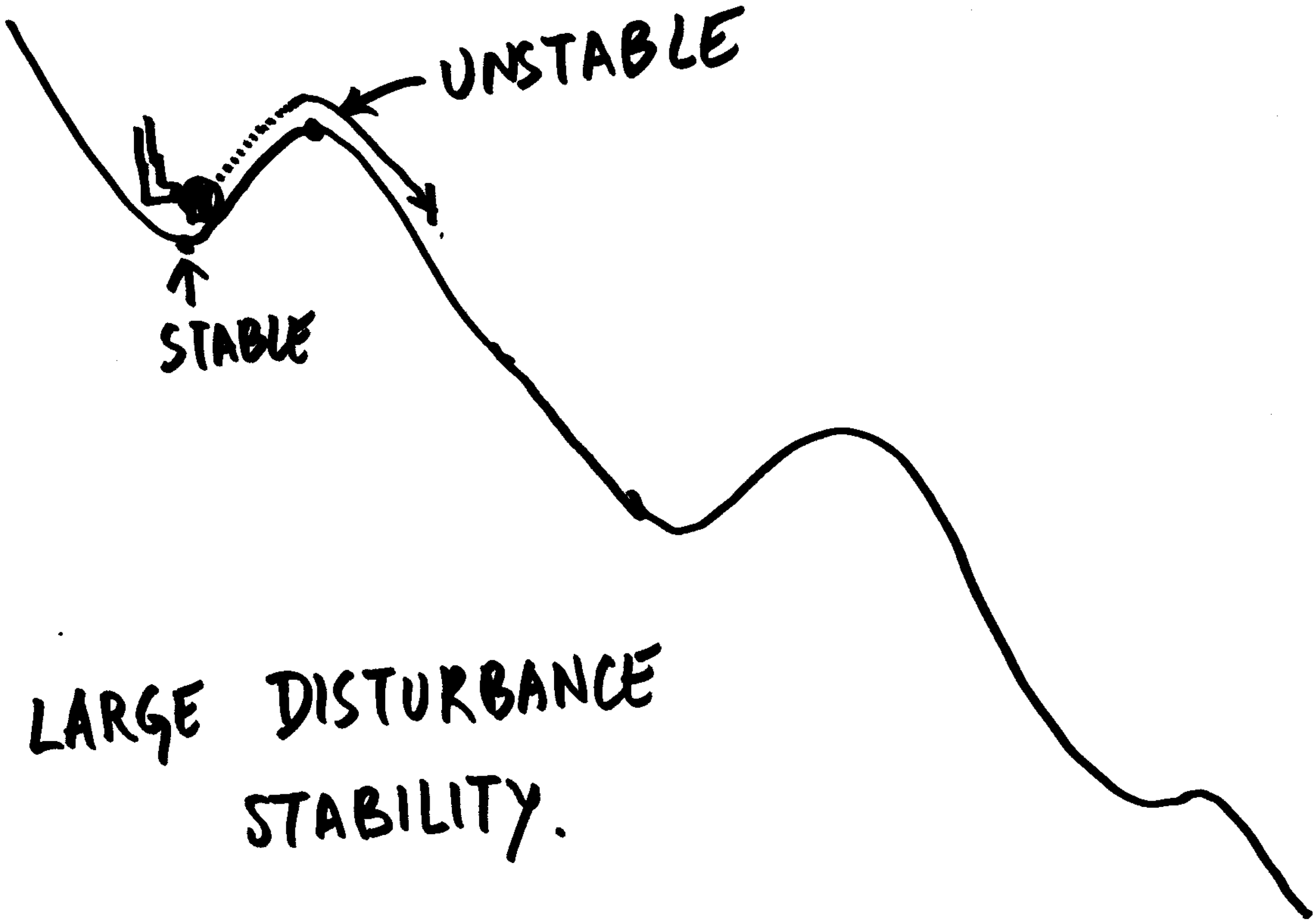
$x = x_2$
AND
$\dot{x} = \frac{dx}{dt} = 0$

2 STATES.

x (POSITION)

$\dot{x}$ (VELOCITY) $= v$

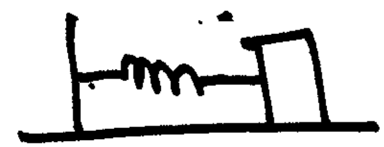
$$\left.\begin{aligned} &\frac{dx}{dt} = 0 \\ &\frac{d(\dot{x})}{dt} = \frac{d^2x}{dt^2} = \frac{dv}{dt} = 0 \end{aligned}\right\}$$

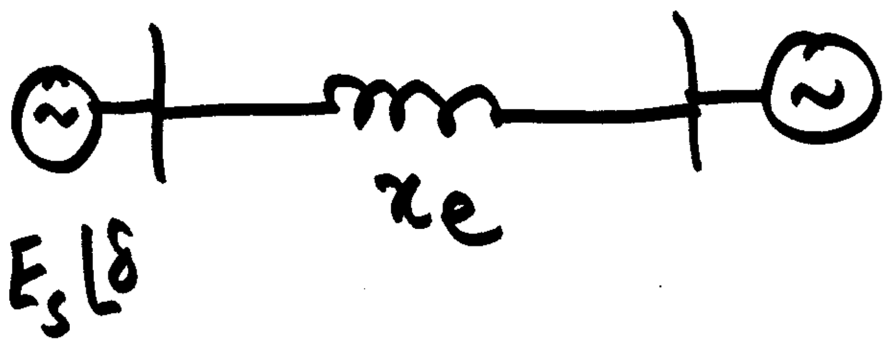


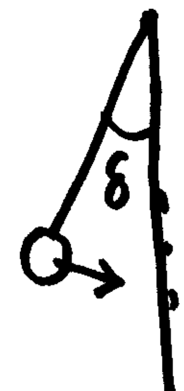
LARGE DISTURBANCE STABILITY.

$$P_e = \frac{E_s\, E \sin\delta}{x_e}$$

$\rightarrow Kx$







TOY MODEL

$$\begin{cases} \dfrac{d\delta}{dt} = \omega - \omega_0 \\[2ex] \dfrac{2H}{\omega_B} \dfrac{d(\omega-\omega_0)}{dt} = P_m - \dfrac{E_s\, E \sin\delta}{x_e} \end{cases}$$

$$\frac{2H}{\omega_B} \frac{d(\omega - \omega_0)}{dt} \cong P_m - P_e$$

mech

electrical power

$$= \cong \underset{(pu)}{T_m} - \underset{(pu)}{T_e}$$

$2\pi \times f_B$

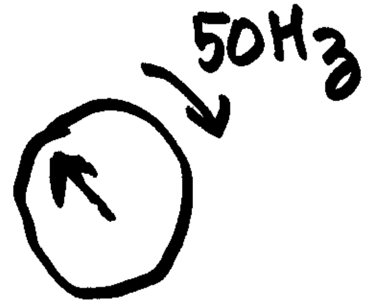
$$\text{Inertia Constant} = \frac{\frac{1}{2} J \omega_m^2}{VA_{base}} \quad : \frac{MJ}{MVA}$$

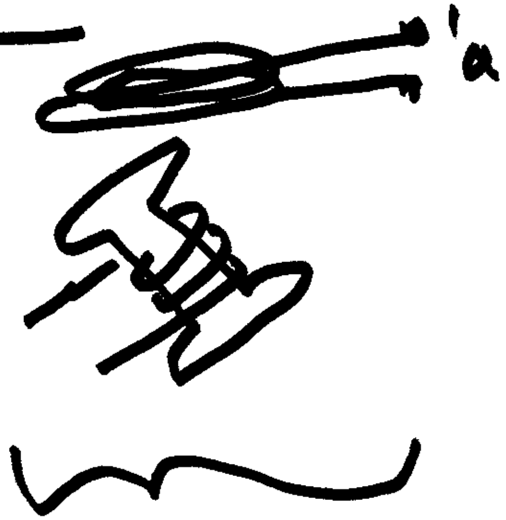
$$\omega \rightarrow \quad \omega_m \cdot \frac{P}{2} = \omega$$

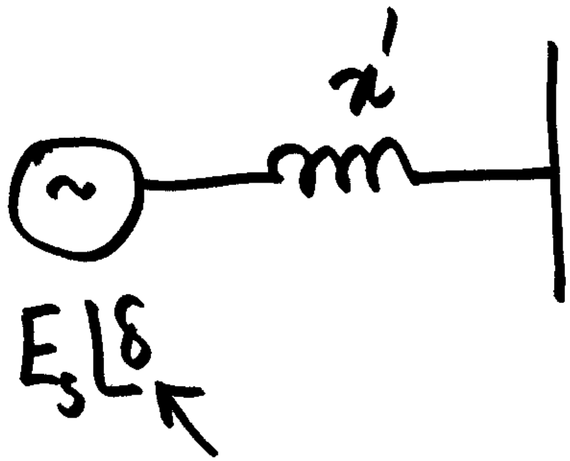
$$\omega_0 \rightarrow 2\pi f_0 \leftarrow$$

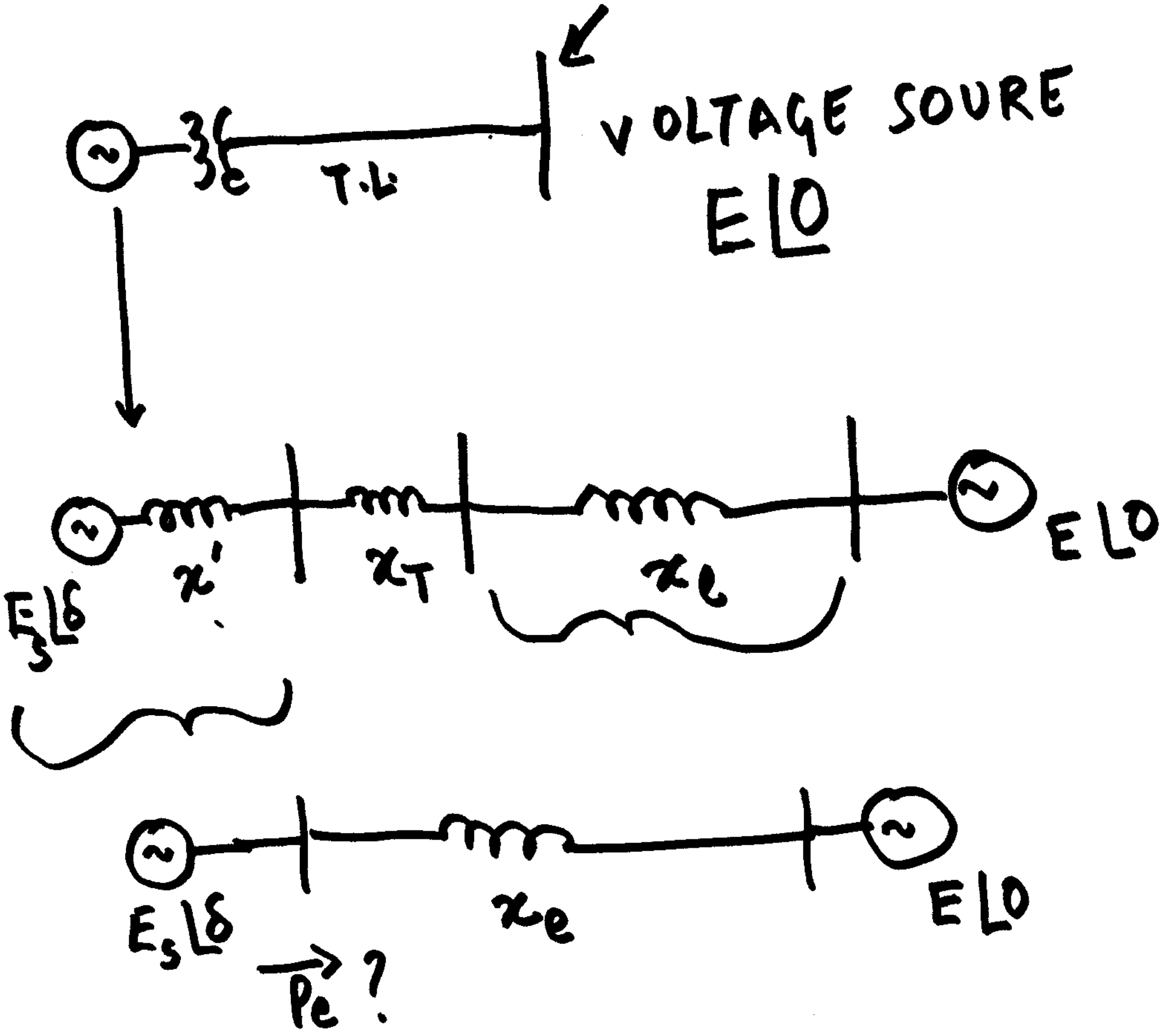
$$\frac{2H}{\omega_B}\frac{d(\omega-\omega_0)}{dt} = P_m - P_e$$

$$\frac{d\delta}{dt} = (\omega-\omega_0)$$







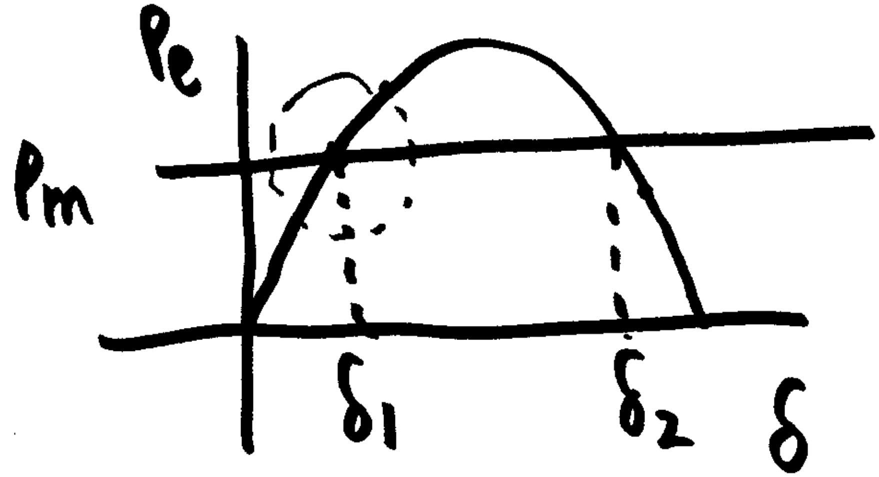

VOLTAGE SOURE
E∠0
T.L
x'
x_T
x_e
E∠0
E_s∠δ
x_e
E∠0
E_s∠δ
Pe ?

$$\frac{d\delta}{dt} = \omega - \omega_0$$

$$\frac{2H}{\omega_B}\frac{d(\omega-\omega_0)}{dt} = P_m - \frac{E_s E \sin\delta}{x_e}$$

$$\frac{d\delta}{dt} = 0 \; , \; \frac{d(\omega-\omega_0)}{dt} = 0$$

EQUILIBRIA $\longrightarrow$ $\omega_e = \omega_0$ and $P_m = \frac{E_s E \sin\delta_e}{x_e}$.



$$\omega_e = \omega_0$$

and

$$\cancel{\delta = \sin^{-1}\left(E_s E\right.}$$

$$\delta_e = \sin^{-1}\left(\frac{P_m \cdot x_e}{E_s E}\right)$$

δ_1 OR δ_2

$$\left.\begin{aligned} d\delta/dt &= 0 \\ \frac{d(\omega - \omega_0)}{dt} &= 0 \end{aligned}\right\}$$

$$\left.\begin{aligned} \omega &= \Delta\omega + \omega_e \\ \delta &= \Delta\delta + \delta_e \end{aligned}\right\}$$

① $$\frac{d\delta}{dt} = \omega - \omega_0 \Rightarrow \frac{d(\delta_e + \Delta\delta)}{dt} = \cancel{\omega_e} + \Delta\omega - \cancel{\omega_0}$$

$$\frac{d\Delta\delta}{dt} = \Delta\omega$$

② $$\frac{2H}{\omega_B} \cdot \frac{d(\omega_e + \Delta\omega - \omega_0)}{dt} = \frac{2H}{\omega_B} \cdot \frac{d\Delta\omega}{dt}$$

$$= P_m - \frac{E_s E}{x_e} \underbrace{\sin(\delta_e + \Delta\delta)} = P_m -$$

$$= P_m - \frac{E_s E}{x_e}\left[\sin\delta_e \cos\Delta\delta + \cos\delta_e \sin\Delta\delta\right]$$

$$= \cancel{P_m - \frac{E_s E}{x_e}\sin\delta_e} - \frac{E_s E}{x_e}\cos\delta_e \cdot \Delta\delta$$

$$\cos\Delta\delta \approx 1 \qquad \sin\Delta\delta \approx \Delta\delta .$$

$$\frac{2H}{\omega_B}\cdot\frac{d\,\Delta\omega}{dt} = -\underbrace{\left\{\frac{E_s E}{x_e}\cos\delta_e\right\}}_{K}\cdot\Delta\delta$$

$$\frac{d\Delta\delta}{dt} = \Delta\omega$$

$$\frac{2H}{\omega_B}\frac{d\Delta\omega}{dt} = -K\,\Delta\delta$$

$\cos\Delta\delta \approx 1$ $\sin\Delta\delta \approx \Delta\delta$

$$K = \frac{E_s E \cos\delta_e}{x_e}$$

$$\Delta\delta = A\sin(\omega_n t + \phi)$$

$$\Delta\omega = \omega_n A\cos(\omega_n t + \phi)$$

$$-\frac{2H}{\omega_B}\omega_n^2 A\sin(\omega_n t + \phi) = -K A\sin(\omega_n t + \phi)$$

$$\omega_n^2 = \left(\omega_B K/2H\right)$$

$$\Delta\delta = A \sin(\omega_n t + \phi)$$

$$\omega_n = \sqrt{\frac{\omega_B K}{2H}}$$

$\underline{\underline{t=0}}$

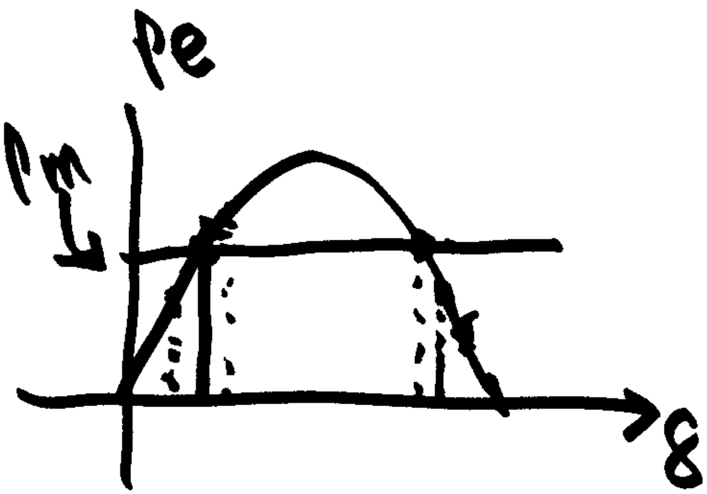
$\Delta\delta = \Delta\delta(0)$ $\Delta\omega = \Delta\omega(0)$

$$\left.\begin{aligned} \Delta\delta(0) &= A\sin(\phi) \\ \Delta\omega(0) &= \omega_n A \cos\phi \end{aligned}\right\} \text{YES.}$$

A , ϕ

$K < 0$ for $\delta_e = \delta_2$

UNSTABLE



$$\boxed{\begin{matrix}\Delta\delta(t)\\ \Delta\omega(t)\end{matrix}} \rightarrow A, \phi$$

$$\Delta\delta = A \sin(\omega_n t + \phi)$$

$$\Delta\omega = A\omega_n \cos(\omega_n t + \phi)$$

$$\omega_n = \sqrt{\frac{\omega_B K}{2H}} \qquad K = \frac{E E_s \cos\delta_e}{x_e}$$

K is negative $\quad \omega_n \rightarrow$ complex

AT THE "OTHER EQUILIBRIUM"

$$\Delta\delta = A \sin(j\Omega t + \phi) \qquad \sin x = \frac{e^{jx} - e^{-jx}}{2j}$$

$\omega_n = j\Omega$

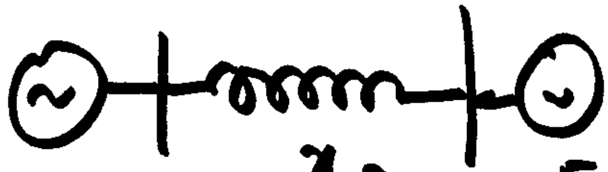
$$= \frac{A \, e^{j(j\Omega t)} e^{j\phi} - e^{-j(j\Omega t)} \cdot e^{-j\phi}}{2j}$$

$$\Delta\delta = \underbrace{k_1 e^{-\Omega t} - k_2 e^{\Omega t}}$$

$\omega_n \to$ complex

$\Omega \to$ real no.

$e^{5t} \to$ GROWS WITH t. $e^{-5t} \to$ DECAYS.



$E_s \angle \delta$ $\quad x_e \quad$ $E \angle 0$

$\delta_e \to 50^\circ$

$\cos 50$

$\approx \cos 45$

$\approx 1/\sqrt{2}$.

$x_e = 0.5$

$$\omega_n = \sqrt{\frac{\omega_B K}{2H}}$$

$$= \sqrt{\frac{\omega_B}{2H} \cdot \frac{E_s E_b \cos\delta_e}{x_e}}$$

$\omega_B \to 2\pi f_B = 314 \qquad H \to 4$

$$\sqrt{\frac{314}{2\times 4}\times\frac{1\times 1\times \cos 50}{0.5}}$$

$$= \sqrt{\frac{314}{8}\cdot\frac{0.7}{0.5}} = \sqrt{\frac{314\times 0.7}{4}}$$

$$= \frac{\sqrt{314\times 0.7}}{2} = \frac{\sqrt{220}}{2} = 7.5 \text{ rad/s}.$$

$$f_n = \underline{1.2} \text{ Hz}.$$