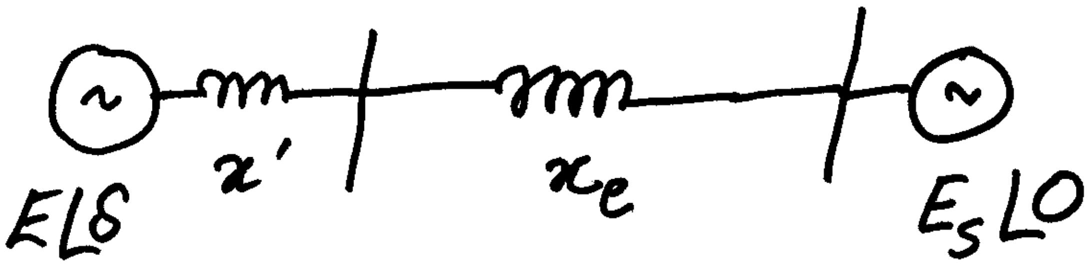
Prof. Kulkarni
Lec. 4 . 16/10/09



SIMPLIFIED MODEL

$$\frac{d\delta}{dt} = (\omega - \omega_0)$$

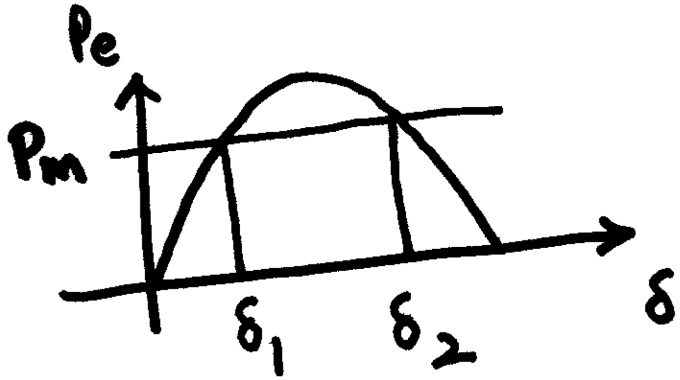
$$\frac{2H}{\omega_B}\frac{d(\omega-\omega_0)}{dt} = P_m - \frac{E_s E}{x}\cdot \sin\delta$$

$$x = x' + x_e$$

EQUILIBRIA

$$\Rightarrow \frac{d\delta}{dt} = 0 \quad \& \quad \frac{d(\omega - \omega_0)}{dt} = 0 \quad \Rightarrow \quad \omega_e = \omega_0$$

$$\& \ \delta_e = \sin^{-1}\left(\frac{P_m x}{E_s E} \right)$$



$\delta_e \rightarrow \delta_1$ or δ_2

STABILITY → SMALL DISTURBANCES (around equilibria)

→ LARGE DISTURBANCES

SMALL DISTURBANCES

$\omega = \omega_e + \Delta\omega \qquad \delta = \delta_e + \Delta\delta$

if $\Delta\omega$ & $\Delta\delta$ are very small, P_m is constant

$$\boxed{\begin{aligned} \frac{d\Delta\delta}{dt} &= \Delta\omega \\ \frac{2H}{\omega_B}\frac{d\Delta\omega}{dt} &= -K\,\Delta\delta \end{aligned}}$$

$\cos\Delta\delta \approx 1$

$\sin\Delta\delta \approx \Delta\delta$

$$K \rightarrow \frac{E_s E \cos\delta_e}{x}$$

equlibrium value

STABILITY ?

RESPONSE FOR SMALL DISTURBANCES

$$\left.\begin{aligned} \Delta\delta &= A \sin(\omega_n t + \phi) \\ \Delta\omega &= A\,\omega_n \cos(\omega_n t + \phi) \end{aligned}\right\} \quad \omega_n = \sqrt{\frac{\omega_B K}{2H}}$$

$A, \phi \longrightarrow$ obtained from initial value of $\Delta\delta$ & $\Delta\omega$

NOTE $K > 0$ if $\delta_e < 90^\circ \longleftarrow$ EQUILIBRIUM $\delta_e = \delta_1,\ \omega_e = \omega_0$

$\downarrow$

OSCILLATORY BEHAVIOR

damping?

FOR THE EQUILIBRIUM $\delta_e = \delta_2$, $\omega_e = \omega_0$

$\delta_2 > 90^\circ \Rightarrow K < 0$

RESPONSE :

$$\Delta\delta = K_1 e^{\Omega t} + K_2 e^{-\Omega t}$$

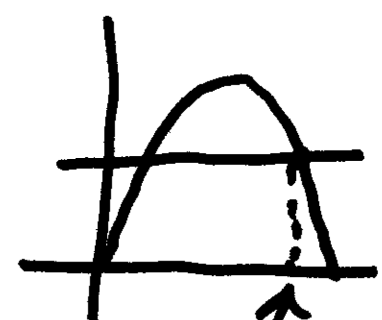
$$\Delta\omega = K_1 \Omega e^{\Omega t} - K_2 \Omega e^{-\Omega t}$$

$\Omega \rightarrow$ real , K_1 , $K_2 \rightarrow$ initial value of deviations

COMBINATION OF GROWING & DECAYING TERMS

in general

UNSTABLE



ONE CANNOT OPERATE AT $\delta_e = \delta_2$, $\omega_e = \omega_0$

(UNSTABLE EQUILIBRIUM)

$\delta_e = \delta_1 \quad \omega_e = \omega_0 \quad \longrightarrow$ OSCILLATORY

(small disturbances)

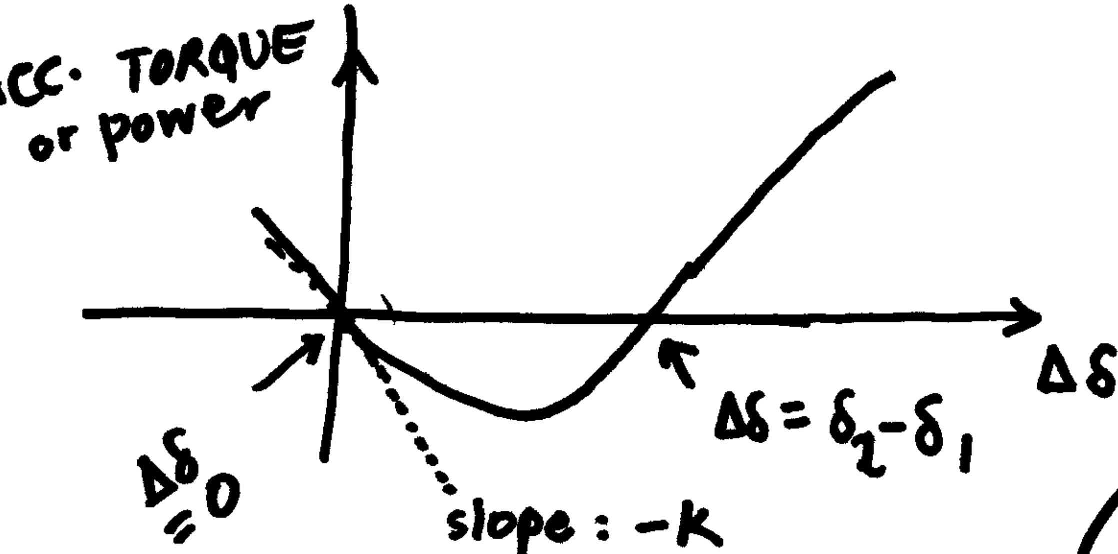
<u>Large disturbances</u>?

$$\frac{d\delta}{dt} = \omega - \omega_0$$

$$\frac{2H}{\omega_B} \frac{d(\omega-\omega_0)}{dt} = \left.\underbrace{P_m - \frac{E_s E}{x}\sin\delta}\right\}$$

ACCELERATING TORQUE

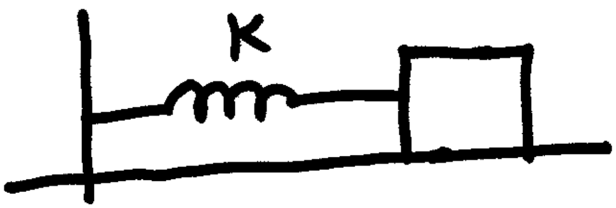
≈ POWER in pu



EQUILIBRIUM $\delta_e = \delta_1$

$$P_m - \frac{E_S E \sin\delta}{x}$$

"RESTORATIVE" near $\delta_e = \delta_1$



if $\Delta\delta(0)$ & $\Delta\omega(0)$
>0 >0

ARE NOT SMALL

AND IF

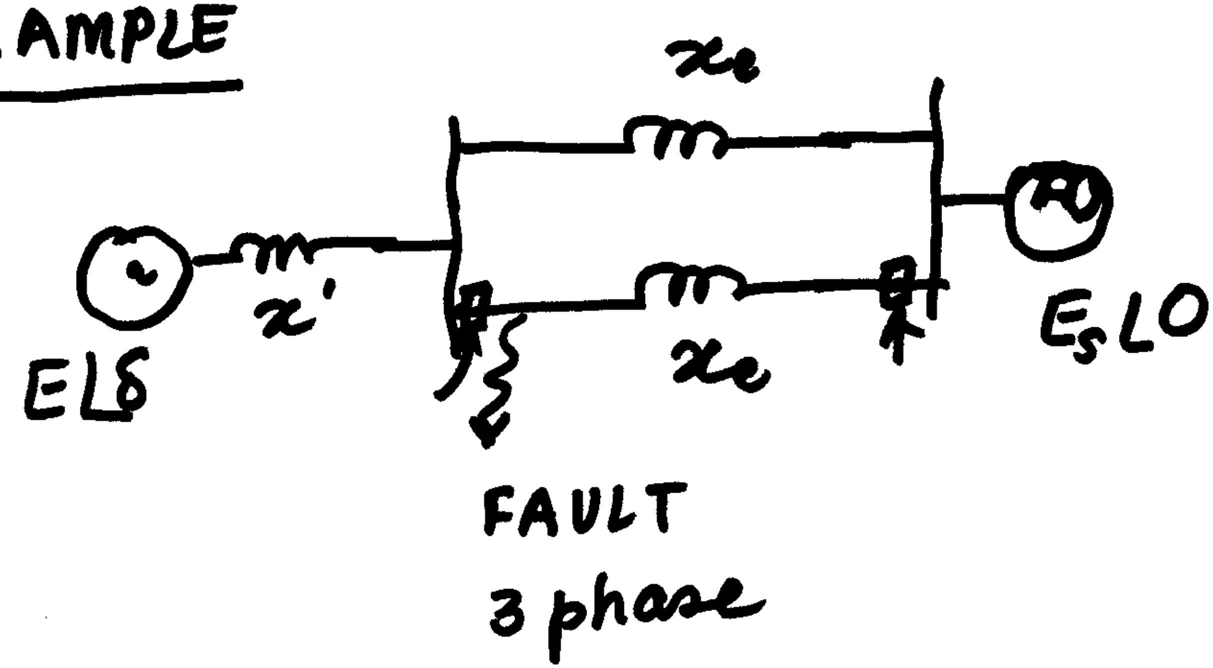
$\Delta\omega$ DOES NOT BECOME ZERO

BEFORE $\Delta\delta = \delta_2 - \delta_1$

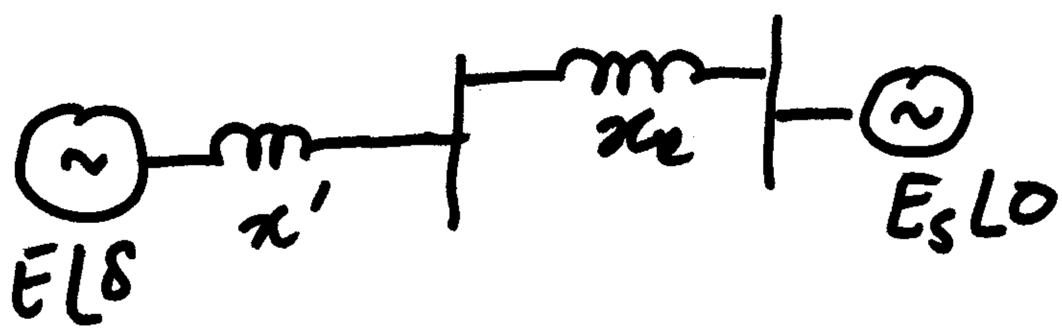
NON-RESTORATIVE

↳ UNSTABLE

EXAMPLE



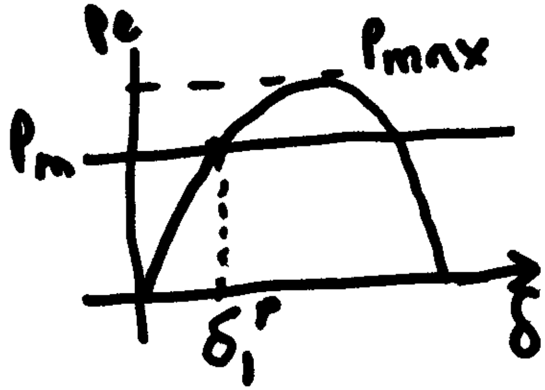
POST FAULT



DUE TO FAULT $\rightarrow$ SIGNIFICANT DEVIATION
$\underline{\Delta\delta}$ & $\underline{\Delta\omega}$

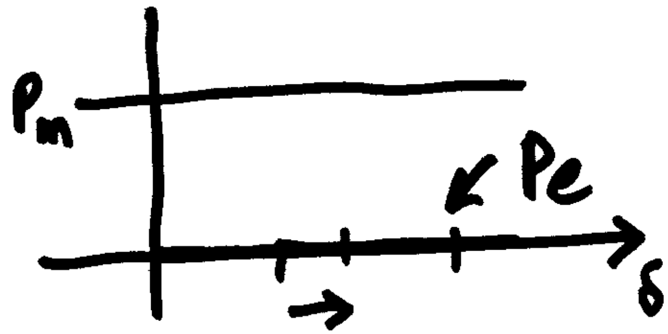
PRE-FAULT

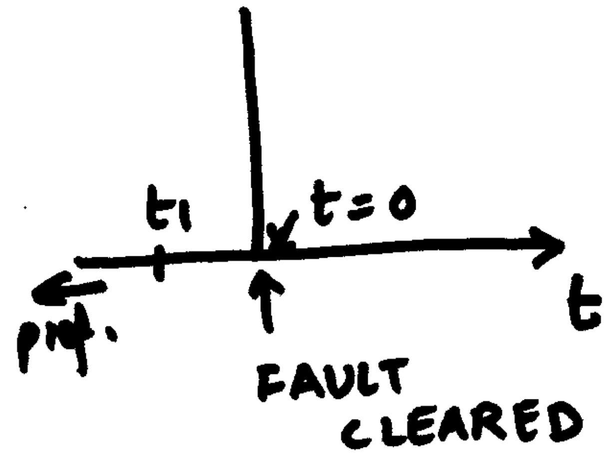
$t < t_1$



FAULTED

$t = t_1$ to 0



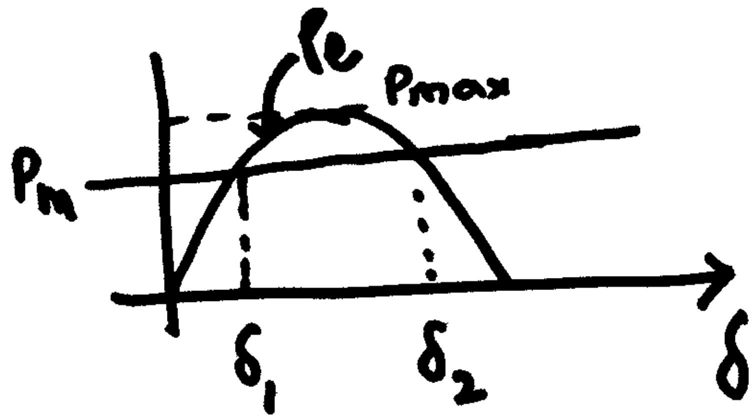


POST FAULT

$t = 0$

$\Delta\delta(0) > 0$

$\Delta\omega(0) > 0$



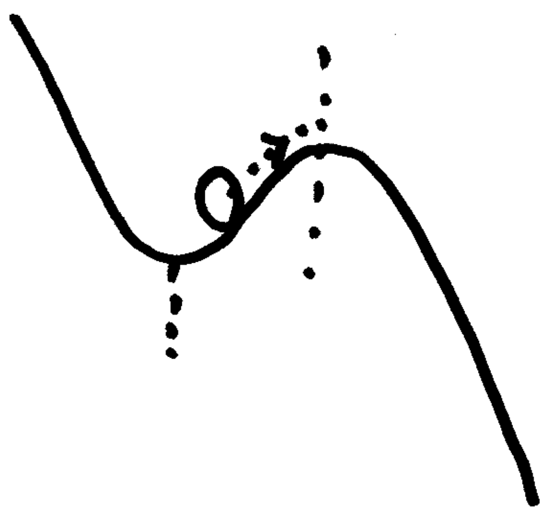
$$P_e = \frac{E_s E}{x'} \sin\delta$$

$$= \frac{E_s E}{x} \sin\delta$$

COMPUTE RESPONSE ?

→ NUMERICAL INTEGRATION - SLIDE

STABILITY EVALUATION ? - NOT EASY



IF ENERGY IS CONSERVED

KE + PE > PE

depends on $\Delta\omega(0)$ & $\Delta\delta(0)$

PE → AT δ_2

$\Rightarrow$ UNSTABLE

SHOW THAT $W =$

$$\frac{1}{2} \cdot \frac{2H}{\omega_B} \cdot (\omega - \omega_0)^2 - P_m \cdot \delta - \frac{E_s E}{x} \cos\delta$$

$x \leftarrow$ POST FAULT

$x = x' + x_e$

$\frac{dW}{dt}$ is conserved

i.e. it is constant

$\therefore$ IF at time $t = 0$ (FAULT CLEARING)

$\delta(0) = \delta_1 + \Delta\delta(0)$

$\omega(0) = \omega_0 + \Delta\omega(0)$

if $W(\delta(0), \omega(0)) > W(\delta_2, \omega_0)$

$\Rightarrow$ UNSTABLE

$$\frac{dW}{dt} = \frac{2H}{\omega_B}(\omega-\omega_0)\cdot\frac{d}{dt}(\omega-\omega_0)$$

$$-\,P_m\cdot\frac{d\delta}{dt} + \frac{E_s E \sin\delta}{x}\frac{d\delta}{dt}$$

$$\frac{d\delta}{dt} = \omega-\omega_0$$

$$\frac{dW}{dt} = \left[\frac{2H}{\omega_B}\frac{d}{dt}(\omega-\omega_0) - \left(P_m - \frac{E_s E}{x}\sin\delta\right)\right]\frac{d\delta}{dt}$$

$$= 0$$

RECAP

$$\dot{x} = \frac{dx}{dt} = \underline{\underline{A}}\,\underline{x} \quad \Big\}\ \text{higher order linear system}$$

LINEAR SYSTEMS : $\dot{x} = \dfrac{dx}{dt} = ax + bu$

NONLINEAR SYSTEM

: $\dot{x} = \dfrac{dx}{dt} = f(x, u)$

$$\dot{\underline{x}} = f(x, u)$$

$$\dot{x}_1 = f_1(x_1, x_2, u_1, u_2) \quad \dot{x}_2 = f_2(x_1, x_2, u_1, u_2)$$

$$\begin{cases} e^{\lambda t} \\ e^{-\lambda t} \end{cases} \qquad t e^{\lambda t} \qquad \sin(\Omega t + \phi)$$

LINEAR SYSTEMS

- INHERENTLY LINEAR
- OBTAINED AS APROXIMATIONS.

  'SMALL' DEVIATIONS FROM EQUILIBRIA OF NONLINEAR SYSTEMS.

FORMAL PROCEDURE

$$\dot{x} = f(x, u) \qquad \begin{aligned} x &= x_e + \Delta x \\ u &= u_e + \Delta u \end{aligned}$$

$$(\dot{x}_e + \Delta\dot{x}) = f(x_e + \Delta x,\ u_e + \Delta u)$$

$$\Delta\dot{x} = f(x_e, u_e) + \left.\frac{\partial f}{\partial x}\right|_{x=x_e,\,u=u_e} \cdot \Delta x + \left.\frac{\partial f}{\partial u}\right|_{x=x_e,\,u=u_e} \Delta u + \underline{\text{higher order}}$$

$f(x_e, u_e) = 0 \leftarrow$ Equlibrium

$$\Delta\dot{x} = \left.\partial f/\partial x\right|_{\substack{x=x_e \\ u=u_e}} \Delta x + \left.\partial f/\partial u\right|_{\substack{x=x_e \\ u=u_e}} \Delta u$$

$$\Delta \dot{x} = \text{"}a\text{"}\Delta x \quad + \text{"}b\text{"}\Delta u$$

$a\ ,\ b$ constants $\rightarrow$ depend on equilibrium point.

$\rightarrow$ $$\underbrace{f(x_e, u_e) = 0}$$

N-R

G-S

$$\boxed{\dot{x} = -x^2} \leftarrow x_e = 0$$

0

is this

$\Delta \dot{x} = 0$ ✗

SMALL DIST STABLE ?

$$x_e \Rightarrow \dot{x} = 0$$

$$-x^2 = 0$$

$$\underline{x_e = 0}$$

$$(\dot{x_e + \Delta x}) = \Delta \dot{x}$$

$$= -(x_e + \Delta x)^2$$

$$= -(\cancel{x_e^2} + 2x_e \Delta x + \cancel{\Delta x^2})$$

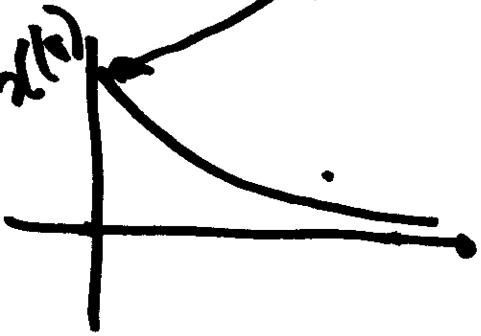
$\dot{x} = a x \qquad x = 0$

$$x(t) = e^{at} x(0) \rightarrow \frac{dx}{dt} = a x$$

$\underline{x(0)}$

SOLUTION

$x(t)\big|_{t=0} = x(0)$



$\underline{a > 0}$ $\qquad$ $\underline{a < 0}$ $\qquad$ STABILITY PROPERTY

$$\dot{x}_e = 0 = a x_e \qquad \boxed{x_e = 0}$$

$$x(0) \neq x_e \rightarrow x(t) = e^{at} x(0)$$

$$\left\{\begin{aligned} \dot{x}_1 &= a_{11} x_1 \\ \dot{x}_2 &= a_{22} x_2 \end{aligned}\right\} \quad \begin{aligned} x_1 &= e^{a_{11} t} x_1(0) \\ x_2 &= e^{a_{22} t} x_2(0) \end{aligned}$$

$$\left\{\begin{aligned} \dot{x}_1 &= a_{11} x_1 + a_{12} x_2 \\ \dot{x}_2 &= a_{21} x_1 + a_{22} x_2 \end{aligned}\right\}$$

"TRANSFORMATION"

$$\left[\begin{aligned} \dot{x}_1 &= x_1 + 0.5x_2 \\ \dot{x}_2 &= 0.5x_1 + x_2 . \end{aligned}\right\}\Bigg]$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 1 & 0.5 \\ 0.5 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} .$$

$$\rightarrow \dot{x}_1 - \dot{x}_2 = 0.5x_1 - 0.5x_2$$

$$\rightarrow \dot{x}_1 + \dot{x}_2 = 1.5x_1 + 1.5x_2 .$$

$$\dot{x}_1 - \dot{x}_2 = 0.5\,(x_1 - x_2)$$
$$\dot{x}_1 + \dot{x}_2 = 1.5\,(x_1 + x_2).$$

$$\left.\begin{aligned} \dot{y}_1 &= 0.5\,y_1 \\ \dot{y}_2 &= 1.5\,y_2 \end{aligned}\right\}$$

$$\left.\begin{aligned} y_1 &= e^{0.5t}\,y_1(0) \\ y_2 &= e^{1.5t}\,y_2(0) \end{aligned}\right\}$$

$$\dot{x} = ax \longrightarrow x(t) = e^{at}x(0)$$

$$\dot{x} = ax + bu \longrightarrow ?$$