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Lec. 9
28/11/09

Euler
(Forward Euler) : $\dot{x} = f(x, t) = f(x, t)$

$$\Rightarrow x_{k+1} = x_k + h f(x_k, t_k)$$

1) ORDER : ONE

2) EXPLICIT

3) SINGLE STEP

Linear system $\quad \dot{x} = ax \Rightarrow$ stable if $a<0$

$h > \left|\frac{2}{a}\right|$ Euler method $\rightarrow$ stable if $|1+ah| < 1$

Backward Euler : $\dot{x} = f(x,t)$

$$x_{k+1} = x_k + h\,\underline{f(x_{k+1}, t_{k+1})}$$

1) ORDER : ONE

2) IMPLICIT

3) SINGLE STEP

Linear system $\dot{x} = ax$ $\Rightarrow$ stable if $a<0$

$\underline{\underline{\text{B}}}$ack-Euler method $\rightarrow$ stable if

$$1/|1-ah| < 1$$

Trapezoidal : $\dot{x} = f(x,t)$

$$\underline{\underline{x}}_{k+1} = \underline{\underline{x}}_k + \frac{h}{2}\left(f(\underline{\underline{x}}_k, t_k) + f(x_{k+1}, t_{k+1})\right)$$

1) ORDER : <u>TWO</u>

2) IMPLICIT

3) SINGIE STEP

Linear System : $\dot{x} = ax \qquad a < 0.$

Trapezoidal Rule : $\dfrac{(|1 + ah/2|)}{(|1 - ah/2|)} < 1$

LINEAR HIGHER ORDER SYSTEMS

$\dot{x} = A x$

stable if

$Re(\lambda) < 0$

all eigenvalues

Euler

$\dot{x}_{k+1} = [I + Ah] x_k$

$|\mu| < 1$

$\Rightarrow \quad |(1 + \lambda h)| < 1$

$$\dot{x} = -5x \qquad x(t) = e^{-5t} x(0)$$

$$h \qquad T = \frac{1}{5} = 0.2$$

$$h = 0.01$$

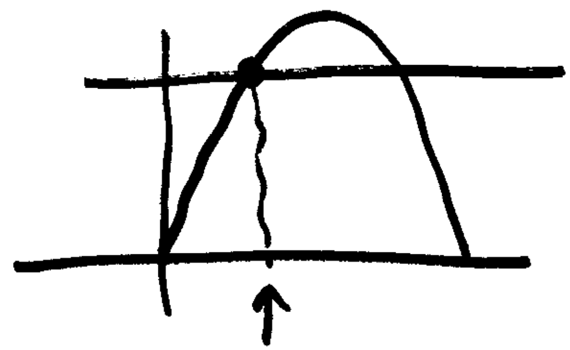
$$x_{k+1} = \underbrace{(1 - 5h)}_{1+ah} x_k$$

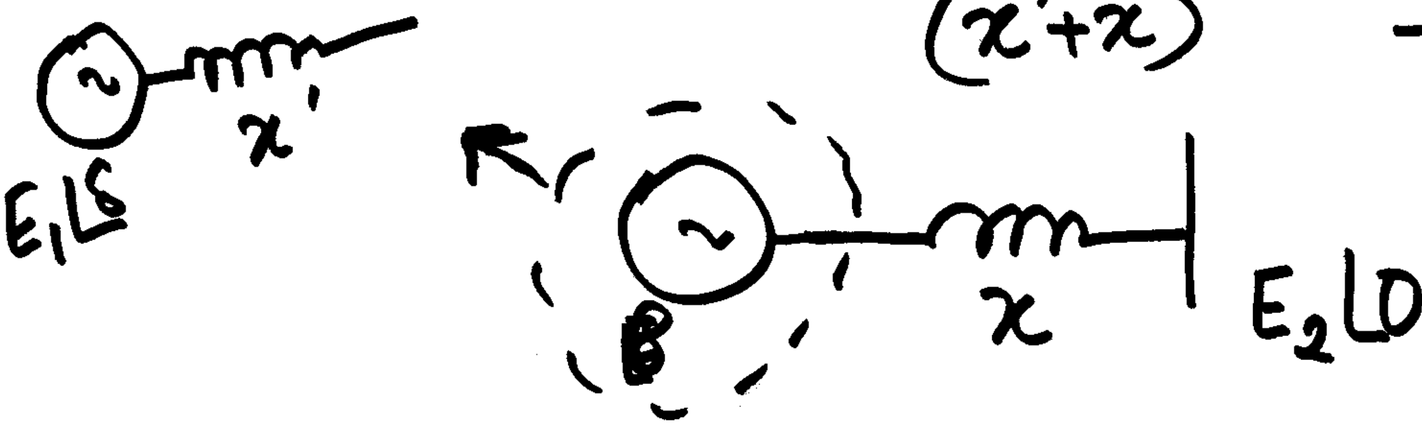
$$\frac{x_{k+1} - x_k}{h} = -5x_k$$

$$\dot{\delta} = \omega - \omega_0$$

$$\frac{d}{dt}(\omega - \omega_0) = \dot{\omega} = \frac{\omega_B}{2H}\left(P_m - K\sin\delta\right)$$

$$K = \frac{E_1 E_2}{(x' + x)}$$



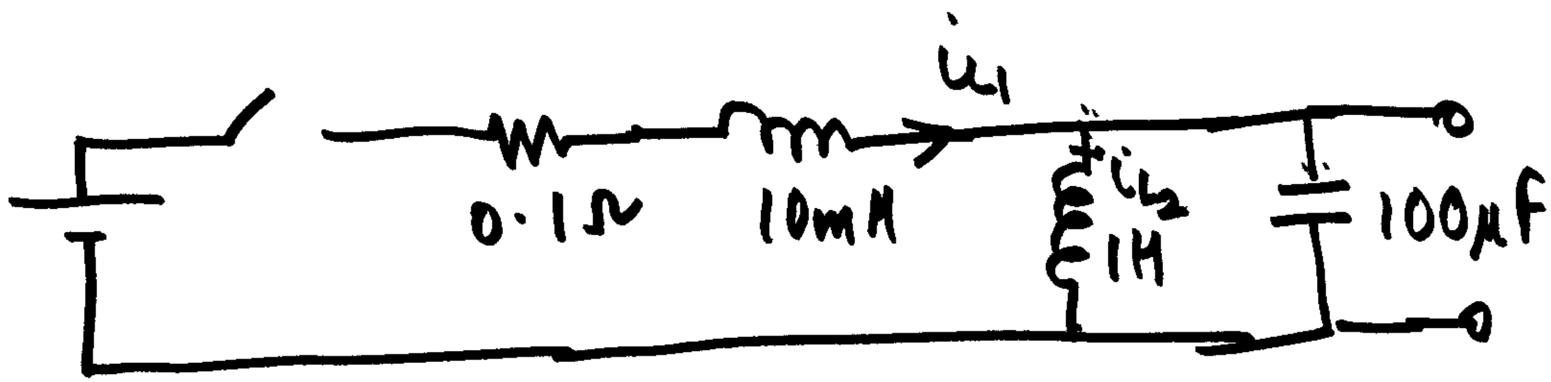


$$\dot{\delta} = \omega - \omega_0$$

$$\delta_{k+1} = \delta_k + \frac{h}{2}\left[\left(\omega_{k+1}^{\approx \omega_k} - \omega_0\right) + \left(\omega_k - \omega_0\right)\right]$$

$$\omega_{k+1} = \omega_k + \frac{h}{2} \cdot \left(\frac{\omega_B}{2H}\right)\left[\left(P_m - K\sin\delta_{k+1}^{\approx \delta_k}\right) + \left(P_m - K\sin\delta_k\right)\right]$$

SOLVE THIS USING G.S.



$$A = \begin{bmatrix} -10 & 0 & -100 \\ 0 & 0 & 1 \\ 10000 & -10000 & 0 \end{bmatrix}$$

$$\lambda_{1,2} = -5 \pm j\,1005$$

$$\lambda_3 = -0.1$$

$$\dot{x} = \underline{\underline{A}}\,x \qquad x = \begin{bmatrix} i_{L1} \\ i_{L2} \\ V_c \end{bmatrix}$$

$$\left|1 + \lambda_i h\right| < 1$$