MANAGERIAL ECONOMICS

Prof. Trupti Mishra
Shailesh J. Mehta School of Management,
IIT Bombay

Lecture No - 19 : Theory of Production

Recap from last session

Long Run Production Analysis
Return to Scale
Isoquants, Isocost
Choice of input combination
Expansion path
Economic Region of Production

Session Outline

•Different kind of Production Functions: Cobb Douglas Production function

•Optimal input combination

– Graphical representation

- Effect of Changes in Input Prices

•Law of diminishing returns -Example

• Return to Scale- Example

Production function -Assumptions:

- Perfect divisibility of both inputs and output;
- Two factors of production – capital (K) and labour (L);
- Limited substitution of one factor for the other;
- A given technology; and,
- Inelastic supply of fixed factors in the short-run.

Forms of production functions - Economic Literature

- Cobb-Douglas production function
- Constant Elasticity of Substitution (CES) production function

The Cobb-Douglas Production Function

$Q = AK^{a}L^{b}$

where a and b are positive fractions.

$Q = AK^{a}L^{(1-a)}$

Properties of the Cobb-Douglas Production Function

- **First, the multiplicative form of the power function can be transformed into its log-linear form as**: $\log Q = \log A + a \log K + b \log L$

In its logarithmic form, the function becomes simple to handle and can be empirically estimated using linear regression techniques.

Properties of the Cobb-Douglas Production Function

Second, power functions are homogeneous and the degree of homogeneity is given by the sum of the exponents a and b as in the Cobb-Douglas function. If a + b = 1, then the production function is homogeneous of degree 1 and implies constant returns to scale.

Properties of the Cobb-Douglas Production Function

Third, a and b represent the elasticity coefficient of output for inputs, K and L, respectively.

The output elasticity coefficient (ε) in respect of capital can be defined as proportional change in output as a result of a given change in K, keeping L constant. Thus,

$$\varepsilon_k = (\partial Q/Q)/(\partial K/K) = (\partial Q/\partial K).(K/Q)$$

Properties of the Cobb-Douglas Production Function

By differentiating $Q = AK^aL^b$, with respect to K and substituting the result into equation , the elasticity coefficient, εk, can be derived: $\boldsymbol{\partial Q/ \partial K = aAK^{(a-1)}L^b}$

Substituting the values for Q and ∂Q/∂K into equation

$\boldsymbol{\varepsilon k = a\,AK^{(a-1)}L^b\,[\,K\,]/AK^aL^b = a}$

Properties of the Cobb-Douglas Production Function

It follows that the **output coefficient for capital, K, is ‘a’**. The same procedure may be applied to show that ‘b’ is the elasticity coefficient of output for labour, L.

Properties of the Cobb-Douglas Production Function

Fourth, the constants a and b represent the relative distributive share of inputs K and L in the total output, Q.

The share of K in Q is given by: $\partial Q/\partial K \cdot K$

Similarly, the share of L in Q :$\partial Q/\partial L \cdot L$

Properties of the Cobb-Douglas Production Function

The relative share of K in Q can be obtained as:

$\partial Q / \partial K \,.\, K.\, 1/Q = a$

and the relative share of L in Q can be obtained as:

$\partial Q / \partial L \,.\, L \,.\, 1/Q = b$

Properties of the Cobb-Douglas Production Function

Finally, the Cobb-Douglas production function in its general form ,

$Q = K^a L^{(1-a)}$, implies that at zero cost, there will be zero production.

Given $\mathbf{Q = AK^aL^b}$

Average Products of L (APL) and K (APK):

$APL = A\ (K/L)^{(1-a)}$

$APK = A\ (L/K)^1$

Marginal Products of L (MPL) and K (MPK):

$MPL = a(Q/L)$

$MPK = (1 - a)Q/K$

Marginal Rate of Technical Substitution of L for K ($MRTS_{L,K}$):

$MRTS_{L,K}$ = MPL/ MPK = a /(1-a) K/L

Note the MRTS L,K is the rate at which a marginal unit of labour, L, can be substituted for a marginal unit of capital, K (along a given isoquant) without affecting the total output.

Effect of Changes in Input Prices on the Optimal Combination of Inputs

Changes in input prices affect the optimal combination of inputs at different magnitudes, depending on the nature of input price change.

If all input prices change in the same proportion, the relative prices of inputs (that is the slope of the budget constraint or isocost line) remain unaffected.

Effect of Changes in Input Prices on the Optimal Combination of Inputs

When input prices change at different rates in the same direction, opposite direction, or price of only one input changes while the prices of other inputs remain constant, the relative prices of the inputs will change.

This change in relative input-prices changes both the input combinations and the level of output as a result of the substitution effect of change in relative prices of inputs.

Effect of Changes in Input Prices on the Optimal Combination of Inputs

A change in relative prices of inputs would imply that some inputs have become cheaper in relation to others.

Cost minimising firms attempt to substitute relatively cheaper inputs for the more expense ones - refers to the **substitution effect of relative input-price changes.**

Effect of Changes in Input Prices on the Optimal Combination of Inputs

$Q = 100K^{0.5} L^{0.5}$

W = Rs 30 r = Rs 40

a. Find the quantity of labor and capital that firm should use in order to minimize the cost of producing 1444 units of output.

b. What is the minimum cost?

Law of Diminishing Returns- Numerical

A firm produces output according to the production function $Q = 10KL - L^3$

Capital is fixed at 10. Find out

a. Derive AP and MP
b. At what level labour does diminishing marginal return set in.
c. At what level labour is the average product of labour at its highest.

Return to Scale : Numerical

$Q = 5K + 8L$

$Q = L^3 + L^2K + K^2L + K^3$

$Q = L^{0.3}K^{0.5}$

Session References

Managerial Economics; D N Dwivedi, 7th Edition

Managerial Economics – Christopher R Thomas, S Charles Maurice and Sumit Sarkar

Micro Economics : ICFAI University Press