NPTEL lectures on

Elementary Numerical Analysis

by

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Assignment 1

1. Let $f : [a, b] \to \mathbb{R}$ be continuous, $x_1, \cdots, x_n \in [a, b]$ and $\alpha_1, \cdots, \alpha_n$ be non-negative real numbers. Show that there exists $c \in [a, b]$ such that

$$\sum_{i=1}^{n} \alpha_i \, f(x_i) = f(c) \sum_{i=1}^{n} \alpha_i.$$

2. Let $f : [a, b] \to \mathbb{R}$ be continuous and $g : [a, b] \to \mathbb{R}$ be integrable. If $g(x) \leq 0, \ x \in [a, b]$, then show that

$$\int_a^b f(x) \, g(x) \, dx = f(c) \int_a^b g(x) \, dx.$$

3. Let

$$f(x) = x^3 - 2x^2 + x.$$

Show that there exists c such that $f(c) = \dfrac{1}{2}$.

4. Let $M > 0$. Show that

$$\frac{M^n}{n!} \to 0 \text{ as } n \to \infty.$$

5. Let $l_i(x)$ denote the ith Lagrange interpolation polynomial based on $x_0, x_1, \cdots, x_n$ with $n \geq 1$. Show that

$$\sum_{i=0}^{n} x_i \, l_i(x) = x.$$