NPTEL lectures on

Elementary Numerical Analysis

by

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Assignment 4

1. Let A be an $n \times n$ matrix. After the first step of Gauss elimination process, without partial pivoting, the following matrix is obtained.

$$\begin{pmatrix} a_{11} & A_{12} \\ 0 & A_{22} \end{pmatrix},$$

where A_{12} is of size $1 \times n-1$ and A_{22} is of size $(n-1) \times (n-1)$.

   (a) If A is positive definite, then show that A_{22} is positive definite.

   (b) If A is diagonally dominant by columns, then show that A_{22} is diagonally dominant by columns.

2. Consider $g(x) = x(2-x)$. Show that for all starting point $x_0 \in (0,2)$, the Picard's fixed-point iteration converges to the fixed point 1. Are sufficient conditions for convergence of Picard's iteration satisfied?

3. Consider the initial value problem

$$y'(x) = -2\, y(x),\ 0 \le x \le 1,\ y(0) = 1.$$

   (a) Find an upper bound for the error in Euler method at $x = 1$ in terms of the step size h.

(b) Solve the difference equation which results from Euler method.

(c) Compare the bound obtained from (a) with the actual error as obtained from (b) at $x = 1$ for $h = 0.1$.

4. Show that Euler and Runge-Kutta methods fail to determine an approximate solution of the initial value problem

$$y' = y^{\alpha},\ \alpha < 1,\ y(0) = 0.$$