NPTEL lectures on

Elementary Numerical Analysis

by

Professor Rekha P. Kulkarni

Department of Mathematics

Indian Institute of Technology Bombay

Assignment 5

1. let $x_0, x_1, \cdots, x_N$ be equally spaced nodes. Derive Simpson's implicit method for the initial value problem $y' = f(x, y),\ y(x_0) = y_0$ by applying Simpson rule to the integral

$$y(x_{n+1}) - y(x_{n-1}) = \int_{x_{n-1}}^{x_{n+1}} f(x, y(x))dx.$$

2. Consider approximate solution of the initial value problem

$$y' = f(x, y),\ y(x_0) = y_0$$

by the formula

$$y_{n+1} = y_n + \int_{x_n}^{x_{n+1}} p_1(x)dx,$$

where $p_1(x)$ is the linear polynomial on $[x_n, x_{n+1}]$ interpolating of $g(x) = f(x, y(x))$ at x_n and x_{n+1}. Choose an appropriate approximation of $g(x_{n+1})$ that leads to the Runge-Kutta method of order 2.

3. Let A be a tridiagonal matrix of order n with diagonal entries equal to zero and $a_{i,i+1} = a_{i+1,i} = 1,\ i = 1, 2, \cdots, n-1$. For $k = 1, \cdots, n$, let $u^{(k)}$ be the $n-$ vector whose $i-$th entry is $\sin[ik\pi/(n+1)], i = 1, \cdots, n$. Prove that

$$Au^{(k)} = 2\cos\left(\frac{k\pi}{n+1}\right) u^{(k)},\ k = 1, \cdots, n.$$

4. If A is a tridiagonal matrix with $a_{ii} = d,\ a_{i,i+1} = a_{i+1,i} = e$, for all i, then the eigenvalues of A consists of the numbers

$$d + 2e\cos\left(\frac{k\pi}{n+1}\right) \quad k = 1, \cdots, n.$$