

NPTEL lectures on
Elementary Numerical Analysis

by

Professor Rekha P. Kulkarni

Department of Mathematics

Indian Institute of Technology Bombay

Quiz 2

Time: 30 minutes

Marks: 10

Use of calculators is not permitted.

1. Let A and B be two $n \times n$ real matrices and for $x \in \mathbb{R}^n$, let $\|x\|_2$ denote the 2– norm. Define

$$\|A\|_2 = \max\left\{\frac{\|Ax\|_2}{\|x\|_2} : x \neq 0\right\}.$$

Show that

$$\|AB\|_2 \leq \|A\|_2 \|B\|_2.$$

(2 marks)

2. Let

$$A = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix}.$$

Find $\|A\|_2$.

(2 marks)

3. Let

$$D = \text{diag} [d_1, d_2, \dots, d_n]$$

be a diagonal matrix. Derive a formula for $\|D\|_1$.

(2 marks)

4. Let $f : [0, 2] \rightarrow \mathbb{R}$ be defined by $f(x) = (1 + x)^{1/5}$. Check whether f satisfies conditions in Picard's fixed point iteration theorem which guarantee existence and uniqueness of a fixed point in $[0, 2]$ and convergence of the iterates to the fixed point. (2 marks)

5. Let $f(x) = x^2 - 2x - 3$ and $x_0 = 2$. Find the first two iterates x_1 and x_2 in Newton's method.

(2 marks)