Note Title 4/30/2012

Assignment 3 : Solution

Q.1 **Trapezoidal Rule** : $\int_0^1 \frac{dx}{x+2} = \frac{\frac{1}{2}+\frac{1}{3}}{2} = \frac{5}{12}$

$\simeq 0.416666\ldots$

Simpson Rule : $\int_0^1 \frac{dx}{x+2} = \frac{1}{6}\left(\frac{1}{2}+4\cdot\frac{2}{5}+\frac{1}{3}\right) = \frac{73}{180}$

$\simeq 0.405555\ldots$

Gauss 2-point Rule : $\int_0^1 \frac{dx}{x+2} = \frac{\frac{1}{\frac{1}{2}-\frac{1}{2\sqrt{3}}+2}+\frac{1}{\frac{1}{2}+\frac{1}{2\sqrt{3}}+2}}{2}$

$= \frac{30}{37} \simeq 0.405405$

Exact: $\ln 3 - \ln 2$

$\simeq 0.405465$

Q.2: $\int_0^{2h} x^{-1/2} f(x)\,dx \simeq \sqrt{2h}\left(w_0 f(0) + w_1 f(h) + w_2 f(2h)\right)$

$f(x) = 1 \Rightarrow 2\sqrt{2h} = \sqrt{2h}\,(w_0 + w_1 + w_2)$

$\Rightarrow w_0 + w_1 + w_2 = 2$

$f(x) = x \Rightarrow \frac{4h}{3}\sqrt{2h} = \sqrt{2h}\,(w_1 h + w_2 \cdot 2h)$

$\Rightarrow w_1 + 2w_2 = \frac{4}{3}$

$f(x) = x^2 \Rightarrow \frac{8h^2}{5}\sqrt{2h} = \sqrt{2h}\,(w_1 h^2 + w_2 \cdot 4h^2)$

$\Rightarrow w_1 + 4w_2 = \frac{8}{5}$

$$w_0 + w_1 + w_2 = 2$$

$$w_1 + 2w_2 = \frac{4}{3}$$

$$w_1 + 4w_2 = \frac{8}{5}$$

$$\Rightarrow \quad w_0 = \frac{11}{15}, \quad w_1 = \frac{18}{15}, \quad w_2 = \frac{1}{15}$$

Q.3 <u>Forward</u> <u>Difference</u> <u>Formula</u> :

$$f'(x_0) \simeq \frac{f(x_0+h)-f(x_0)}{h} = \frac{f(0.6)-f(0.5)}{0.1}$$

$$= \frac{0.5646 - 0.4794}{0.1} = 0.852$$

<u>Central</u> <u>Difference</u> <u>Formula</u>

$$f'(x_1) \simeq \frac{f(0.7)-f(0.5)}{0.2} = \frac{0.6442 - 0.4794}{0.2}$$

$$= 0.824$$

Backward Difference Formula

$$f'(x_2) \simeq \frac{f(0.7) - f(0.6)}{0.1} = \frac{0.6442 - 0.5646}{2}$$

$$= 0.796$$

Q.4 $k(AB) = \|AB\| \, \|(AB)^{-1}\|$

$$= \|AB\| \, \|B^{-1}A^{-1}\|$$

$$\leq \|A\| \, \|B\| \, \|B^{-1}\| \, \|A^{-1}\|$$

$$= k(A)k(B)$$

Q.5 Recall that $\|A\|_F = \left(\sum_{i=1}^{n} \sum_{j=1}^{n} a_{ij}^2 \right)^{1/2}$

$$\begin{aligned} k_F(A) &= \|A\|_F \, \|A^{-1}\|_F \\ &\geq \|A A^{-1}\|_F \\ &= \|I\|_F = \sqrt{n} \end{aligned}$$