NPTEL lectures on

Elementary Numerical Analysis

by

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Model Solutions: Quiz 1

1. Let
$$f(x) = 198x^4 + 27x^3 - 10x^2 + 47x + 13.$$
Find the divided difference $f[1 \quad 2 \quad 3 \quad 4 \quad 5]$. (1 mark)

Ans.: 198

Explanation: $f[1 \quad 2 \quad 3 \quad 4 \quad 5] = \dfrac{f^{(4)}(c)}{4!}$.

2. Let
$$x_0 = 1,\ x_1 = \frac{4}{3},\ x_2 = \frac{5}{3},\ x_3 = 2$$
and for $i = 0, 1, 2, 3$, let $\ell_i(x)$ be the Lagrange interpolation polynomial of degree 3 such that
$$l_i(x_i) = 1,\ l_i(x_j) = 0,\ \text{for } i \neq j.$$
Evaluate
$$l_0\left(\frac{3}{2}\right) + l_1\left(\frac{3}{2}\right) + l_2\left(\frac{3}{2}\right) + l_3\left(\frac{3}{2}\right).$$
(1 mark)

Ans.: 1

Explanation: $\displaystyle\sum_{i=0}^{3} l_i(x) = 1.$

3. Let $f : [0, 1] \to \mathbb{R}$ be such that
$$f(0) = 1,\ f'(0) = 3,\ f(1) = 7,\ f'(1) = 10,$$
where $f'(x)$ denotes the derivative of f at x. Find the cubic polynomial which interpolates f and f' at 0 and at 1. (2 marks)

Ans.: $1 + 3x + 3x^2 + x^2(x - 1)$.

4. Let $f : [0, 7] \to \mathbb{R}$ be such that

$$f(0) = 3,\ f(1) = 16,\ f(3) = 108,\ f(7) = 724.$$

Find

(a) a polynomial of degree ≤ 2 which interpolates f at $0, 1, 3$,
Ans.: $3 + 13x + 11x(x-1)$

(b) a polynomial of degree ≤ 3 which interpolates f at $0, 1, 3, 7$.
Ans.: $3 + 13x + 11x(x-1) + x(x-1)(x-3)$. **(1+1 marks)**

5. Let $f(x) = \dfrac{1}{x},\ x \in [1, 3]$ and $p_2(x)$ be the quadratic polynomial which interpolates f at 1, 2, 3. Find the best possible upper bound for $\|f - p_2\|_\infty = \max\limits_{x\in[1,3]} |f(x) - p_2(x)|$. (2 marks)

Ans.: $\frac{2}{3\sqrt{3}}$.
Explanation:

$$f(x) - p_2(x) = \frac{f^{(3)}(c_x)}{3!}(x-1)(x-2)(x-3),\ \|f^{(3)}\|_\infty = 6,$$

and

$$\max_{x\in[1,3]} |(x-1)(x-2)(x-3)| = \frac{2}{3\sqrt{3}}.$$

6. Evaluate

$$\int_0^4 (x-1)(x-2)(x-4)dx.$$

(2 marks)

Ans. $-\frac{16}{3}$.
Explanation: Apply Simpson Rule.