Quiz 2 : Solution

Note Title 3/29/2011

1. Note that $\|Ax\|_2 \le \|A\|_2 \, \|x\|_2$.

$$\|ABx\|_2 \le \|A\|_2 \, \|Bx\|_2 \le \|A\|_2 \, \|B\|_2 \, \|x\|_2$$

$\Rightarrow$ for $x \neq \bar{0}$, $\dfrac{\|ABx\|_2}{\|x\|_2} \le \|A\|_2 \, \|B\|_2$

$$\Rightarrow \max_{x \neq \bar{0}} \frac{\|ABx\|_2}{\|x\|_2} = \|AB\|_2 \le \|A\|_2 \, \|B\|_2$$

2. $x = [x_1, x_2]^t$, $Ax = \left[\dfrac{x_1+x_2}{\sqrt{2}}, \dfrac{x_1-x_2}{\sqrt{2}}\right]^t$

$$\|Ax\|_2 = \left\{\left(\frac{x_1+x_2}{\sqrt{2}}\right)^2 + \left(\frac{x_1-x_2}{\sqrt{2}}\right)^2\right\}^{1/2} = (x_1^2 + x_2^2)^{1/2} = \|x\|_2$$

$\Rightarrow \|A\|_2 = 1$

3. $x = [x_1, x_2, \cdots, x_n]^t$, $\quad Dx = [d_1 x_1, \cdots, d_n x_n]^t$

$$\|Dx\|_1 = \sum_{j=1}^{n} |d_j|\,|x_j| \le \max_{1 \le j \le n} |d_j| \, \|x\|_1$$

Let $|d_k| = \max\limits_{1 \le j \le n} |d_j|$

$$\|De_k\|_1 = |d_k| \Rightarrow |d_k| \le \|D\|_1$$

Thus $\|D\|_1 = \max\limits_{1 \le j \le n} |d_j|$.

4. For $x \in [0, 2]$, $1 \le f(x) \le 3^{1/5} < 2$.
Hence $f : [0, 2] \longrightarrow [0, 2]$, continuous on $[0, 2]$

differentiable on $(0, 2)$, $|f'(x)| = \dfrac{1}{5(1+x)^{4/5}} \le \dfrac{1}{5} < 1$

5 $f(x) = x^2 - 2x - 3$, $f'(x) = 2x - 2$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} = x_n - \frac{(x_n^2 - 2x_n - 3)}{2x_n - 2}$$

$\vdots$

$\left(\frac{1}{2}\right)$

$$= \frac{x_n^2 + 3}{2x_n - 2}$$

$$x_0 = 2 \Rightarrow x_1 = \frac{4+3}{4-2} = \frac{7}{2}$$

$$x_2 = \frac{\left(\frac{7}{2}\right)^2 + 3}{7-2} = \frac{55}{20} = \frac{11}{4}$$