Complex Analysis - Video course

COURSE OUTLINE

Complex numbers, the topology of the complex plane, the extended complex plane and its representation using the sphere. Complex functions and their mapping properties, their limits, continuity and differentiability, analytic functions, analytic branches of a multiple-valued function. Complex integration, Cauchy's theorems, Cauchy's integral formulae. Power series, Taylor's series, zeroes of analytic functions, Rouche's theorem, open mapping theorem. Mobius transformations and their properties. Isolated singularities and their classification, Laurent's series, Cauchy's residue theorem, the argument principle.

COURSE DETAIL

S. No	Modules/ Lectures	Topics	No of Lectures
1.	**Introduction (1 lecture)**	Introduction and overview of the course, lecture-wise description.	1
2.	**The Algebra Geometry and Topology of the Complex Plane (5 lectures)**	Complex numbers, conjugation, modulus, argument and inequalities	1
		Powers and roots of complex numbers, geometry in the complex plane, the extended complex plane	1
		Topology of the complex plane: Open sets, closed sets, limit points, isolated points, interior points, boundary points, exterior points, compact sets, connected sets, sequences and series of complex numbers and convergence.	3
	Complex Functions:	Introduction to complex functions.	1
		Limits and continuity.	1
		Differentiation and the Cauchy-Riemann equations, analytic functions, elementary functions and their mapping properties, harmonic	4

NP-TEL

NPTEL

http://nptel.iitm.ac.in

Mathematics

Pre-requisites:

Single and multi-variable real analysis.

Coordinators:

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3.	**Limits, Continuity and Differentiation (8 lectures)**	properties, harmonic functions.	
		Complex logarithm multi-function, analytic branches of the logarithm multi-function, complex exponent multi-functions and their analytic branches, complex hyperbolic functions.	1
		Problem Session	1
4.	**Complex Integration Theory (10 lectures)**	Introducing curves, paths and contours, contour integrals and their properties, fundamental theorem of calculus.	2
		Cauchy's theorem as a version of Green's theorem, Cauchy-Goursat theorem for a rectangle, The anti-derivative theorem, Cauchy-Goursat theorem for a disc, the deformation theorem.	4
		Cauchy's integral formula, Cauchy's estimate, Liouville's theorem, the fundamental theorem of algebra, higher derivatives of analytic functions, Morera's theorem.	3
		Problem Session	1
5.	**Further Properties of Analytic Functions (7 lectures)**	Power series, their analyticity, Taylor's theorem.	3
		Zeroes of analytic functions, Rouche's theorem.	2
		Open mapping theorem, maximum modulus theorem.	2
6.	**Mobius Transformations (3 lectures)**	Properties of Mobius transformations.	2
		Problem Session	1
		Isolated singularities, removable singularities.	1

7.	**Isolated Singularities and Residue Theorem (6 lectures)**	Poles, classification of isolated singularities.	2
		Casoratti-Weierstrass theorem, Laurent's theorem.	1
		Residue theorem, the argument principle	1
		Problem Session	1
		Total	**40**

References:

- H. A. Priestley, Introduction to Complex Analysis, 2nd edition (Indian), Oxford, 2006.
- L. V. Ahlfors, Complex Analysis, 3rd edition, McGraw Hill, 2000.
- J. E. Marsden and M. J. Hoffman, Basic Complex Analysis, 3rd edition, W.H. Freeman, 1999.
- J. W. Brown and R. V. Churchill, Complex Variables and Applications, 7th edition, McGraw Hill, 2003.
- J. H. Mathews and R.W. Howell, Complex Analysis for Mathematics and Engineering, 3rd edition, Narosa, 1998.
- T. Needham, Visual Complex Analysis, Oxford, 1997.