LINEAR PROGRAMMING AND ITS EXTENSIONS (NPTEL) ASSIGNMENT

1. Consider the standard LPP(3) with extreme points X_1, X_2, X_3, X_4 and extreme directions $\vec{d}_1, \vec{d}_2$ *and* $\vec{d}_3$ such that
$C^t X_1 = 5,\ C^t X_2 = 7,\ C^t X_3 = 4 = C^t X_4,\ C^t \vec{d}_1 = 0,\ C^t \vec{d}_2 = 0$
and $C^t d_3 = 4$. Characterise all the optimal solutions, of the LPP..
Note: Finite optimal solutions exist even though directions are present in the feasible region. Try to explain why ?

2. Use a linear programming formulation to show that the constraints
$$2x_1 - x_2 - x_3 + 2x_4 + x_5 \le 3$$
$$-3x_4 + x_2 + 4x_3 - 5x_4 - 2x_5 \le -4,\quad x_j \ge 0 \forall j$$
imply $-6x_4 + 8x_2 + 7x_3 - 9x_4 - 5x_5 > -18.$

3. Obtain the set of alternate optimal solutions given the following optimal tableau:

x_1	x_2	x_3	x_4	x_5	x_6	RHS
0	0	0	0	2	3	9
1	0	2	-1	-1	1	4
0	1	-2	1	2	3	5

4. Consider the problem:

$$\text{Min } 2x_1 - x_2 - 5x_3 - 3x_4$$
$$s.t.\ x_1 + 2x_2 + 4x_3 - x_4 = 6$$

$$2x_1 + 3x_2 - x_3 + x_4 = 12$$
$$x_1 \quad + x_3 + x_4 = 4 \qquad x_1, x_2, x_3, x_4 \geq 0$$

Find a bfs with the basic variables as x_1, x_2 & x_4. Is this solution optimal?

5. The starting and current tableaux of a given LPP are shown. Find the values of the unknowns a through ℓ

Starting Tableau

	z	x_1	x_2	x_3	x_4	x_5	RHS
$z_j - c_j$	1	a	1	-3	0	0	0
	0	b	C	d	1	0	6
	0	-1	2	e	0	1	1

Current Tableau

	Z	x_1	x_2	x_3	x_4	x_5	RHS
$z_j - c_j$	1	0	$-\frac{1}{3}$	i	k	l	-4
	0	g	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{1}{3}$	0	f
	0	h	I	$-\frac{1}{3}$	$\frac{1}{3}$	1	3

6. Read each of the following statements carefully and check whether it is true or false. Justify your answer by constructing a

simple illustrative example, if possible, or by a simple proof. In all these statements, problem (1) is the LP.

$$\begin{aligned} &\textit{Minimize} \quad z(x)=cx \\ &\textit{Subject to} \quad Ax=b \\ &\qquad\qquad\qquad x \geqq 0 \end{aligned}$$

where A is a matrix of order $m \times n$. **K** is the set of feasible solutions of this problem (1) $f(b)$ is the minimum objective value in this problem, as a function of the right – hand side constant vector, b.

1. The number of positive x_j by BFS of (1) can never exceed the rank of A.
2. The unboundedness criterion will never be satisfied while solving the Phase I problem.
3. If (1) is feasible, termination occurs in solving the Phase I problem associated with (1) only when all the artificial variables leave the basic vector.
4. If a tie occurs in the pivot row choice during a pivot step while solving (1) by the simplex algorithm, the BFS obtained after this pivot step is degenerate.
5. Simplex algorithm moves from an extreme point of K only in a no degenerate pivot step.
6. In solving an LP by the simplex algorithm, a new feasible solution is generated after every pivot step.
7. Let the rank of A in (1) be m. If (1) has an optimum solution, there must exist an optimum basis. Every basis consists of m column vectors and a pivot step changes one column vector in a basis. Hence starting from a feasible basis, the simplex algorithm finds an optimum basis in at most m pivot steps.
8. Every feasible solution of (1) in which m variables are positive is a BFS.

9. If $\overline{x}$ and $\overline{y}$ are two adjacent BFSs of (1), the total number of variables that are positive in either $\overline{x}$ or $\overline{y}$, or both , is at most m+1.
10. No feasible solution of (1) in which m+2 or more variables are positive can lie on an edge of K.
11. Every convex set has an extreme point.
12. Every nonempty convex polytope has an extreme point.
13. If (1) has an optimum solution, K must be a bounded set.
14. An LP can have more than one optimum solution iff it is degenerate.
15. The total number of optimum feasible solutions of any LP is always finite.
16. The total number of BFSs of any LP is always finite.
17. The set of optimum solutions of any LP is always a bounded set.
18. If (1) has an optimum solution in which m+1 or more variables are positive, it must have an uncountable number of optimum solutions.
19. If rank of A is m, and (1) is nondegenerate, every feasible solution in which exactly m variables are positive, must be a BFS.
20. While solving (1) by the simplex algorithm, if the BFS in the beginning of a pivot step is degenerate, the objective value remains unchanged in this pivot step.
21. Can a vector which has just left the basis in the simplex algorithm recenter on the very next pivot? Hint. If x_{B_r} is the out going variable, compute its new $C_{B_r} - Z_{B_r}$ and show that it will be positive.