Linear Programming and its Extensions (NPTEL)
Assignment

1. Consider the following product mix-problem:

$$Minimise:\ Z = -2x_1 - 3x_2 - 2x_3 - x_4$$

Subject to:

$$x_1 + 3x_2 + x_4 \le 7 \quad (amount\ of\ raw\ material\ 1\ available)$$

$$2x_1 + x_2 \le 4 \quad (amount\ of\ raw\ material\ 2\ available)$$

$$x_2 + 4x_3 + x_4 \le 6 \quad (amount\ of\ raw\ material\ 3\ available)$$

$$x_1,\ x_2,\ x_3,\ x_4 \ge 0$$

x_5, x_6, x_7 are the slack variables corresponding to the three inequality constraints. (x_1, x_3, x_2) constitute the optimal basic variables and let B denote the corresponding basis.

$$B^{-1} = \begin{bmatrix} -1/5 & 3/5 & 0 \\ -1/10 & 1/20 & 1/4 \\ 2/5 & -1/5 & 0 \end{bmatrix}.$$

Answer the following questions giving reasons for your answers.

(a) If you are given the choice of increasing the availability of only one of the materials by one unit, which one will you choose ? Why ?

(b) What is an optimum solution if availability of b_2 is increased by 31 units ?

(c) If total of 10 units of 3rd raw material can be made available, i.e. 4 units more, what is the maximum amount you can afford to pay for it ?

(d) Obtain the interval for c_1 such that the current solution remains optimal for all values of c_1 in that interval. Do the same for c_2.

(e) Let x_8 denote the number of a new product that the company has the option to produce at Rs.10/- per unit. In the objective function the cost coefficient will be -10 since we are minimizing the objective function. Let $A_8^t = (20,\ 20-3\lambda,\ -24+\frac{\lambda}{5})$. At what value of λ will it become profitable to manufacture the new product ?

(f) For what range of values of θ will the current solution remain optimal , if the input-output coefficient a_{22} is changed to $a_{22}+\theta$?

2. Consider the parametric RHS problem:

$$\begin{aligned} &Min\ Z = C^t x \\ &Subject\ to\ Ax = b + \lambda b^*, \\ &\qquad\qquad x \geqq 0. \end{aligned}$$

Suppose the problem is known to be infeasible for $\lambda = \lambda_0$. Prove that the problem will be infeasible for either all values of $\lambda \leq \lambda_0$ or for all values of $\lambda \geq \lambda_0$.

3. Consider the following parametric LPP in tableau form:

x_1	x_2	x_3	x_4	x_5	RHS	
3	4	7	-1	1	0	C
0	8	0	0	-2	0	C^*
5	-4	14	-2	1	16	
1	-1	5	-1	1	8	

Objection function is to be minimized . Solve the problem for all values of the parameter μ, starting with the bfs solution consisting of the basic variables (x_1, x_5). Show that the corresponding basis (A_1, A_5) is not optimal for any value of μ. Proceed from there to solve for all values of μ.