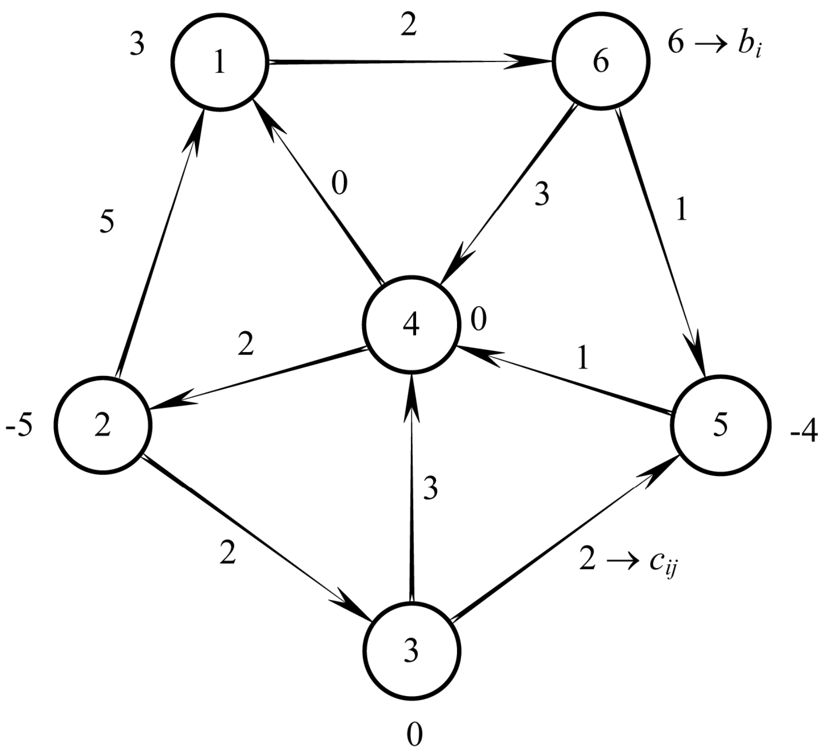
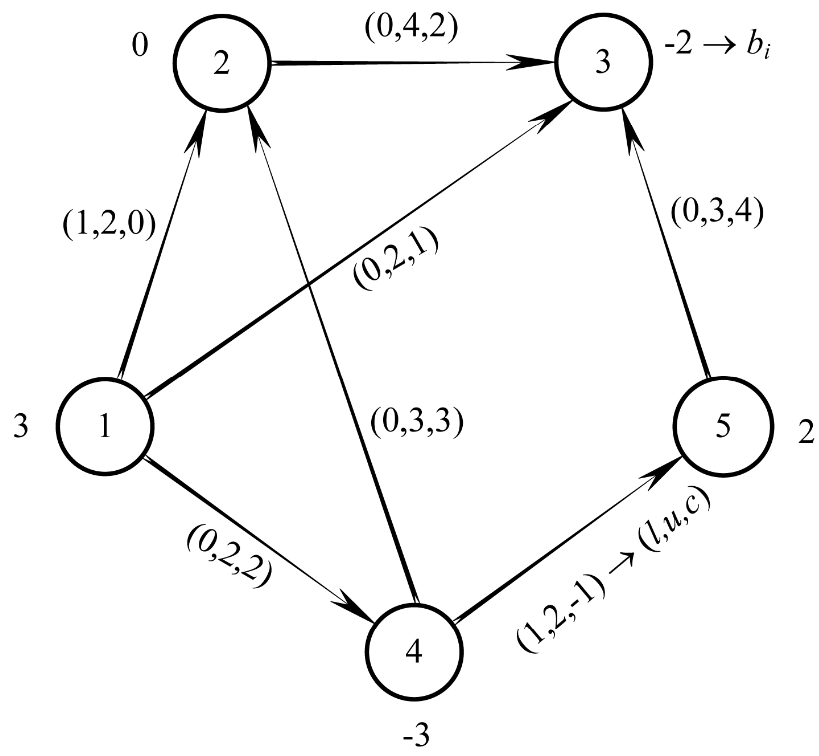
Linear Programming and its Extensions (NPTEL)
Assignment

1. Transform the given un-capacitated min-cost flow problem to a transportation problem.

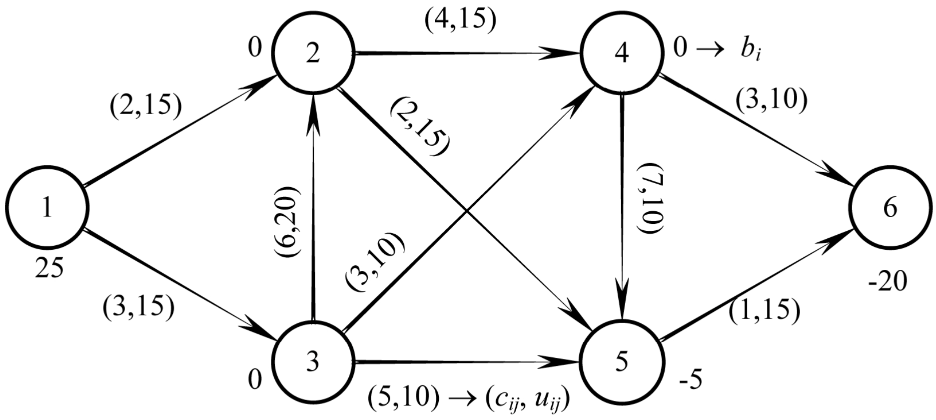


2. Consider the min-cost flow problem given below with positive lower bounds on arc flows. Transform it to a min-cost flow problem with ‘0’ lower bounds

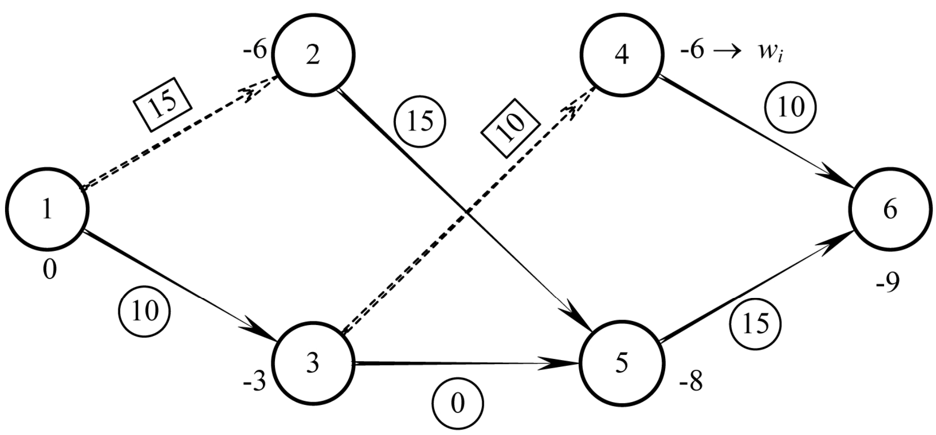


Since $\ell_{45} = 1$, i.e. atleast one unit of flow should be on arc (4, 5), we make the transformation $y_{35} = x_{35} - 1$. Then $y_{35} = 0$ when $x_{35} = 1$ and $y_{35} \geq 0$. The transformation will change $b_4 \to -4$ and $b_5 = 2+1 = 3$. Similarly make the transformation for arc (1,2) to reduce the problem to a min-cost flow problem with ‘0’ lower bounds.

3. Following min-cost flow problem was worked out in the 29th lecture.



The network given below depicts an optimal solution.



Post-optimality Analysis.

(i) By how much can c_{12} and c_{35} change one at a time so that the current solution remains optimal.
Remember, for arc (1,2), $\bar{c}_{12} = c_{12} + \Delta_{12} - w_1 + w_2 \leq 0$
and for arc (3,5), $\bar{c}_{35} = c_{35} + \Delta_{35} - w_3 + w_5 \geq 0$

(ii) Find an interval for Δ_{25} such that the current solution will remain optimal when $c_{25} \to c_{25} + \Delta_{25}$

(iii) Suppose $b_1 \to b_1 + \delta$, i.e. $\hat{b}_1 = 25 + \delta$ and $\hat{b}_4 \to -\delta$, i.e. $\hat{b}_4 = -\delta$.
Find an interval for δ such that the current solution remains optimal.

(iv) Consider $\hat{b}_1 = 25 + \delta$ and $\hat{b}_2 = -\delta$. In this case also determine an interval for δ such that the current solution remains optimal.

(v) Show one iteration of the dual-simplex algorithm when δ exceeds either of its limits in (iv).

(vi) Suppose $u_{12} \to 20$. Since $\bar{c}_{12} < 0$, increase of flow on arc (1,2) will improve the cost. Thus the current solution may no longer be optimal. Obtain the new optimal solution.
Arc (2,5) is a basic arc at its upper bound. Suppose u_{25} decreases to 10. The current solution is no larger feasible. Obtain an optimum solution using the dual-simplex pivot on the current optimum solution.