Probability and Statistics
Test Set 1

1. Find $\liminf_n E_n$ and $\limsup_n E_n$, if E_n is given by

(i) $E_n = \begin{cases} \left(0, \frac{1}{n}\right), & \text{if n is odd,} \\ \left(1 - \frac{1}{n}, 1\right), & \text{if n is even}, \quad n = 1, 2, 3, ... \end{cases}$

(ii) $E_n = \begin{cases} \left(0, 1 - \frac{1}{n}\right), & \text{if n is odd,} \\ \left(\frac{1}{n}, 1\right), & \text{if n is even}, \quad n = 1, 2, 3, ... \end{cases}$

(iii) $E_n = \left\{0, \frac{1}{n}, \frac{2}{n}, ..., \frac{n-1}{n}, 1\right\}, \quad n = 1, 2, ...$

(iv) $E_n = \begin{cases} \{(x, y) : 0 \le x \le \frac{3n+4}{4n}, \frac{n-2}{2n} \le y \le 1\}, \text{if n is odd,} \\ \{(x, y) : \frac{n-4}{4n} \le x \le 1, 0 \le y \le \frac{n+2}{2n}\}, \text{if n is even.} \end{cases}$

Identify if $\lim_n E_n$ exists?

2. Prove or disprove the following statements:
   (i) A nonempty class of sets closed under the formation of intersections, proper differences and disjoint unions is a ring. (A-B is called a proper difference if B is a proper subset of A.)
   (ii) A nonempty class of sets closed under the formation of intersections and differences is a ring.
   (iii) A nonempty class of sets closed under the formation of intersections and symmetric differences is a ring.
   (iv) A nonempty class of sets closed under the formation of countable intersections and symmetric differences is a σ-ring.

3. Let **R** be a ring of subsets of a set X and **A** be the class of all sets E for which either $E \in R$ or $E^C \in R$. Check if **A** is an algebra.

4. Show that the intersection of two rings is a ring but their union is not necessarily so.

5. Let X be an uncountable set and **A** be the class of all subsets of X which are either countable or have countable complements. Show that **A** is an algebra.

6. Identify the following classes of subsets of X (X is an uncountable set) as ring and/or algebra:

(i) $\{\phi, X\}$ (ii) $\{\phi, A, X\}$ (iii) $\{\phi, A, A^C, X\}$ (iv) $P(X)$

(v) The class of all finite subsets of X, (vi) The class of all countable subsets of X.