Probability and Statistics
Test Set 10

1. Let X ~ Bin (1, p), 1/4 $\leq$ p $\leq$ 3/4. Find the MLE and MME of p. Are these unbiased.

2. Consider a random sample $X_1,\ldots,X_n$ from a double exponential distribution with density

$$f(x,\theta,\sigma) = \frac{1}{\sigma}\exp\{-\frac{|x-\theta|}{\sigma}\}\ ,\ \ x > \theta\ ,\ \theta \in R\ ,\ \sigma > 0.$$

   Find MME's and MLE's of θ and σ. Discuss the unbiasedness of MME's.

3. Let $X_1,\ldots,X_n$ be a random sample from a $N(1, \sigma^2)$ population. Find the MME and the MLE of σ^2. Check their unbiasedness and consistency.

4. Assume the number of emails received per minute by a service centre is a Poisson random variable with parameter λ. If in 10 minutes 20 emails are received, find the MLE of λ.

5. The waiting time X at a railway crossing follows a uniform U (0, b) distribution. On twelve random occasions the waiting times (in minutes) as 12, 14, 9, 7, 10, 12, 7, 9, 9, 11, 9, 12. (a) Find an unbiased estimate for the average waiting time. (b) Find an unbiased estimate for Var (X). (c) Find an unbiased estimate for b. (d) Find an estimate for standard deviation. of X. Check if it is unbiased.

6. The time (in hours) X taken for degradation of certain waste material varies due to slight variations in the amount of water used and the potency of the ingredients in the waste. Assume that X is a gamma random variable with p and α unknown. On the basis of 14 observations on X given below find the MME's of p, α, E(X) and Var(X): 1.1, 1.8, 2.0, 1.9, 1.2, 2.4, 2.2, 2.5, 1.2, 1.9, 2.5, 1.6, 2.1, 3.2.

7. The lifetimes of a component are assumed to be exponential with parameter λ. Twenty of these identical components were placed on test independently. Due to a technical error, the only information that was saved was that four components failed before 120 hours. Find the MLE of λ.

8. Find a **95%** confidence interval for the mean of a normal distribution with σ **= 2**, given the sample **(2.4, – 0.6, – 0.3, – 1.1)**. Determine the confidence interval if σ is unknown?

9. Twenty measurements on temperature of a compound under certain conditions yield a standard deviation 1.04^0 Celsius. Determine a **95%** confidence interval for the true standard deviation of such measurements.

10. Measurements on the range of wiping the glass for a random sample of 100 wipers used in cars yield a sample standard deviation of 0.02 inch. Construct a 99% confidence interval on the standard deviation.

11. Two different medicines are given to six patients to control their LDL each for three months with a gap of one year between the two trials. The decrease in LDL is recorded as follows:

Patient	1	2	3	4	5	6
Medicine 1	24	22	23	26	22	25
Medicine 2	26	25	23	25	21	28

Find a 95% confidence interval for the mean difference in the reduction of LDL by the two medicines.

12. Independent random samples of sizes 64 and 36 are taken from two normal populations having known standard deviations of 2.4 and 3.0 respectively. Determine a 95% confidence interval for the difference in the means of two populations.

13. Assume that there is a constant probability p that a printer tray manufactured by machine in a factory will be defective. If in 60 printers selected at random from output, it is found that 5 are defective, find a 90% confidence interval for p.