Probability and Statistics
Test Set 2

1. If 6 balls are placed at random into 6 cells, find the probability that exactly one cell remains empty.

2. Let event E be independent of events F, $F \cup G$ and $F \cap G$. Show that E is independent of G.

3. A pair of dice is rolled until a sum of 7 or an even number appears. Find the probability that a 7 appears first.

4. In a certain colony, 65% of the families own a car, 35% own a computer and 25% own both a car and a computer. If a family is randomly chosen, what is the probability that this family owns a car or a computer but not both?

5. A survey of people in given region showed that 25% drank regularly. The probability of death due to liver disease, given that a person drank regularly, was 6 times the probability of death due to liver disease, given that a person did not drank regularly. If the probability of death due to liver disease in the region is 0.005, what is the probability of death due to liver disease given that a person drank regularly?

6. Show that if $P(A \mid B) = 1$, then $P(B^C \mid A^C) = 1$.

7. If $P(A^c) = 0.4$, $P(B) = 0.3$ and $P(A \cap B^c) = 0.3$, find $P(B / A \cup B^c)$.

8. In a ternary communication channel, let T_i denote the event "Digit i is transmitted" and let R_i denote the event "Digit i is received" for i = 1, 2, 3. Assume that a 3 is transmitted three times more frequently than a 1, and a 2 is transmitted twice as often as 1. (i) If a one has been received, what is the probability that a 1 was sent? (ii) Find the probability of a transmission error. (iii) Find the probability that digit i is received for i = 1, 2, 3.

9. In any given year a male automobile policyholder will make a claim with probability p_m and a female automobile policyholder will make a claim with probability p_f, where $p_m \neq p_f$. The fraction of policyholders that are male is α, $0 < \alpha < 1$. A policyholder is randomly chosen and A_i denotes the probability that this policyholder will make a claim in the year i, i = 1, 2,…. Find $P(A_1)$ and $P(A_2|A_1)$ and show that $P(A_2|A_1) > P(A_1)$.

10. Which of the following statements is true?
    (i) $P(A) = 0.3$, $P(B) = 0.7$, $P(A \cup B) = 0.5$, $P(A \cap B) = 0.5$.
    (ii) $P(A) = 0.5$, $P(A \cup B) = 0.7$, A and B are independent, $P(B) = 0.4$.
    (iii) $P(A) = 0.2$, $P(A \cup B) = 0.9$, A and B are disjoint, $P(B) = 0.6$.
    (iv) none of these.

11. Four players A, B, C and D are distributed thirteen cards each at random from a complete deck of 52 cards. What is the probability that player C has all four Jacks?

12. Each coefficient in the equation $ax^2 + bx + c = 0$ is determined by throwing an ordinary die. Find the probability that the equation will have real roots.

13. An Integrated M.Sc. student has to take 5 courses a semester for 10 semesters. In each course he/she has a probability 0.5 of getting an ‘Ex’ grade. Assuming the grades to be independent in each course, what is the probability that he/she will have all ‘Ex’ grades in at least one semester.

14. If $P(A) > 0$, show that $P(A \cap B \mid A) \geq P(A \cap B \mid A \cup B)$.

15. Let $S = \{1, 2, \ldots, n\}$ and suppose that A and B are, independently, equally likely to be any of the 2^n subsets (including) the null set and S itself) of S. Let X denote the number of elements of B. Find $P(X = i)$, $P(A \subset B \mid X = i)$, $P(A \subset B)$ and deduce that $P(A \cap B = \phi) = \left(\frac{3}{4}\right)^n$.

16. A question paper consists of eight True-False and two multiple choice (A, B, C, D) questions. Each question carries one mark for the correct answer and zero for wrong answer. Assume that an unprepared student answers all questions independently with guess. What is the probability that he/she will score at least 9 marks?

17. Four tennis players A, B, C, D have the probabilities of winning a tournament as $P(A) = 0.35$, $P(B) = 0.15$, $P(C) = 0.3$, $P(D) = 0.2$. Before the tournament, the player B is injured and withdraws. Find the new probabilities of winning the tournament for A, C and D.

18. In 2n tosses of a fair coin, what is the probability that more tails occur than the heads?