Probability and Statistics
Test Set 3

1. Check if the following functions define cdf's :

(a) $F(x)=\begin{cases}0, & x<0,\\ \frac{x}{2}, & 0\le x\le \frac{2}{3},\\ 1, & x>\frac{2}{3}.\end{cases}$

(b) $F(x) = \sin x,\quad 0 < x < \pi/2.$

(c) $F(x)=\begin{cases}0, & x\le 1,\\ 1-\frac{1}{x^2}, & x>1.\end{cases}$

(d) $F(x)=\begin{cases}0, & x\le 0,\\ \frac{1}{2}+\frac{1}{2}e^{-x}, & x>0.\end{cases}$

2. Let X be a r.v. with the cdf given by :

$$F(x)=\begin{cases}0, & x<0,\\ \frac{x}{8}, & 0\le x\le 1,\\ \frac{x+1}{8}, & 1\le x<2,\\ \frac{2x+1}{8}, & 2\le x<3,\\ 1, & x\ge 3.\end{cases}$$

Find $P(1/2 \le X \le 5/2)$, $P(1< X < 3)$, E(X), V(X) and median of X.

3. Let X and Y be two independent random variables with respective m.g.f.'s given by $M_X(s)=\frac{(1+e^s)^3}{8}$ and $M_Y(t)=e^{e^t-1}$. Find $P(X + Y = 1)$.

4. Let X be a continuous r.v. with pdf given by

$$f(x)=\begin{cases}0, & x<0,\\ \frac{x}{4}, & 0\le x<1,\\ \frac{3}{4}, & 1\le x<2,\\ \frac{3-x}{4}, & 2\le x<3,\\ 1, & x\ge 3.\end{cases}$$

Find cdf, mean, variance and the median of X.

5. Suppose that an organism produces one, two or three other organisms with equal probability and suppose that each of these second stage organisms produce one, two or three organisms with equal probability. Find the distribution of the number of organisms at the third stage.

6. To determine whether or not they have a certain blood disease, 100 people are to have their blood tested. However, rather than testing each individual separately, it has been decided first to group the people in batches of 10. The blood samples of the 10 people in each batch will be pooled and analyzed together. If the test is negative, one test will suffice, else each of the ten people will be individually tested. Assuming the incidence of disease to be 1% for the population, find the expected number of tests required for each batch.

7. Let X be a discrete random variable with $p_X(-1) = \frac{1-3d}{4}$, $p_X(0) = \frac{1+d}{4}$, $p_X(1) = \frac{1-2d}{4}$ and $p_X(2) = \frac{1+4d}{4}$. For what values of d, does $p_X(x)$ describe a valid probability mass function? Further determine the value of d for which Var(X) is a minimum.

8. Let X denote the number of typographical errors in a large office per month having probability mass function

$$p_X(x) = k\left[\frac{1}{x+1} - \frac{1}{x+2}\right], \quad x = 0, 1, 2, \ldots$$

Find the value of k, cdf of X, E(X) and the median of X.

9. Let the distribution of marks on a class test have mean 75 and s.d. 3. Use Chebyshev's inequality to show that the probability of a student having marks above 90 or below 60 is at most 1/25.