Probability and Statistics
Test Set 4

1. Ruby and Mini tied for the first place in a beauty contest. The winner is to be decided by the majority opinion of a panel of three judges chosen at random from a group of nine judges. If six of these judges favour Ruby and three favour Mini, what is the probability that Ruby will be declared the winner.
2. A missile can successfully hit a target with probability 0.75. If three successful hits can destroy the target completely, how many missiles must be fired so that the probability of the completely destroying the target is not less than 0.95?
3. A mechanical system consists of n components each of which function independently with probability p. The entire system will be able to operate effectively, if at least one-third of its components function. For what values of p, a 6-component system more likely to operate effectively than a 3-component system?
4. The pen refills produced by a certain company are defective with probability 0.01, independently of each other. The company sells the refills in packets of size 10 and offers a money-back guarantee if more than one of the 10 refills in the packet is found to be defective. If 3 packs are purchased, what is the probability that at most one pack will be returned?
5. The number X of refrigerators that a store sells in a day follows the Poisson distribution with $\lambda = 5$. The profit on each refrigerator is Rs. 1000.00. If at the time of opening the store 10 refrigerators are in stock (with no replenishment during the daytime), the profit from the sale of the refrigerators during the day is Y = 1000 min(X, 10). Find the probability distribution of Y.
6. The number of times that an individual contracts viral infection in a given year is a Poisson random variable with parameter $\lambda = 5$. Using a new health regime, 75% of the population reduces the Poisson parameter λ to 1. If an individual does not get viral infection for a year, what is the probability that he/she followed the new health regime?
7. The average number of typographical errors in a large book is 300. What is the probability that no more than 2 errors will be found in randomly selected 1% of the pages?
8. A boy and a girl decide to meet between 4 and 6 p.m. in a park. They decide not to wait for the other for more than 10 minutes. Assuming arrivals to be independent and uniformly distributed, find the probability that they will meet.
9. A small industrial unit has 20 machines whose lifetimes are independent exponentially distributed with mean 100 months. If all the machines are under use at a time, find the probability that even after 200 months there are at least two machines working.
10. The time to failure in hours, X, of the IEDs produced at two manufacturing plants I and II follows exponential distribution with means 6 and 2 hours respectively. Plant II produces four times as many IEDs as plant I. The IEDs are intermingled and supplied. What is the probability that an IED selected at random will work for at least 6 hours?
11. A series system has n independent components. The lifetime X_i of the i^{th} component is exponentially distributed with mean 2^i. If the system fails before time t what is the probability the failure was caused only by component j (j = 1,…,n).

12. The time to failure of a certain system has a gamma distribution with a mean of 20 days and a standard deviation of 10 days. Determine the probability of a failure within 15 days of the start of operations.
13. The lifetime X in hours of a component is modelled as a Weibull distribution with $\beta = 3$. Starting with a large number of components it is observed that 90% of the components that have lasted 5 months fail before 6 months. Determine the parameter α. Further determine the probability that a component is working after 8 months.
14. In an examination, a student is considered to have failed, secured grade 'C', grade 'B' and grade 'A', according as he scores less than 45%, between 45% and 60%, between 60% and 75% and above 75% respectively. In a particular year 10% of students failed in the examination and 5% students got 'A' grade. Assuming the marks to be normally distributed, find the percentage of students who get grade 'B' and second class 'C' respectively.
15. The specifications for the diameter of the upper end of chalk pieces are set as 3.0 ± 0.01 cm. The diameter has a normal distribution with mean 3 cm. and s.d. 0.005 cm. What proportion of chalk pieces will be declared defective?
16. The width of a mobile handset is (in cm) normally distributed with $\mu = 5.0$ and $\sigma = 0.03$. The specification limits were given as 5.0 ± 0.05. What percentage of handsets will be defective? What is the maximum allowable value of σ that will permit no more than 1 defective in 100 when the widths are $N(05.0, \sigma^2)$?
17. The distance that an Olympic standard male long jumper clears at every attempt, is a normal random variable with mean 8 meters and s.d. 10 cm. What is the maximum distance that he will clear with probability 0.95? What is the distance that he will clear only 10% of the time?
18. The marks of students in an exam are distributed as normal with mean score 74 and variance 62.41. Determine
    a) the lowest passing grade if the bottom 10% of the students are declared failed
    b) the last student's score to get 'A', if the top 5% of the students are given A's;
    c) the last score of 'B' if the top 10% of the students are given A's and the next 25% are given B's.
19. Let Y denote the length in cm. of certain type of rods. Assume that Y has a log-normal distribution with parameters $\mu = 0.8$ and $\sigma = 0.1$. Find the probability that a randomly selected rod has length more than 2.7 mm. Between what two values will Y fall with probability 0.95?