Probability and Statistics
Test Set 5

1. Let X be a continuous random variable with density function given by

    $f_X(x) = b(x + 1)$, $-1 < x < 2$
          $= 0$,   otherwise.
    Find b, E(X), V(X) and the density function of $Y = X^2$.

2. Let X be continuous random variable with density given by

    $$f_X(x) = \begin{cases} kx, & 0 \leq x < 1 \\ k, & 1 < x \leq 2 \\ k(3\text{-}x), & 2 < x \leq 3 \end{cases}$$

    Determine the value of k, median and other quartiles of the distribution of X. Also find the density of $Y = (X - 3/2)^2$.

3. Let X be a random variable with density function given by

    $f_X(x) = 3x^2/\pi^3$, $0 < x < \pi$
          $= 0$,   otherwise.

    Find the distribution of $Y = \sin(X)$.

4. Let $X \sim NegBin(r, p)$. Find the p.m.f. of each of the following functions of X :

    (a) $Y_1 = 2X + 3$; (b) $Y_2 = X + 3$; (c) $Y_3 = X^2 + 1$; (d) $Y_4 = X^3$.

5. Let $X \sim Beta(a, b)$. Find the distributions of $Y_1 = 1/(1 + X)$ and $Y_2 = 1 - X$.

6. Let C denote the temperature in degree Celsius to which a computer will be subjected to in the field. Assume that C is uniformly distributed over the interval (15,21). Let F denote the field temperature in degrees Fahrenheit so that $F = (9/5)C + 32$. Find the density of F.

7. Let a continuous random variable X have the density given by

    $f_X(x) = cx^2 Exp\{-\beta x^2\}$, $x > 0, \beta > 0$.

    Find c and the density of $Y = X^2$.

8. Let X be a random variable with the pdf
    $f(x) = k(x + 1)$, $-1 \leq x \leq 1$
          $= k(3 - x)$, $1 \leq x \leq 3$.
    Find k, measures of skewness and kurtosis of the distribution of X. Also find the distribution of $Y = |X|$.