Probability and Statistics
Test Set 7

1. Let (X,Y) have the joint pmf

Y\ X	-1	1	2
-2	1/8	1/12	1/6
1	1/10	1/12	1/8
2	1/15	1/10	3/20

   Find the joint pmf of (U,V) where U = $|X|$ and V = |Y|.

2. Let X and Y be independent standard normal r.v.'s and represent a point in a two-dimensional plane. Let (X_1, Y_1) and (X_2, Y_2) be two observations on it.and Z be the distance between them. Find the distribution of Z^2 ?

3. Let X and Y be independent random variables each with negative exponential distribution with mean σ. Find the joint and marginal distributions of U = X/Y and V = X +Y.

4. Let X_1, X_2 be i.i.d. N(0,1) and $Y_1 = X_1^2 + X_2^2$, $Y_2 = X_1/X_2$. Find distributions of Y_1 and Y_2 . Are they independent?

5. Let X_1 and X_2 have independent gamma distributions with parameters (m, λ) and (n, λ). Find the distribution of $Y = X_1/(X_1+X_2)$. Is Y independent of $Z = X_1+X_2$? Is Z independent of $U = X_1/X_2$?

6. Let X_1, X_2, X_3 be independent exponential random variables with the probability density $f(x) = e^{-x}$, $x > 0$. Define random variables Y_1, Y_2 and Y_3 as $Y_1 = X_1 + X_2 + X_3$, $Y_2 = \dfrac{X_1 + X_2}{X_1 + X_2 + X_3}$, $Y_3 = \dfrac{X_1}{X_1 + X_2}$.

   Find the joint and marginal densities of Y_1, Y_2 and Y_3. Are they independent?

7. Let independent random variables U, V, W be such that $U = \ln X_1 \sim N(3, 2)$; $V = \ln X_2 \sim N(4, 1.5)$; $V = \ln X_3 \sim N(2.5, 1.5)$. Find the distribution of $P = e^{1.8} X_1^3 X_2^{2.3} X_3^{2.2}$. Determine L_1 and L_2 such that $P(L_1 \le P \le L_2) = 0.90$.
8. Let (X, Y) have bivariate normal distribution with density function

$$f(x,y) = \frac{1}{\pi\sqrt{2}} e^{-\frac{1}{3}(x^2 - xy + y^2)}, \quad -\infty < x, y < \infty.$$

   Find the correlation coefficient between X and Y, $P(-1 < X < 1.5|Y=1)$, V(3X + Y) and $P(-3 < 2X + Y < 4)$.

9. A metal sheet consists of two sections **A** and **B**, each of which is manufactured independently on a different machine. The length (in inches) of section **A** is normally distributed with mean **25** and variance **0.02** and the length of section **B** is

normally distributed with mean **18** and variance **0.03**. The sheet is formed by joining the two sections together by placing one after the other. Suppose that the sheets can be used in the construction of an side wall of a ship if its total length is between **32.6** to **34.4** inches. What is the probability that the rod can be used in the construction?