Probability and Statistics
Test Set 8

1. The life of a pencil cell is a random variable with mean 60 hrs and standard deviation 15 hrs. A cell is used until it fails, at which point it is replaced by a new one. Assuming 20 such cells are available with independent lives find the approximate probability (using central limit theorem) that over 1300 hrs of use can be obtained.

2. Let X, the lives of electric coil used in heater have mean μ and s.d. 120 hours. n of these coils are put on test till they fail, resulting in observations $X_1,\dots,X_n$ find the minimum value of n so that the probability that $\overline{X}$ differs by μ by less than 60 hours is at least 0.95?

3. Let $X_1, \dots, X_{11}$ be i.i.d. $N(\mu, \sigma^2)$, find $P(\sqrt{11}\left|\overline{X}-\mu\right| \le 1.796\, S)$.

4. Let $X_1, \dots, X_n, X_{n+1}, X_{n+2}$ be i.i.d. $N(\mu, \sigma^2)$ and let $\overline{X}$ and S^2 denote the sample mean and sample variance based on $X_1, \dots, X_n$. Find the distribution of $\sqrt{\dfrac{n}{2(n+2)}}\left(\dfrac{X_{n+1}+X_{n+2}-2\overline{X}}{S}\right)$.

5. Let $X_1, \dots, X_m$ be a random sample from $N(\mu_1, \sigma^2)$ population and $Y_1, \dots, Y_n$ be another independent random sample from $N(\mu_2, \sigma^2)$ population. Let $\overline{X}, \overline{Y}, S_1^2, S_2^2$ be the sample means and sample variances based on X and Y-samples respectively. Determine the distribution of
$$U = \frac{2(\overline{X}-\mu_1)+3(\overline{Y}-\mu_2)}{S\sqrt{(4/m+9/n)}}, \text{ where } S^2 = \frac{(m-1)S_x^2+(n-1)S_y^2}{(m+n-2)}.$$

6. Consider two independent samples- the first of size 12 from a normal population with variance 6 and the second of size 6 from a normal population with variance 3. Compute the probability that the sample variance from the second sample exceeds the one from the first.

7. The temperature at which certain chemical reaction takes place is normally distributed with variance σ^2. A random sample of size n has the sample variance S^2. How many observations are required to ensure that $P(S^2/\sigma^2 \le 1.8) \ge 0.99$?

8. Let X_1, X_2, X_3, X_4 be independent $N(0, \sigma^2)$ random variables. Find $P(X_1^2+X_2^2+X_3^2+X_4^2 \le \sigma^2)$.

9. Let $X_1, \dots, X_n$ be a random sample from $N(\mu, \sigma^2)$ population. For $1 < k < n$, define
$$U = \frac{1}{k}\sum_{i=1}^{k} X_i,\ V = \frac{1}{n-k}\sum_{i=k+1}^{n} X_i,\ S^2 = \frac{1}{k-1}\sum_{i=1}^{k}(X_i-U)^2,\ T^2 = \frac{1}{n-k-1}\sum_{i=k+1}^{n}(X_i-V)^2.$$

Find the distributions of

$$W_1 = U - V, W_2 = \frac{(k-1)S^2 + (n-k-1)T^2}{\sigma^2}, W_3 = \frac{S^2}{T^2}, W_4 = \frac{\sqrt{k}(U-\mu)}{T},$$

$$W_5 = \frac{\sqrt{(n-k)}(V-\mu)}{T} \text{ and } W_6 = \sqrt{\frac{k(n-k)}{n(n-2)W_2}}(U-V).$$