Probability and Statistics
Hints/Solutions to Test Set 1

1. Find $\liminf_n E_n$ and $\limsup_n E_n$, if E_n is given by

   (i) $\liminf_n E_n = \phi$, $\limsup_n E_n = \phi$. Hence $\lim_n E_n$ exists.

   (ii) $\liminf_n E_n = (0, 1)$, $\limsup_n E_n = (0, 1)$. Hence $\lim_n E_n$ exists.

   (iii) $\liminf_n E_n = \{0, 1\}$, $\limsup_n E_n = \{x : x \text{ is rational in } [0, 1]\}$. Hence $\lim_n E_n$ does not exist.

   (iv) $\liminf_n E_n = \left\{(x, y) : \frac{1}{4} \le x \le \frac{3}{4}, y = \frac{1}{2}\right\}$,

   $\limsup_n E_n = \left\{(x, y) : 0 \le \frac{3}{4}, \frac{1}{2} \le y \le 1\right\} \bigcup \left\{(x, y) : \frac{1}{4} \le x \le 1, 0 \le y \le \frac{1}{2}\right\}$.

2. Prove or disprove the following statements:

   (i) It is a ring. Use definition to prove the statement.

   (ii) Not necessarily a ring. Example: $\Pi = \{\phi, \{1\}, \{2\}\}$.

   (iii) It is a ring. Use definition to prove the statement.

   (iv) Not necessarily a σ-ring.

3. It is an algebra. Use definition to prove the statement.

4. Use definition to prove the first statement. Example for the second part:

   Let $□_1 = \{\phi, \{a\}\}$, $□_2 = \{\phi, \{b\}\}$. Then $□_1$ and $□_2$ are rings but $□_1 \bigcup □_2$ is not a ring.

5. Use definition to prove the statement.

6. Identify the following classes of subsets of X (X is an uncountable set) as ring and/or algebra:

   (i) ring and algebra

   (ii) $\{\phi, A, X\}$ ring but not algebra

   (iii) $\{\phi, A, A^C, X\}$ ring and algebra

   (iv) $P(X)$ ring and algebra

   (v) The class of all finite subsets of X is ring but not algebra

   (vi) The class of all countable subsets of X is a ring but not algebra.