Probability and Statistics
Hints/Solutions to Test Set 2

1. Required probability $= \dfrac{6\times 5\times \binom{6}{2}\times 4!}{6^6} = \dfrac{25}{108}$.
2. Use definitions.
3. Let A → 7 appears, B → even number appears. Then
   A = {(1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1)}. Also B will have 18 elements. So
   $P(A) = \frac{1}{6}, P(B) = \frac{1}{2}$. Now the P(7 appears first) $= \frac{1}{6} + \frac{1}{3}.\frac{1}{6} + \left(\frac{1}{3}\right)^2.\frac{1}{6} + ... = \frac{1}{4}.$.
4. Given P(Car) = 0.65, P(Computer) = 0.35, and $P(\text{Car} \cap \text{Computer}) = 0.25$.
   The required probability $= P(\text{Car} \cap \text{Comp}^C) + P(\text{Car}^C \cap \text{Comp})$
   $= P(\text{Car}) - P(\text{Car} \cap \text{Comp}) + P(\text{Comp}) - P(\text{Car} \cap \text{Comp}) = 0.5$
5. Let A → person drank regularly, D → death due to liver disease. Given P(A) = 0.25, P(D) = 0.005 and $P(D|A) = 6\, P(D|A^C) = 6\,p$, say. Use theorem of total probability to get 6 p = 1/ 75.
6. Use addition rule and the definition of the conditional probability.
7. 0.3.
8. (i) Use Bayes theorem, Reqd prob. $= \frac{5}{8}$.
   (ii) Use theorem of total probability. Reqd prob. $= \frac{23}{120}$.
   (iii) Use theorem of total probability,
   P(digit 1 was received) $= \frac{1}{5}$,
   P(digit 2 was received) $= \frac{79}{240}$,
   P(digit 3 was received) $= \frac{113}{240}$.
9. Let C → an automobile policyholder makes a claim,
   M → automobile policyholder is male
   F → automobile policyholder is female
   Given P(M) = α, P(F) = 1 – α., 0 < α < 1. $P(C|M) = p_m$, $P(C|F) = p_f$.
   Now $P(A_1) = P(C \mid M)\, P(M) + P(C \mid F)\, P(F) = \alpha\, p_m + (1 - \alpha)\, p_f$.
   $$P(A_2 \mid A_1) = \frac{P(A_2 \cap A_1)}{P(A_1)} = \frac{\alpha p_m^2 + (1-\alpha) p_f^2}{\alpha p_m + (1-\alpha) p_f}.$$
10. (i) False (ii) True (iii) False (iv) False

11. $\binom{13}{4}\Big/\binom{52}{4}$.

12. Count the cases to find the required probability as 43/216.

13. P(getting all 'Ex' in one semester) = $\frac{1}{2^5}=\frac{1}{32}$. Reqd. prob. $=1-\left(\frac{31}{32}\right)^{10}=0.272$.

14. Use definition.

15. $P(X=i)=\binom{n}{i}\Big/2^n$, $i=0,1,\ldots,n$. $P(A\subset B\mid X=i)=2^{i-n}$. $P(A\subset B)=\left(\frac{3}{4}\right)^n$.

Also $P(A\cap B=\phi)=P(A\subset B^C)$.

16. P(student scores at least 9 marks) = P(scores 9 marks) + P(scores 10 marks) = $\frac{7}{2^{11}}$

17. 7/17, 6/17, 4/17.

18. Let X $\rightarrow$ number of tails, Y $\rightarrow$ number of heads.

$$\begin{aligned}
p &= P(X>Y)=P(X=n+1)+P(X=n+2)+\ldots+P(X=2n)\\
&=\left(\frac{1}{2}\right)^{2n}\left[\binom{2n}{n+1}+\ldots+\binom{2n}{2n}\right]=\left(\frac{1}{2}\right)^{2n}\left[\binom{2n}{0}+\ldots+\binom{2n}{n-1}\right]\\
&=P(Y>X)
\end{aligned}$$

$$r=P(X=Y)=\binom{2n}{n}\left(\frac{1}{2}\right)^{2n}.$$

As $2p+r=1$, we get $p=\frac{1}{2}\left\{1-\left(\frac{1}{2}\right)^{2n}\binom{2n}{n}\right\}$.