

Probability and Statistics
Hints/Solutions to Test Set 5

1. Integrating $f(x)$ over the range we get $b = 2/9$. $E(X)$ and $V(X)$ can be easily calculated. To find the distribution of Y , note that the range of Y is 0 to 4. For $0 \leq y \leq 1$, there are two inverse images \sqrt{y} and $-\sqrt{y}$. For $1 < y < 4$, there is one inverse image \sqrt{y} . Using these we get the density of Y as

$$\begin{aligned} f_Y(y) &= \frac{2}{9} y^{-1/2}, & 0 \leq y \leq 1, \\ &= \frac{1}{9} (1 + y^{-1/2}), & 1 < y < 4, \\ &= 0, & \text{elsewhere.} \end{aligned}$$

2. Integrating $f(x)$ over the range we get $k = 1/2$. Median and quartiles can be evaluated using direct calculations. For the distribution of Y , the argument is same as in Problem 1.
3. Here also Y is two- to-one function of X in the given domain.

Problem 4 to 7 are straightforward..

8. We get $k = 1/4$. The density of Y is obtained as

$$\begin{aligned} f_Y(y) &= \frac{1}{2}, & 0 \leq y \leq 1, \\ &= \frac{3-y}{4}, & 1 < y \leq 3, \\ &= 0, & \text{elsewhere.} \end{aligned}$$