Probability and Statistics
Hints/Solutions to Test Set 7

1. $p_{U,V}(1,1)=\frac{11}{60}, p_{U,V}(1,2)=\frac{3}{8}, p_{U,V}(2,1)=\frac{1}{8}, p_{U,V}(2,2)=\frac{19}{60}.$

2. $Z^2=(X_1-X_2)^2+(Y_1-Y_2)^2.$ $\frac{X_1-X_2}{\sqrt{2}}\sim N(0,1), \frac{Y_1-Y_2}{\sqrt{2}}\sim N(0,1).$ So $\frac{Z^2}{2}\sim\chi_2^2.$
   Hence Z^2 has a negative exponential distribution with mean 4.

3. $f_{U,V}(u,v)=\frac{1}{\sigma^2}ve^{-v/\sigma}.\frac{1}{(1+u)^2}, u>0, v>0.$ The densities of U and V can be derived easily now.

4. $f_{Y_1,Y_2}(y_1,y_2)=\frac{1}{2}e^{-y_1/2}.\frac{1}{\pi(1+y_2)^2}, y_1>0, y_2\in\mathbb{R}$. Clearly Y_1 and Y_2 are independent.

5. Similar. Y and Z are independent. Z and U are independent.

6. Similar. Y_1, Y_2 and Y_3 are independent.

7. Consider ln P and use linearity property of independent normal variables. Now use the symmetry of normal distribution to get values of L_1 and L_2.

8. Use the properties of a bivariate normal population.

9. Use the linearity property of normal distribution.