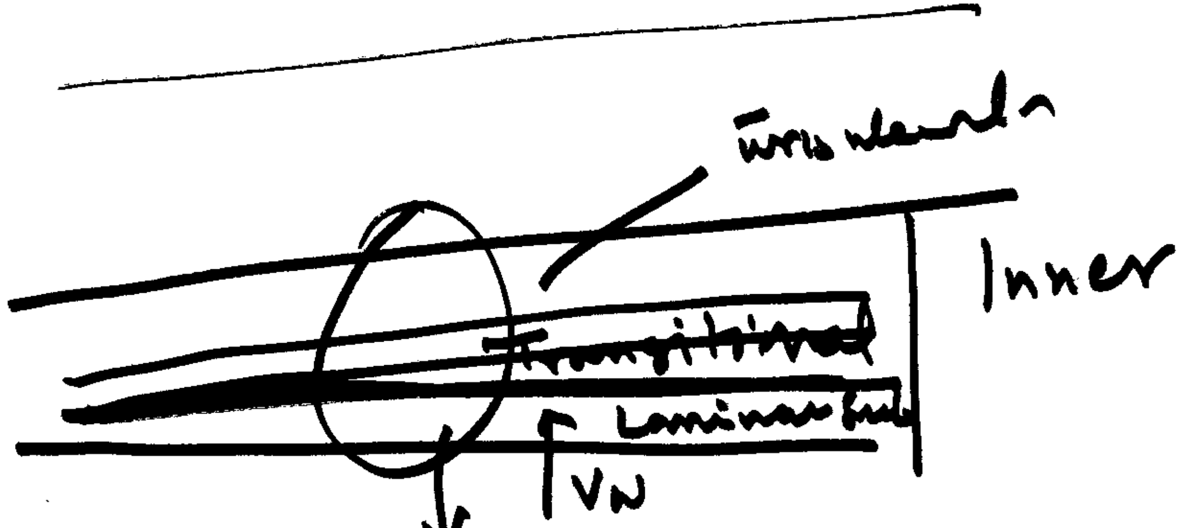
Prof. A.W. Date
Lec. No. 24
Date: 2/3/11



$$u\frac{\partial u}{\partial x} \simeq 0$$

$$v\frac{\partial u}{\partial y} \simeq V_W\frac{\partial u}{\partial y}$$

$$\tau_{tot} = \tau_w = \tau_l + \cancel{\tau_t}^{\,0} = \mu \frac{\partial u}{\partial y}\Big|_{y=0}$$

$$u = \frac{\tau_w}{\mu} y + C \qquad u = 0 \text{ at } y = 0 \quad \therefore C = 0$$

$$u = \frac{\tau_w \, y}{\mu} \times \frac{\rho}{\rho}$$

$$\frac{u}{u_\tau} = \frac{u_\tau^2}{u_\tau} \cdot \frac{y}{\nu} = \frac{y u_\tau}{\nu}$$

$$\underline{\underline{u^+ = y^+}} \text{ in } \underline{\text{LSL}}$$

$f = 2\frac{u_\tau^2}{\bar{u}^2}$

$\sqrt{\frac{f}{2}} = \frac{u_\tau}{\bar{u}}$

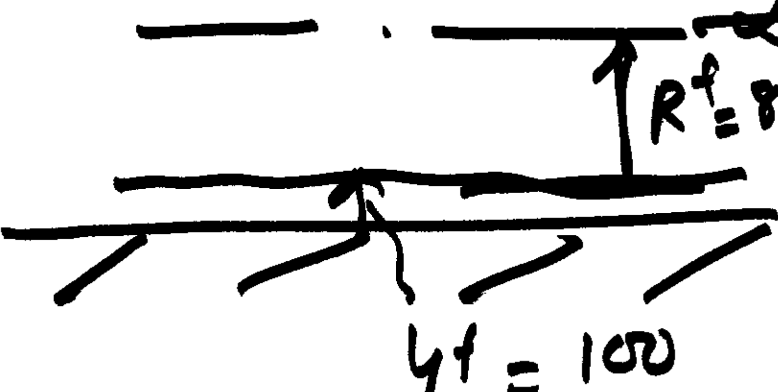
$f = \frac{\tau_w}{\rho u_0^2/2} = .046\, Re^{-0.2}$

30,000

$\frac{2u_\tau^2}{\bar{u}^2} = .046\left[\frac{\bar{u}.D}{\nu}\right]^{-.2}$

$= .046\left[\frac{2\bar{u}}{\nu}.\frac{R u_\tau}{\nu}.\frac{\nu}{u_\tau}\right]^{-.2}$

$= .046\left[2\frac{\bar{u}}{u_\tau}.R^+\right]^{-.2}$



$2\left(\frac{u_\tau}{\bar{u}}\right)^{1.8} = .046\times2^{-.2}.R^{+-.2}$

$2\left(\frac{f}{2}\right)^{.9} = .046\times2^{-.2}\,R^{+-.2}$

$\frac{2^{1.9}}{.046}.\left(\frac{f}{2}\right)^{.9} = R^{+-.2}$

$\frac{y^+}{R^+} \simeq \frac{100}{800}$ or $Re = 309000$ $R^+ \simeq 811$

$\simeq \frac{100}{1000}$

$u^+ = F(y^+)$ for the turbulent layer

$$\frac{1}{u_\tau}\frac{\partial u}{\partial y} = \frac{\partial F}{\partial y^+}\cdot\frac{\partial y^+}{\partial y} = \frac{\partial F}{\partial y^+}\cdot\frac{u_\tau}{\nu}.$$

$$\therefore \frac{\partial u}{\partial y} = \frac{u_\tau^2}{\nu}\cdot\left[\frac{\partial F}{\partial y^+}\right] \qquad \mu_t >> \mu$$

$$\therefore \frac{\partial F}{\partial y^+} \propto \frac{\nu}{u_\tau y} \propto \frac{1}{y^+}$$

$$\frac{\partial u^+}{\partial y^+} \propto \frac{1}{y^+} \rightarrow u^+ = \frac{1}{K}\ln y^+ + C$$

$$\frac{\partial u}{\partial y} \propto \frac{u_\tau}{y} \qquad y^+ = u_\tau \cdot \frac{y}{\nu}$$

$$\frac{\partial u}{\partial y} = \frac{1}{K_{tr}} \frac{u_\tau}{y}$$

$$\cancel{u = \frac{u_\tau}{K_{tr}} \cdot \ln y + C_{tr}}$$

$$\frac{\partial u^+ \cdot u_\tau}{\partial y^+ \cdot \frac{\nu}{u_\tau}} = \frac{1}{K_{tr}} \cdot \frac{u_\tau}{y^+ \frac{\nu}{u_\tau}}$$

$$\frac{\partial u^+}{\partial y^+} = \frac{1}{K_{tr}} \cdot \frac{1}{y^+}$$

$$\underset{\text{LSL}}{u^+ = y^+} \xrightarrow{\;y^+\;} \boxed{u^+ = \frac{1}{K_{tr}} \ln y^+ + C_{tr}}$$

Transitional layer