Prof. Date
Lec - 5
Date - 16-3-10

$$LHS = u^* \frac{\partial T^*}{\partial x^*} + v^* \frac{\partial T^*}{\partial y^*}$$

$$= 1 \frac{1}{1} + \delta^* \frac{1}{\delta^*}$$

$$RHS: = \frac{1}{RePr}\left[\cancel{\frac{1}{1^2}} + \frac{1}{\delta^{*2}}\right]$$

$$\frac{\partial^2 T^*}{\partial x^{*2}} \ll \frac{\partial^2 T^*}{\partial y^{*2}}$$

$$P_r = \frac{\mu C_p}{K} = \frac{\mu/\rho}{(K/\rho C_p)}$$

$$= \frac{\nu}{\alpha} = \frac{\text{Kinematic Vis}}{\text{Thermal Diff}}$$

$$T^* = \frac{T - T_\infty}{\Delta T_0}$$

$$\frac{\rho_m C_{pm} \Delta T_0 U_\infty}{L}\left[u^* \frac{\partial T^*}{\partial x^*} + v^* \frac{\partial T^*}{\partial y^*}\right]$$

$$= \frac{k \cdot \Delta T_0}{L^2}\left[\frac{\partial^2 T^*}{\partial x^{*2}} + \frac{\partial^2 T^*}{\partial y^{*2}}\right]$$

$$+ \frac{\mu \cdot U_\infty^2}{L^2}\left[2\left(\frac{\partial u^*}{\partial x^*}\right)^2 + 2\left(\frac{\partial v^*}{\partial y^*}\right)^2 + \left(\frac{\partial u^*}{\partial x^*} + \frac{\partial v^*}{\partial y^*}\right)^2\right]$$

$$\frac{\mu U_\infty^2}{\cancel{L^2}\, L} \cdot \frac{\cancel{L}}{\rho_m c_{p,m} \Delta T_0 U_\infty}$$

$$\frac{U_\infty^2}{c_{p_m} \Delta T_0} \cdot \frac{\mu}{\rho_m U_\infty L} \qquad E_c \equiv \frac{U_\infty^2}{c_{p_m} \Delta T_0}$$

$$\cdot \frac{1}{Re}$$

$$U_\infty^2 = \left(\frac{m^2}{s^2}\right)$$

$$c_{p_m} \Delta T = \frac{J}{kg \cdot K} \cdot K$$

$$= \frac{J}{kg} = \frac{N \cdot m}{kg} = \frac{\cancel{kg} \cdot m/s^2 \cdot m}{\cancel{kg}}$$

$$= m^2/s^2$$

$$u^* \frac{\partial T^*}{\partial x^*} + v^* \frac{\partial T^*}{\partial y^*}$$

$$= \frac{k_m \cancel{\Delta T_0}}{L^{\cancel{2}}} \times \frac{\cancel{L}}{\rho_m c_m \cancel{\Delta T_0} U_\infty}$$

$$= \frac{\alpha_m}{L U_\infty}$$

$$= \frac{\alpha_m}{\nu_m} \cdot \frac{\nu_m}{L U_\infty}$$

$$= \frac{1}{Pr} \cdot \frac{1}{Re}$$

$$\cancel{\rho_m \frac{\partial h_m}{\partial t}} + \rho_m \left[u \frac{\partial h_m}{\partial x} + v \frac{\partial h_m}{\partial y} \right] = \rho_m c_{pm} \left[u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right]$$

$$= k \left[\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right] - \frac{\partial}{\partial x} \left[\cancel{\Sigma \dot{m}''_{k} h_k} \right] - \frac{\partial}{\partial y} \left[\cancel{\Sigma \dot{m}''_{k} h_k} \right]$$

$$+ \mu \left[2 \left(\frac{\partial u}{\partial x} \right)^2 + 2 \left(\frac{\partial v}{\partial y} \right)^2 + \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right)^2 \right]$$

$$+ \frac{\partial p}{\partial t} + u \frac{\partial p}{\partial x} + v \frac{\partial p}{\partial y} + \underline{\underline{\dot{Q}_{chem}}} + \dot{Q}_{rad.}$$

$$c_{pk} = c_{pm}$$

$$h_k = h^0_{f,k} + c_{pm} (T - T_{ref})$$

$$h_m = \Sigma \omega_k h_k = \Sigma \omega_k h^0_{fk} + c_{pm} (T - T_{ref}) \underbrace{\Sigma \cancel{\omega_k}}_{=1}$$

$$= \Sigma \omega_k h^0_{fk} + c_{pm} (T - T_{ref})$$

$$= \Delta H_c + c_{pm} (T - T_{ref})$$

$$h_m = c_{pm} (T - T_{ref})$$

$$u^*\frac{\partial v^*}{\partial x^*}+v^*\frac{\partial v^*}{\partial y^*}=-\frac{\partial p^*}{\partial y^*}+\frac{1}{Re}\left[\frac{\partial^2 v^*}{\partial x^{*2}}+\frac{\partial^2 v^*}{\partial y^{*2}}\right]$$

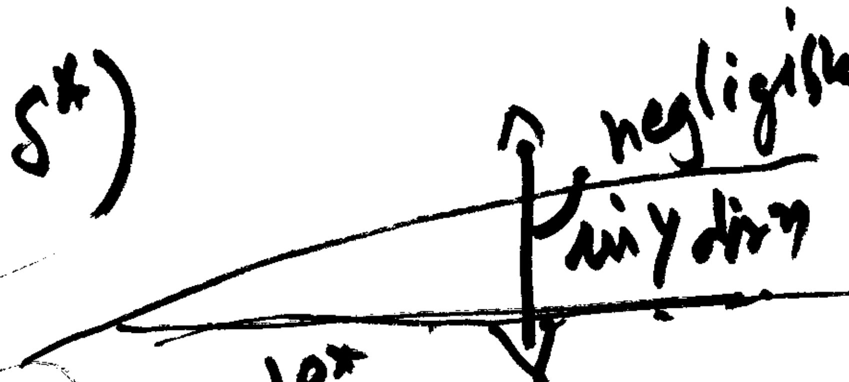
$$\underbrace{1\,\frac{\delta^*}{1}+\delta^*\,\frac{\delta^*}{\delta^*}}_{O(\delta^*)}=\qquad +\delta^{*2}\left[\frac{\delta^*}{1}+\frac{\delta^*}{\delta^{*2}}\right]$$

$$+\delta^{*3}\left[1+\frac{1}{\delta^{*2}}\right]$$

$$+O(\delta^*)$$

$$\frac{\partial p^*}{\partial y^*}\simeq O(\delta^*)$$

$$=0$$



$$\frac{\partial p^*}{\partial x^*}=-\frac{dp^*}{dx^*}=-\frac{dp^*_\infty}{dx^*}$$

$$=-\frac{dp^*_w}{dx}$$

$$u^* \frac{\partial u^*}{\partial x^*} + v^* \frac{\partial u^*}{\partial y^*} = -\frac{\partial p^*}{\partial x^*} + \boxed{\frac{1}{Re}\left[\cancel{\frac{\partial^2 u^*}{\partial x^{*2}}} + \frac{\partial^2 u^*}{\partial y^{*2}}\right]}$$

$$O\{1\} + O\{1\} = \quad O(1) \qquad \frac{1}{\delta^{*2}}$$

$$\frac{1}{Re} \sim O(\delta^2)$$

$$Re \propto \frac{1}{\delta^2}$$

$$O(1)$$

$$\rho\left[u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right]=-\frac{\partial p}{\partial x}+\mu\left[\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right]$$

$$\frac{\rho U_\infty^2}{L}\left[u^*\frac{\partial u^*}{\partial x^*}+v^*\frac{\partial u^*}{\partial y^*}\right]=-\frac{\rho U_\infty^2}{L}\frac{\partial p^*}{\partial x^*}+\frac{\mu U_\infty}{L^2}\left[\frac{\partial^2 u^*}{\partial x^{*2}}+\frac{\partial^2 u^*}{\partial y^{*2}}\right]$$

$$u^*\frac{\partial u^*}{\partial x^*}+v^*\frac{\partial u^*}{\partial y^*}=-\frac{\partial p^*}{\partial x^*}+\left(\frac{\mu U_\infty}{L^2}\cdot\frac{L}{\rho U_\infty^2}\right)\left[\frac{\partial^2 u^*}{\partial x^{*2}}+\frac{\partial^2 u^*}{\partial y^{*2}}\right]$$

$$\frac{\mu}{\rho U_\infty L}=\frac{1}{Re}$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$\frac{u_\infty}{L}\frac{\partial u^*}{\partial x^*} + \frac{u_\infty}{L}\frac{\partial v^*}{\partial y^*} = 0$$

$$\frac{u_\infty}{L}\left[\frac{\partial u^*}{\partial x^*} + \frac{\partial v^*}{\partial y^*}\right] = 0$$

$$\frac{\partial(\rho_m u_i)}{\partial t} + \frac{\partial}{\partial x_j}(\rho_m u_j u_i)$$

$$= \cancelto{0}{\rho_m \frac{\partial u_i}{\partial t}} + \rho_m u_j \frac{\partial u_i}{\partial x_j} + \rho_m u_i \cancelto{0}{\frac{\partial u_j}{\partial x_j}}$$

$$= \rho_m u_j \frac{\partial u_j}{\partial x_j}$$

$$= \rho_m \left[u_1 \frac{\partial u_1}{\partial x_1} + u_2 \frac{\partial u_1}{\partial x_2} \right] \text{ — in } x_1\text{-dir}^n$$

$$= \rho_m \left[u_1 \frac{\partial u_2}{\partial x_1} + u_2 \frac{\partial u_2}{\partial x_2} \right] \text{ — in } x_2\text{-dir}^n$$

$$\frac{\partial}{\partial x_j}\left[\mu \frac{\partial u_i}{\partial x_j}\right]$$

$$= \mu \frac{\partial^2 u_i}{\partial x_j^2}$$

$$= \mu \left[\frac{\partial^2 u_1}{\partial x_1^2} + \frac{\partial^2 u_1}{\partial x_2^2}\right]$$

$$= \mu \left[\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right]$$

$$\frac{\partial}{\partial x_j}\left[\mu \frac{\partial u_j}{\partial x_i}\right]$$

$$= \mu \frac{\partial^2 u_j}{\partial x_j \partial x_i}$$

$$= \mu \frac{\partial}{\partial x_i}\left[\cancelto{0}{\frac{\partial u_j}{\partial x_j}}\right]$$