2 Laminar Boundary Layers- l5 to l13

2.1 Similarity Solutions

1. Starting with Falkner-Skan equation $z^{'''} + z\,z^{''} + \beta\,(1 - z^{'2}) = 0$, show that

$$f^{'''} + 0.5\,(1+m)\,f\,f^{''} + m\,(1 - f^{'2}) = 0 \qquad (1)$$

where $f^{'}\,(\eta) = z^{'}\,(\overline{\eta}) = (u/U_\infty)$, $\eta = y\,\sqrt{U_\infty/(\nu\,x)} = \sqrt{(2-\beta)}\,\overline{\eta}$ and m $= \beta/(2-\beta)$. Hence, show that

   (a) the distribution of transverse velocity v is given by

$$\frac{v}{U_\infty}\,Re_x^{0.5} = -\,(\frac{m+1}{2})\left\{f + (\frac{m-1}{m+1})\,\eta\,f^{'}\right\}$$

   (b) Boundary conditions are: f (0) = - 2 $B_f/(1+m)$ = constant, $f^{'}\,(0) = 0$ and $f^{'}\,(\infty) = 1$ where $B_f = (v_w/U_\infty)\,Re_x^{0.5}$ and $Re_x = U_\infty\,x/\nu$.

2. Show that the enrgy eqn derives to

$$\theta^{''} + Pr\,\left[(\frac{m+1}{2})\,f\,\theta^{'} - \gamma\,f^{'}\,\theta + 2\,Ec_x\,(f^{''})^2\right] = 0 \qquad (2)$$

where Ec $= (U_\infty^2(x)/2)/(Cp\,\Delta T_{ref}\,x^\gamma)$ = constant. If Ec $\neq 0$, $\gamma = 2$m and the boundary conditions are $\theta(0) = 1$ and $\theta(\infty) = 0$

3. Write a computer program to solve equations 1 and 2 using the *shooting method* described in lectures l7 and l8.

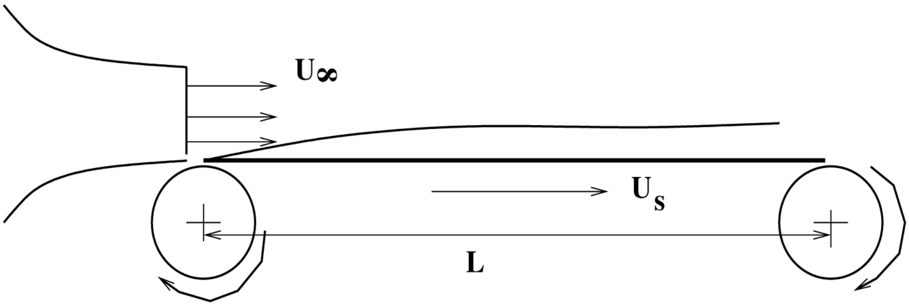

Figure 4: Boundary Layer Development over a Moving Surface

4. Consider development of a laminar boundary layer on a *flat plate* which is moving in the streamwise direction at the rate U_s while the free-steam velocity U_∞ = constant (see figure 4). Show that for this case, similarity equation 1 is applicable with m = 0 such that U_∞ is repalced by relative velocity $U_{rel} = \mid U_\infty - U_s \mid$ (see, Abraham and Sparrow, International Journal of Heat and Fluid Flow, vol 26, (2005), p 289295). Also, write the applicable boundary conditions at $\eta = 0$ and ∞. Adapt the shooting-method computer program to determine $f''(0)$ for $0 < (U_\infty/U_s) < \infty$. Can $f''(0) < 0$? Does this imply flow separation ?

5. The equation governing development of thermal boundary layer in a low speed incompressible flow with uniform properties is given by

$$\rho C_p \left[u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} \right] = k\,\frac{\partial^2 T}{\partial y^2} + \mu\,(\frac{\partial u}{\partial y})^2 \tag{3}$$

   (a) Consider the case of $U_\infty = C \times x$ and T_w = constant and V_w = 0. Examine if a similarity solution is possible in this case. How should T_w vary with x for similarity solutions to be possible ?

   (b) Now, neglect viscous dissipation and assume U_∞ = constant, V_w = 0 and q_w = constant. Is similarity solution possible ?

   (c) Consider case of Pr << 1 and $V_w = 0$ with T_w = constant and $U_\infty = C \times x$. Making suitable approximations, show that $Nu_x = K \times \sqrt{Re_x\, Pr}$ and determine the value of K

(d) Water flows over a flat plate with $U_\infty = 0.3$ m/s. Determine the location (x_{trans}) of the transition point if $(U_\infty\, \delta_1/\nu)_{transition} = 645$. Plot variation of $\tau_{w,x}$ and h_x upto $x = x_{trans}$. Evaluate fluid properties at 30^0C.

2.2 Integral Solutions

1. Consider development of laminar boundary layer on a flat plate without suction or blowing. Evaluate δ_1/δ, δ_2/δ , δ_4/δ and v/U_∞ when following velocity profiles are assumed:

$$\begin{aligned}
\frac{u}{U_\infty} &= \eta \\
&= 2\,\eta - \eta^2 \\
&= \sin\,(\frac{\pi}{2} \times \eta) \\
&= 2\,\eta - 2\,\eta^3 + \eta^4
\end{aligned}$$

where $\eta = (y/\delta)$. In each case, compare your evaluations with the similarity solutions. Which profile assumption mimics the similarity solution best ?

2. Consider laminar boundary layer development of liquid metal over a flat surface with $V_w = \mathrm{Ec} = 0$. *Constant Wall Heat Flux* is applied at the surface. Assume 4th order polynomial for temperature profile and $(u/U_\infty) = 1$. Show that $(\Delta/\delta) \simeq 0.553\, Pr^{-0.5}$.

3. Repeat the above problem for a stagantion point flow and show that $St_x \simeq 0.7746\ (Re_x\, Pr)^{-0.5}$.

4. Show that the velocity profile for a laminar boundary layer with rbitrary variation of $V_w(x)$ and $U_\infty(x)$ can be approximated as

$$\frac{u}{U_\infty} = 3\,\eta + \eta^3 - 3\,\eta^2 + e\,(\eta^4 - 3\,\eta^3 + 3\,\eta^2 - \eta) \tag{4}$$

$$
\begin{aligned}
e &= \frac{3\,V_w^* - \lambda + 6}{6 + V_w^*} & \eta = \frac{y}{\delta} \qquad (5)\\
V_w^* &= \frac{v_w\,\delta}{\nu} & \lambda = \frac{\delta^2}{\nu}\,\frac{dU_\infty}{dx} \qquad (6)
\end{aligned}
$$

5. Consider laminar boundary layer development of a fluid (Pr $= 10$) with $U_\infty = C\,x^{0.333}$. Assume $V_w =$ Ec $= 0$ and $T_w =$ const. For this case, $St_x = K\,Re_x^{-0.5}$. Estimate the value of K from both the Similarity and Integral methods.

6. Air at atmospheric pressure flows past a surface such that $U_\infty = 2\,(1 - 0.236\,x)$ m/s where x is measured in meters. Find the location of the separation point and the value of the heat transfer coefficient at that point. Evaluate fluid properties at 50^0C.

7. For a flow across a cylinder, the free-stream velocity varies as $U_\infty = 2\,V_a\,\sin{(\theta)}$ where V_a is the approach velocity and θ is measured from the forward stagnation point. Assuming $T_w =$ const and Pr $=$ 0.7, calculate and plot $Nu_\theta = h_\theta\,R/k$ for $\theta \leq \pi/2$.
Hence, show that $Nu_{\theta=0} \simeq 0.7\,(V_a\,R/\nu)^{0.5}$