ME-662 CONVECTIVE HEAT AND MASS TRANSFER

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LECTURE-14 LAMINAR INTERNAL FLOWS

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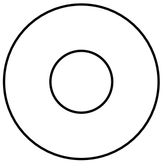
1. Relevance
2. Important Definitions
3. Prediction of Developing Flow

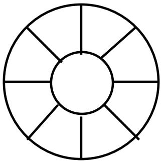
Relevance - L14($\frac{1}{17}$)

1. In Heat Exchangers, it is important to have knowledge of pressure drop (or friction factor f) and heat transfer coefficient (or Nusselt Number Nu) on the *Tube Side* to facilitate their design
2. Modern Heat exchangers employ ducts of both Circular and Non-Circular cross-section . Sometimes Curved Ducts are preferred or are necessitated to conserve space.
3. Duct passages with Internal Insertions such as Twisted tape or Coils are also popular. Optimally Internally Structured Surfaces such as rib-roughnesses, grooves and indentations are used for augmentation of Nu
4. Solution of Transport Equations of mass, momentum and energy provide means for obtaining f and Nu. In simple ducts, analytical solutions are possible. In more complex ones, CFD solutions become necessary.

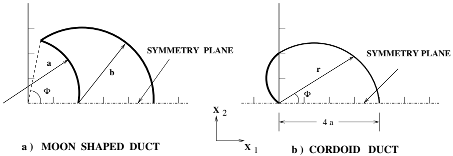
Non-circular Cross-Sections - L14($\frac{2}{17}$)



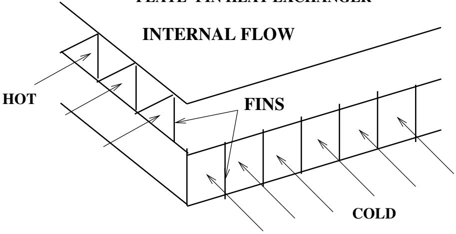
ANNULUS



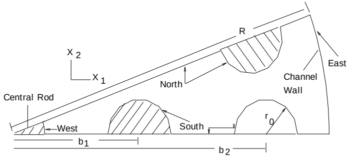
ANNULAR SECTOR DUCT



PLATE–FIN HEAT EXCHANGER

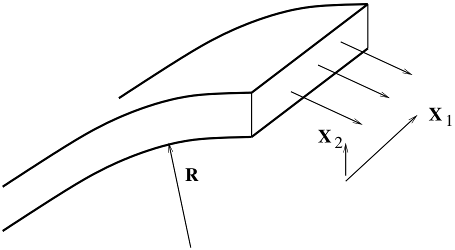


SQAURE AND TRIANGULAR CROSS–SECTION

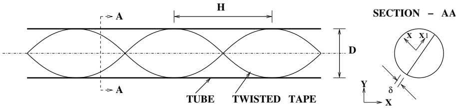


Nuclear Rod Cluster

Curved Ducts - L14($\frac{3}{17}$)

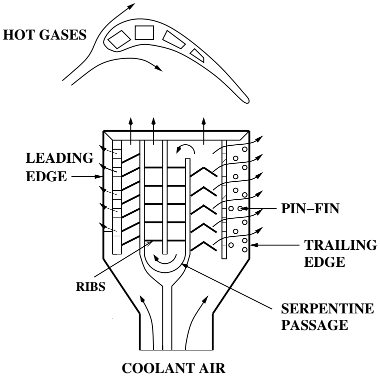


SPIRAL PLATE HEAT EXCHANGER

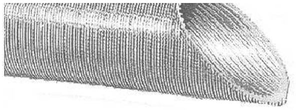


TUBE WITH A TWISTED TAPE INSERT

Internally Structured Surfaces L14($\frac{4}{17}$)

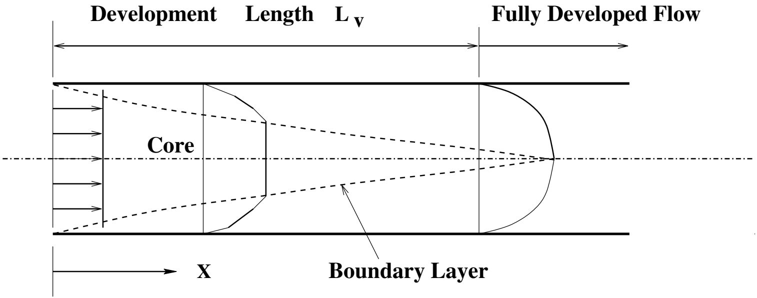


Non-circular ducts with Internal Ribs, Pin-Fins and Wall Perforations



Internally and Externally Spiral Groove Tube

F D and Developing Flows - L14($\frac{5}{17}$)



1. It is of interest to determine L_v = F (Re)
2. Analytical treatment difficult except in simple cases (example follows)
3. Fully-developed flow is identified with $\partial u/\partial x$ = 0 and dp/dx = const.

Simple Developing Flow - L14($\frac{6}{17}$)

1. Consider laminar flow between infinite parallel plates separated by distance 2b.
2. In the entrance region , using BL approximations, the governing eqns and Boundary conditions are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u\,\frac{\partial u}{\partial x} + v\,\frac{\partial u}{\partial y} = -\frac{1}{\rho}\,\frac{d\,p}{d\,x}(x) + \nu\,\frac{\partial^2 u}{\partial y^2} \tag{2}$$

$$u\,(0, y) = \overline{u} \quad , \quad v(0, y) = 0 \;\; \text{(inlet)} \;\; \overline{u} = \frac{1}{b}\int_0^b u\,dy$$

$$\frac{\partial u}{\partial y}\,(x, b) = 0 \quad , \quad v\,(x, b) = 0 \;\; \text{(symmetry)}$$

$$u\,(x, 0) = 0 \quad , \quad v(x, 0) = 0 \;\; \text{(plate wall)} \tag{3}$$

Dimensionless Eqns - L14($\frac{7}{17}$)

$$\frac{\partial u^*}{\partial x^*} + \frac{\partial v^*}{\partial y^*} = 0 \qquad (4)$$

$$Re\left[\frac{\partial(u^*\, u^*)}{\partial x^*} + \frac{\partial(u^*\, v^*)}{\partial y^*}\right] = -Re\,\frac{d\,p^*}{d\,x^*} + \frac{\partial^2 u^*}{\partial {y^*}^2} \qquad (5)$$

$$u^* = \frac{u}{\overline{u}} \quad v^* = \frac{v}{\overline{u}} \qquad p^* = \frac{p}{\rho\,\overline{u}^2} \qquad (6)$$

$$x^* = \frac{x}{D_h} \quad y^* = \frac{y}{D_h} \qquad (7)$$

$$Re = \frac{\overline{u}\,D_h}{\nu} \quad D_h = 4b \qquad (8)$$

Eqn 5 shows that pressure drop in the duct-entrance-length is caused by viscous friction as well as momentum change caused by changes in velocity profiles

Solution by Linearisation - L14($\frac{8}{17}$)

1. Analytical solutions are not possible because of the copupling involved in non-linear convection terms. Therefore, following Langhaar[1], let

$$Re\left[\frac{\partial(u^*\,u^*)}{\partial x^*}+\frac{\partial(u^*\,v^*)}{\partial y^*}\right]=\beta^2(x^*)\,u^* \tag{9}$$

2. Hence, the momentum eqn can be written as

$$\frac{\partial^2 u^*}{\partial {y^*}^2}-\beta^2\,u^*=Re\,\frac{d\,p^*}{d\,x^*} \tag{10}$$

where $d\,p^*/d\,x^* = f_l$ the local Fanning Friction factor.

[1] Langhaar H , *Steady Flow in the Transition Length of a Straight Tube*, J Appl Mech, vol 9, p 55-58, (1942)

Further Manipulations - I L14($\frac{9}{17}$)

1. To make further progress, Define

$$u^{'} = u^{*} + \frac{Re}{\beta^2}\,\frac{d\,p^{*}}{d\,x^{*}}$$

2. Then, the momentum eqn will read as

$$\frac{\partial^2 u^{'}}{\partial {y^{*}}^2} - \beta^2\,u^{'} = 0 \qquad (11)$$

with $u^{*} = 0$ at $y^{*} = 0$ and $\partial u^{'}/\partial y^{*} = 0$ at $y^{*} = 1/4$

3. This is the familiar *Fin-Equation* with a solution

$$u^{'} = C_1 \exp(\beta\,y^{*}) + C_2 \exp(-\beta\,y^{*}) \qquad (12)$$

$$C_1 = \frac{(Re/\beta^2)\,(d\,p^{*}\,/\,d\,x^{*})}{1 + exp\,(\beta/2)} \quad C_2 = C_1\,exp\,(\beta/2) \qquad (13)$$

Evaluation of $d\,p^*/d\,x^*$ L14($\frac{10}{17}$)

1. To evaluate $d\,p^*/d\,x^*$, we use definition of $\overline{u}$. This gives

$$\int_0^{1/4} u^*\, dy^* = \int_0^{1/4} (u^{'} - \frac{Re}{\beta^2}\,\frac{d\,p^*}{d\,x^*})\, dy^* = \frac{1}{4}$$

2. Substitution for $u^{'}$ gives

$$Re\,\frac{d\,p^*}{d\,x^*} = f_l\,Re = \beta\,\left[4\,C_1\,\left\{\exp\left(\beta/2\right) - 1\right\} - 1\right] \qquad (14)$$

Centerline Velocity u_c - L14($\frac{11}{17}$)

Consider equation 10 again. Then at $y^* = 1/4$ (or at centerline)

$$(\frac{\partial^2 u^*}{\partial {y^*}^2})_{1/4} - \beta^2 \, u_c^* = Re \, \frac{d \, p^*}{d \, x^*} \quad (15)$$

where, it can be shown that $(\partial^2 u^* / \partial {y^*}^2)_{1/4} = 2 \, C_1 \, \beta^2 \, \exp(\, \beta/4 \,)$ and hence,

$$u_c^* = -C_1 \, \left[exp(\, \beta/4 \,) - 1\right]^2 \quad (16)$$

Final Solution $\beta \sim$ x L14($\frac{12}{17}$)

1. Integrating equation 5 and noting that $u^*_{y^*=0} = v^*_{y^*=1/4} = 0$ gives

$$Re \frac{d}{d\,x^*} \int_0^{1/4} (u^*\,u^*)\,d\,y^* = -\left(\frac{Re}{4}\frac{d\,p^*}{d\,x^*} + \frac{\partial u^*}{\partial y^*}\Big|_{y^*=0}\right) \quad (17)$$

2. Substitution gives

$$Re \frac{d\,F_1(\beta)}{d\,x^*} = F_2\,(\beta) \quad \text{or} \quad x^* = Re \int_{F_1\,(x^*=0)}^{F_1\,(x^*=x^*)} \frac{1}{F_2}\,d\,F_1 \quad (18)$$

where $F_1 = C_1^2\,[\,I_1 + I_2 - I_3\,]$
$I_1 = (\exp(\beta/2) + 1)^2/4.0 \quad I_2 = (\exp\beta + \beta\exp(\beta/2) - 1)/(2\,\beta)$
$I_3 = 2\,(exp(\beta) - 1)/\beta$
$F_2 = -\beta\,C_1\,[\,\beta\,\{1 + \exp(\beta/2)\} + 1 - \exp(\beta/2)\,]$

Evaluation of the Integral - L14($\frac{13}{17}$)

1. Objective: To evaluate

$$x^* = Re \int_{F_1\,(x^*=0)}^{F_1\,(x^*=x^*)} \frac{1}{F_2}\, d\, F_1$$

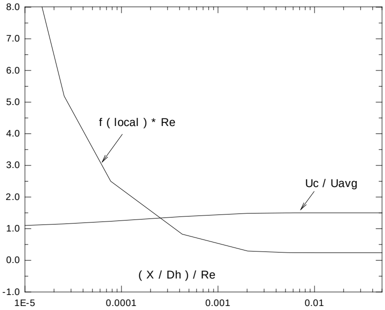
2. We assign different *numerical values* to β and generate functions $F_1(\beta)$ and $F_2(\beta)$
3. Then, integration is performed by Trapezoidal rule
4. Here, $0 < \beta < 60$ were chosen in steps of 1 and found to be sufficient. Note that as $\beta \to \infty$, $x^* \to 0$ and as $\beta \to 0$, $x^* \to \infty$
5. For each β, solutions u_c^* and f_l Re are also evaluated
6. Solutions for select values of β are shown on the next slide

Tabulated Solution - L14($\frac{14}{17}$)

β	C_1	$(x/Dh)\ /\ Re$	u_c^*	$f_l \times Re$
60.0	-1.002e-13	4.60e-6	1.071	1928
50.0	-1.509e-11	6.82e-6	1.0869	1358
40.0	-2.290e-9	1.178e-5	1.111	888
30.0	-3.529e-7	2.50e-5	1.1525	519
20.0	-5.670e-5	7.74e-5	1.233	250
10.0	-0.011	4.26e-4	1.3825	82.59
5.0	-1.19	2.03e-3	1.486	29.38
1.0	-18.57	5.08e-3	1.498	24.60
0.75	-35.24	5.51e-3	1.4991	24.33
0.50	-84.58	6.153e-3	1.4996	24.15
0.30	-247.26	7.01e-3	1.49986	24.053
0.10	-2340.6	1.02024e-2	1.49998	24.006
0.0		∞	1.50	24.0

Comments on the Solution - L14($\frac{15}{17}$)

1. Development length is $(L_v/D_h)/Re \simeq 0.01$
2. Fully Developed Friction Factor is $(f\ Re)_{fd} = 24.0$
3. Fully Developed Centerline velocity is $u_c/\overline{u} = 1.5$
4. These are well-known results from UG Texts
5. More results on L_v on the next slide



Sometimes Apparent Friction Factor is evaluated as

$$f_{app} = -\frac{1}{2}\left(\frac{p_x - p_{x=0}}{x}\right)\frac{D_h}{\rho\,\overline{u}^2} = \frac{1}{x}\int_0^x f_l\,dx \qquad (19)$$

Flow Development Lengths - L14($\frac{16}{17}$)

Duct Cross-section	Geometry parameter	Value of parameter	$L_v/D_h/Re_{D_h}$
Circular			0.05
Annulus	Radius ratio r_i/r_o	0.05	0.01944
		0.10	0.01792
		0.25	0.01679
		0.50	0.01968
		1.0	0.01
Rectangular	Ratio of sides (b / a)	0.0	0.01
		0.125	0.0227
		0.25	0.0427
		0.50	0.066
		0.75	0.0736
		1.0	0.0752
Semi-circle			0.0622

Some References - L14($\frac{17}{17}$)

1. Sparrow E M and Lin S H *Flow development Lengths in the Hydrodynamic Entrance region of Tubes and Ducts*, Physic of Fluids, vol7(1), p 338 (1964)
2. Han L S *Hydrodynamic Entrance Lengths for Incompressible Laminar Flow in Rectangular Ducts*, Trans ASME, Jnl Appl Mech, p 403 (1960)
3. Lundgren T S, Sparrow E M and Starr J B *Pressure Drop due to the Entrance Region in Ducts of Arbitrary Cross-Section* Trans ASME, Jnl of Basic Engg, p 620 (1964)
4. Heaton H S, Reynolds W C and Kays W M *Heat Transfer in Annular Passages, Simultaneous Development of Velocity and Temprature Fields in Laminar Flow*, Int Jnl Heat Mass Transfer, vol 7, p 763, (1964)