ME-662 CONVECTIVE HEAT AND MASS TRANSFER

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LECTURE-24 NEAR-WALL TURBULENT FLOWS-1

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1. Law of the wall for Inner Layer
2. Prandtl's mixing length and van-Driest Hypothesis
3. Treatment of Outer layer

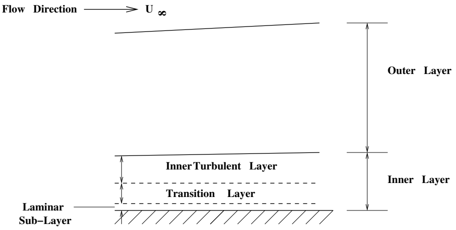
Main Postulate L24($\frac{1}{18}$)

1. So far we considered turbulent flows in which large eddy structure dominates and *diffusive* influence of μ is small. Such flows occur *away from the wall* where y / δ or y / R > 0.1 .
2. Greatest resistance to heat and mass transfer rates, however, is confined to near-wall viscosity affected region .
3. It is also a fortunate occurance that the more significant characteristics of this region are almost *universal*
4. What are the characteristics of this Inner layer ?

Law of the wall - 1 - L24($\frac{2}{18}$)

The Inner layer $y/\delta \approx 0.15$ comprises 3 layers

1. Laminar-like **viscous sub-layer**. In reality, this layer is characterised by a repeated but infrequent fluid bursts.
2. Next, the **transition layer** - likened to the inertial sub-range of the energy spectrum
3. Next, the **inner turbulent layer**



Phenomenologically,

$$u = F\,(y,\ \tau_w,\ \mu,\ \rho,\ others)$$

where *others* include parameters - BL thickness δ (or radius R), dp/dx, v_w, wall roughness height y_r

Law of the wall - 2 - L24($\frac{3}{18}$)

1. Experimental evidence, however, shows that for a smooth, impermeable surface the inner layer is almost completely free of all the *other* parameters.
2. Independence from δ suggests that no information travels from the outer parts to the inner region.
3. Independence from dp/dx suggests that the inner region is independent of the *history* of the flow except that τ_w may depend on the upstream events . The structure of turbulence is thus presumed to be in *local equilibrium*; that is, the timescale of the eddies $<<$ the time taken by the mean flow to change its structure appreciably in response to dp/dx.
4. This assumption of *local equilibrium* is valid for adverse and mildly favourable dp/dx , but not when *re-laminarisation* is encountered in highly accelerated BLs ($\nu/U_\infty^2 \, dU_\infty/dx > 3 \times 10^{-6}$).

Law of the wall - 3 - L24($\frac{4}{18}$)

μ and y are relevant because at the wall, $\tau_w = \mu \, \partial u / \partial y$. ρ is included due to the importance of momentum transfer resulting from velocity fluctuations in the transition and the fully turbulent layers. Therefore, dimensional analysis gives

$$\frac{\rho \, u^2}{\tau_w} = F\,(\frac{\rho \, y^2 \, \tau_w}{\mu^2}) \qquad \text{Define}$$

$$u_\tau \equiv \sqrt{\tau_w / \rho} \quad \text{(Friction velocity)} \quad u^+ \equiv \frac{u}{u_\tau} \quad y^+ \equiv \frac{y \, u_\tau}{\nu}$$

$$\text{or } u^+ = F\,(y^+) \quad \text{(universal 'law of the wall')}$$

We now seek form of $F\,(y^+)$ in three parts of the inner layer.

Variation of shear stress - L24($\frac{5}{18}$)

In the inner layer, the BL form of RANS eqn will read as

$$\rho \left[u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right] = -\frac{d\,p}{d\,x} + \frac{\partial \tau_{tot}}{\partial y} - \rho \left[\frac{\partial}{\partial x} (\overline{u'^2} - \overline{v'^2}) \right]$$

$$\tau_{tot} = \tau_l + \tau_t = \mu \frac{\partial u}{\partial y} - \rho \overline{u'\,v'}$$

The terms in sq brackets on RHS are important only in hightly accelerated flows - hence ignored. Then, since $u\,(\partial u/\partial x) \simeq 0$ and $v \simeq v_w$,

$$\frac{\partial \tau_{tot}}{\partial y} \simeq \frac{d\,p}{d\,x} + \rho\, v_w \frac{\partial u}{\partial y} \rightarrow \text{ intergration gives}$$

$$\frac{\tau_{tot}}{\tau_w} = 1 + \frac{y}{\tau_w} \frac{d\,p}{d\,x} + \frac{\rho\, v_w\, u}{\tau_w} = 1 + p^+\,y^+ + v_w^+\,u^+$$

$$p^+ \equiv \frac{\nu}{\rho\, u_\tau^3} \frac{dp}{dx} \qquad v_w^+ \equiv \frac{v_w}{u_\tau} \quad \text{(Definitions)}$$

Forms of F (y^+) - 1 - L24($\frac{6}{18}$)

To begin with, we assume that dp/dx = v_w = 0 . Then, $\tau_{tot} = \tau_w$ = const. Hence,

1. In Laminar sub layer $\tau_{tot} = \tau_w = \tau_l = \mu\, \partial u / \partial y$ or, upon integration, $u = (\tau_w\, y)/\mu + C$ where C = 0 since, u = 0 at y = 0. Hence, rearrangement gives $u/u_\tau = y\, u_\tau/\nu$ or, $u^+ = y^+$.
2. When dp/dx is moderate , eqn for $\partial \tau_{tot}/\partial y$ shows that 2nd and 3rd dervatives of u w.r.t. y will be nearly zero. Hence, expanding in Taylor's series about $y^+ = 0$, yields:

$$u^+ = y^+ + \frac{{y^+}^4}{4\,!}\,\frac{\partial^4 u}{\partial {y^+}^4} + \ldots$$

3. This equation shows that for small values of y^+, equation $u^+ = y^+$ holds; but at some critical distance away from the wall, u^+ must abruptly depart from linearity.

Forms of F (y^+) - 2 - L24($\frac{7}{18}$)

1. In the Transition Layer , there are no simple arguments because viscous and turbulent stresses are equally important.
2. There is however similarity between the inertial sub-range of the energy spectrum and the transitional layer.
3. If u' is a representative fluctuation then, the viscous lengthscale is $(\nu/u') << \delta$ if the $Re_t = (u'\,\delta)/\nu$ is high.
4. A layer covering a range of values of y can therefor be imagined in which the turbulence structure is independent of both δ and the viscous lengthscale ν/u'.
5. Thus, $\partial u/\partial y$ can only depend on (u'/y). Now, if local value of u' is taken as u_τ then, $\partial u/\partial y \propto u_\tau/y$. Hence

$$u^+ = \frac{1}{\kappa_{tr}}\ln(y^+) + C_{tr} = \frac{\ln(E_{tr}\,y^+)}{\kappa_{tr}} \rightarrow C_{tr} = \frac{\ln(E_{tr})}{\kappa_{tr}}$$

Expected departure from linearity is indeed obsreved.

Forms of F (y^+) - 3 - L24($\frac{8}{18}$)

1. In Turbulent Layer , the universal law can be written as $(\partial u/\partial y) = (u_\tau^2/\nu)\,\partial F/\partial y^+$
2. $(\partial u/\partial y)$ cannot be expexted to be influenced by μ because $\tau_t >> \tau_l$. As such, $\partial F/\partial y^+$ must be independent of ν and, from dimensional considerations, proportional to $\nu/u_\tau\, y$ or $1/\,y^+$.
3. Therefore

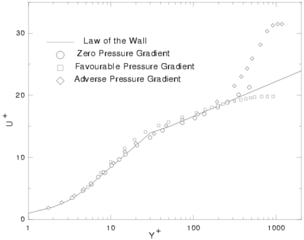
$$\frac{\partial u^+}{\partial y^+} \propto \frac{1}{y^+} \quad \rightarrow u^+ = \frac{1}{\kappa}\ln(y^+) + C = \frac{1}{\kappa}\ln(\,E\,y^+)$$

4. Like the transition layer, the turbulent layer law is again logarithmic.

Universal Law of the wall - L24($\frac{9}{18}$)

Exp data for pr gr param

$$K = \frac{\delta_2}{U_\infty} \frac{d\, U_\infty}{d\, x}$$



$K = -1.434 \times 10^{-3}$ (Adv pr gr)
K = 0 (zero pr gr)
$K = 1.44 \times 10^{-3}$ (fav pr gr)

$$\begin{aligned} u^+ &= y^+ \ \text{for} \ (y^+ \leq 5) \ \text{Lam SL} \\ u^+ &= 5.0 \ln(y^+) - 3.05 \ \text{for} \ (5 \leq y^+ \leq 30) \ \text{Trans L} \\ u^+ &= 2.44 \ln(y^+) + 5.4 \ \text{for} \ (y^+ \geq 30) \ \text{Turb L} \end{aligned}$$

Valid upto $y^+ \simeq 700$ for K = 0; upto $y^+ \simeq 100$ for K $>$0; and upto $y^+ \simeq 300$ for K $<$ 0. For pipe flow (mildly fav pr gr), valid upto $y^+ \simeq 700$. In general, valid for $y^+ \leq 100$.

Thickness of Inner Layer - L24($\frac{10}{18}$)

1. The general limit of $y^+ \leq 100$ corresponds to $\sim$ 15 % of the width of the shear layer.
2. For example, in a pipe flow, $f = 0.046\, Re_D^{-0.2} = 2\,(u_\tau/\overline{u})^2$.
   Then for Re_D = 30,000 (say),
   $R^+ = R\, u_\tau/\nu = 0.0758 \times Re_D^{0.9}$ = 811.
   Therefore, y_{inner}/R = 100 / 811 = 0.12 or 12 %.
3. The constants in the logarithmic region are:
   $\kappa_{tr} = 0.2$, C_{tr} = - 3.05 and E_{tr} = 0.543 (Transition).
   $\kappa = 0.41$,C_{tr} = 5.4 and E = 9.512 (Turbulent)
4. Rather than a 3-layer universal law, we now seek a continuous law of the wall from theory. Recall that the 3-layer law is based on ignorance of the bursting phenomenon in the Lam sub layer which, in turn, will influence the heat/mass transfer rates at the wall. Also, effects of dp/dx and v_w were ignored.

Prandtl's mixing length-1 - L24($\frac{11}{18}$)

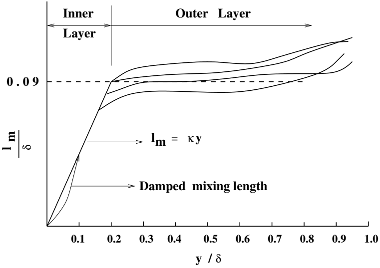
1. In analogy with the Stokes's law for laminar shear stress $\tau_l = \mu\, \partial u / \partial y$, we introduce a model due to Boussinesq, $\tau_t = -\,\rho\, \overline{u'\, v'} = \mu_t\, (\partial u / \partial y)$.
2. Prandtl suggested that

$$
\begin{aligned}
\mu_t &= \rho\, l_m\, v' \quad \rightarrow \quad v' \simeq l_m \left| \frac{\partial u}{\partial y} \right| \\
\tau_t &= \rho\, l_m^2 \left| \frac{\partial u}{\partial y} \right| \frac{\partial u}{\partial y}
\end{aligned}
$$

where v' is velocity fluctuation responsible for transverse momentum transfer and l_m is mean eddy size in the inner layer. Note that unlike μ, turbulent viscosity μ_t is a property of the flow.

Prandtl's mixing length-2 - L24($\frac{12}{18}$)

Transition layer is characterised neither by δ nor by ν/v'. The only relevent scale is y . Prandtl extended this argument to the entire inner layer and proposed that $l_m = \kappa y$.



The Fig shows Exp data for BLs with diff dp/dx and v_w inferred from measurement of $\tau_t = -\rho \overline{u' v'}$ and $\partial u/\partial y$. For $0.2 < y/\delta < 0.9$, the values show a scatter about $l_m/\delta = 0.09$

For $(y/\delta) < 0.2$, l_m is nearly $\propto$ y with $\kappa \sim 0.41$. Very close to the wall, however, l_m is somewhat lower (damped) than that suggested by $l_m = \kappa y$.

Van-Driest Hypothesis - L24($\frac{13}{18}$)

1. In the region where $l_m = \kappa\, y$ holds, $\tau_t = \tau_w = \rho\,(\kappa\, y)^2\,(\,\partial u/\partial y\,)^2$ or, taking the sq root, $\partial u^+/\partial y^+ = \,1/\kappa\, y^+$. This integrates to $u^+ = \ln\,(E\, y^+)/\kappa$ for the turbulent inner layer.
2. To include effects of fluctuations on the transtion and laminar sub layer , Van-Driest proposed

$$
\begin{aligned}
l_m &= \kappa\, y\,\left[1 - \exp(-\frac{y^+}{A^+})\right] \\
\text{or } \mu_t &= \rho\,(\kappa\, y)^2\,\left[1 - \exp(-\frac{y^+}{A^+})\right]^2 \frac{\partial u}{\partial y}
\end{aligned}
$$

where for a smooth wall, $A^+ \simeq 26$. Note that μ_t is zero only at the wall and in regions where viscosity is influential ($y^+ < 30$), l_m is smaller than Prandtl's mixing length. The amplitude of fluctuations decrease exponentially as $y \rightarrow 0$.

Continuous Law - L24($\frac{14}{18}$)

1. To prduce a continuous law of the wall, recall that

$$\begin{aligned} \frac{\tau_{tot}}{\tau_w} &= 1 + p^+ y^+ + v_w^+ u^+ \\ &= \left[1 + \left[\kappa y^+ \left\{1 - \exp(-\frac{y^+}{A^+})\right\}\right]^2 \frac{\partial u^+}{\partial y^+}\right] \frac{\partial u^+}{\partial y^+} \end{aligned}$$

2. However, if the stress-ratio is *assumed to be unity* then, the effects of p^+ and v_w^+ can be absorbed in a suitably defined A^+.
3. Thus, with $\tau_{tot}/\tau_w = 1$, we obtain

$$\frac{\partial u^+}{\partial y^+} = \frac{-1 + \sqrt{1 + 4\,a}}{2\,a} \quad a = \left[\kappa y^+ \left\{1 - \exp(-\frac{y^+}{A^+})\right\}\right]^2$$

$$u^+ = \int_0^{y^+} \frac{\partial u^+}{\partial y^+}\, dy^+ \quad \text{(Continuous Law)}$$

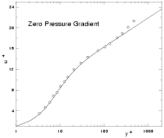
Numerical integration is required.

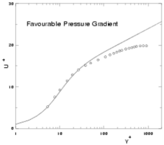
Predictions - L24($\frac{15}{18}$)

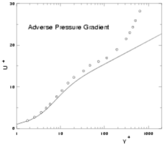
Exptl data for different BLs with different dp/dx and v_w are matched with predcitions tuning A^+ in each case . Kays and Crawford propose

$$A^+ = \frac{25}{a\left[v_w^+ + b\left\{\frac{p^+}{1+c\,v_w^+}\right\}\right] + 1}$$

with a = 7.1, b = 4.25, c = 10
and if $p^+ > 0, \;\; b = 2.9, \;\; c = 0$
or, if $v_w^+ < 0, \;\; a = 9$







Predictions agree very well
upto $y^+ = 500$ for Zero pr. gr.,
upto $y^+ \simeq 100$ for Fav pr. gr.
and
upto $y^+ \simeq 200$ for Adv. pr. gr.

Outer Layers-1 - L24($\frac{16}{18}$)

1. Outer layers can hardly be expected to br universal because the large eddy structure there is severly influenced by dp/dx and other body forces. We need turbulence models for this region.
2. Short-cut methods have been developed. For example, for zero pr. gr. BL, vel profiles at different axial locations can be unified by

$$\frac{u_\infty - u}{u_\tau} = F\left(\frac{y}{\delta}\right) = 1 - \cos\left(\pi \frac{y}{\delta}\right) \text{ (wake function)}$$

$$u^+ = \frac{1}{\kappa} \ln(y^+) + C + \frac{A}{\kappa} \left\{1 - \cos\left(\pi \frac{y}{\delta}\right)\right\}$$

where, $A \simeq 0.55$, $\kappa = 0.4$ and $C = 5.1$.

Outer Layers-2 - L24($\frac{17}{18}$)

1. There are limitations. For example, from expt data for ducted flows, A $\simeq$ 0. Similarly, for finite dp/dx, A = F (x).
2. Also, although u^+ profile is unified, measured v/u_τ and τ_w cannot be. Thus, conditions for similarity cannot be established for turbulent BLs as was possible with laminar BLs.
3. However, only u-profile similarity can be established from computer curve-fitting of exptl data.

$$\frac{u_\infty - u}{u_\tau} = F\,(\frac{y}{\delta_3}) \quad \rightarrow \quad \delta_3\,(x) = -\int_0^\infty F\,dy$$

$$G(x) = \int_0^\infty F^2\,d\,(\frac{y}{\delta_3})$$

4. For $U_\infty = C\,x^m$ BLs and $m < 0$, $G \simeq 6.2\,(\beta + B + 1.43)^{0.482}$ for $(-1 < \beta + B < 12)$ where, $\beta = (\delta_1/\tau_w)\,dp/dx$ and B = $\rho\,v_w\,u_\infty/\tau_w$

Summary - L24($\frac{18}{18}$)

1. We have shown that although the Inner Layer universality can be established for a wide variety of turbulent flows, Outer layer similarity is difficult to establish.
2. For complete description of outer layers, we need to solve the RANS equations via turbulence models . The inner layer universality can be exploited in two ways
    1. To derive approximate correlations for f and Nu
    2. To specify wall-boundary conditions at $y^+ \simeq A^+$ when outer layers are computed by RANS equations. This achieves computational economy.
3. To prepare the ground for studying turbulence models, in the next lecture, we shall explore the likely Interaction between Inner and Outer Layers .