ME-662 CONVECTIVE HEAT AND MASS TRANSFER

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LECTURE-30 PREDICTION OF TURBULENT HEAT TRANSFER

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1. Prediction of St_x (Ext Bls)
   1. Use of law of the Wall
   2. Analogy Method
   3. Integral Method
   4. Effects of V_w and Roughness
2. Prediction of Nu (Internal Flows)
   1. Use of Law of the wall
   2. Analogy Method

Use of Wall Law - Ext BLs - 1 L30($\frac{1}{17}$)

1. From lecture 28, the temperature law of the wall is written as

$$T^+_\infty = Pr_T\,(U^+_\infty + PF_\infty) \quad \text{where}$$

$$U^+_\infty = \frac{U_\infty}{u_\tau} = \frac{U_\infty}{\sqrt{\tau_w/\rho}} = \sqrt{\frac{2}{C_{f,x}}}$$

$$T^+_\infty = \frac{-\,(T_\infty - T_w)}{q_w/(\rho\, C_p\, u_\tau)} = \frac{\rho\, C_p\, U_\infty}{h_x} \times \frac{u_\tau}{U_\infty} = \frac{\sqrt{C_{f,x}/2}}{St_x}$$

$$St_x = \frac{\sqrt{C_{f,x}/2}}{Pr_T(\sqrt{2/C_{f,x}} + PF_\infty)} \quad (Pr_T \simeq 0.9)$$

2. For Pr = 1, $PF_\infty = 0$. Reynolds used Pr_T = 1. Hence, $St_x = C_{f,x}/2$ (Perfect Reynolds Analogy) . For near-unity Pr, $St_x \simeq (C_{f,x}/2) \times Pr^{-0.4}$. Hence, for zero Pr Gr, $\rightarrow$ $St_x = 0.0286\, Re_x^{-0.2}\, Pr^{-0.4}$.
3. For rough surface , appropriate $C_{f,x}$ and PF_∞ to be used

Analogy Method - 1 - Ext BLs - L30($\frac{2}{17}$)

1. Recall that $Pr_{eff} = dT^+/du^+ = (dT^+/dy^+)/(du^+/dy^+)$. Hence, using $(\tau_{tot}/\tau_w) \simeq 1 = (1 + \nu_t/\nu)\,(du^+/dy^+)$ gives

$$\frac{dT^+}{dy^+} = \frac{(1+\nu_t/\nu)(du^+/dy^+)}{Pr^{-1} + (\nu_t/\nu)/Pr_T} = \left[\frac{1}{Pr} + (\frac{1}{du^+/dy^+} - 1)\,\frac{1}{Pr_T}\right]^{-1}$$

2. Integrating from y = 0 to ∞, and using the 3-layer law for u^+, and hence for (du^+/dy^+) it follows that

$$\begin{aligned} T^+_{sl} - 0 &= Pr\, y^+_{sl} = Pr\, u^+_{sl} = 5\, Pr \\ T^+_{trl} - T^+_{sl} &= 5\, Pr_T \ln\,(1 + 5\,\frac{Pr}{Pr_T})\ \ (y^+_{trl} = 30\ \ \text{used}) \\ T^+_{\infty} - T^+_{trl} &= 2.5\, Pr_T \ln\left[\frac{1 + (Pr/Pr_T)\,(\delta^+/2.5 - 1)}{1 + 11\,(Pr/Pr_T)}\right] \end{aligned}$$

Analogy Method - 2 - Ext BLs - L30($\frac{3}{17}$)

Adding the last 3 Eqns, it can be shown that

$$\frac{\sqrt{C_{f,x}/2}}{St_x} = 5Pr + 5\,Pr_T \ln\left(1 + 5\,\frac{Pr}{Pr_T}\right) + 2.5\,Pr_T \ln\left\{\frac{1 + (Pr/Pr_T)\,(\delta^+/2.5 - 1)}{1 + 11\,(Pr/Pr_T)}\right\}$$

where using the Power law,

$$\delta^+ = \left(\frac{U_\infty^+}{8.75}\right)^7 = \left(\frac{\sqrt{2/C_{f,x}}}{8.75}\right)^7$$

The $C_{f,x}$ is evaluated using Integral method of lecture 29.

Use of Int Energy Eqn - 1 - L30($\frac{4}{17}$)

1. When U_∞ and $(T_w - T_\infty)$ vary arbitrarily with x

$$\frac{1}{U_\infty\,(T_w - T_\infty)}\,\frac{d}{dx}\,[\Delta_2\, U_\infty\,(T_w - T_\infty)] = St_x$$

2. For further analysis, let[1] $St_x = C\, Re_x^{-n}$
3. Then for const U_∞ and $(T_w - T_\infty)$ BL , that is flat plate

$$\begin{aligned}
\frac{d\,\Delta_2}{dx} &= St_x = C\,(\frac{U_\infty\, x}{\nu})^{-n} \quad \text{integration gives} \\
\Delta_2 &= \frac{C}{1-n}\,(\frac{U_\infty}{\nu})^{-n}\, x^{1-n} \quad \text{using } \Delta_2 = 0 \ \text{ at } \ x = 0 \\
St_x &= C\,(\frac{1-n}{C} \times \frac{U_\infty\,\Delta_2}{\nu})^{\frac{n}{n-1}}
\end{aligned}$$

[1] Ambroke G S   Sov. Phy. Tech. Phy.,vol2, p 1979, 1957

Use of Int Energy Eqn - 2 - L30($\frac{5}{17}$)

1. We assume validity of the last relationship regardless of the previous history of the BI . Then, IEE becomes

$$\frac{d}{dx}\left[\Delta_2\, U_\infty\,(T_w - T_\infty)\right] = U_\infty (T_w - T_\infty)\, C\left(\frac{1-n}{C} \times \frac{U_\infty\,\Delta_2}{\nu}\right)^{\frac{n}{n-1}}$$

or, integration gives

$$\Delta_2 = \frac{C\,\nu^n}{(1-n)\,U_\infty\,(T_w - T_\infty)}\left[\int_0^x U_\infty\,(T_w - T_\infty)^{1/(1-n)}\,dx\right]^{1-n}$$

2. Using $St_x \sim \Delta_2$ relation from previous slide

$$St_x = \frac{C\,\nu^n\,(T_w - T_\infty)^{n/(1-n)}}{\left[\int_0^x U_\infty\,(T_w - T_\infty)^{1/(1-n)}\,dx\right]^n}$$

Use of Int Energy Eqn - 3 - L30($\frac{6}{17}$)

Assuming flat plate data C = 0.0284 $Pr^{-0.4}$ and n = 0.2, expression of previous slide becomes

$$St_x = \frac{0.0284\, Pr^{-0.4}\, \nu^{0.2}\, (T_w - T_\infty)^{0.25}}{\left[\int_0^x U_\infty\, (T_w - T_\infty)^{1.25}\, dx\right]^{0.2}}$$

$$\simeq 0.0295\, Pr^{-0.4}\, Re_x^{-0.2} \left[1 - \frac{165}{St_x} \left(\frac{\nu}{U_\infty^2} \frac{dU_\infty}{dx}\right)\right]$$

These expressions give remarkably good fit to Exptl data for pr gr parameter $(\nu/U_\infty^2)\,(dU_\infty/dx) < 10^6$ (Crawford and Kays)

Effect of v_w - 1 - L30($\frac{7}{17}$)

1. For Flat Plate and $(T_w - T_\infty)$ = const , Crawford and Kays show that for finite v_w,

$$\frac{St_{x,v_w}}{St_{x,v_w=0}} = \frac{\ln(1+B_h)}{B_h} \rightarrow B_h = \frac{v_w/U_\infty}{St_{x,v_w}}$$

$$\text{or } St_{x,v_w} = 0.0284\, Pr^{-0.4}\, Re_x^{-0.2} \left[\frac{\ln(1+B_h)}{B_h}\right]$$

2. For arbitrary variation of v_w IEE reads as

$$\frac{d\,\Delta_2}{dx} = St_{x,v_w} + \frac{v_w}{U_\infty} = St_{x,v_w}(1+B_h) \text{ or}$$

$$= \left[0.0284\, Pr^{-0.4} \ln(1+B_h)\, \frac{(1+B_h)}{B_h}\right] Re_x^{-0.2}$$

Effect of v_w - 2 - L30($\frac{8}{17}$)

1. For B_h = const , and using Δ_2 = 0 at x = 0, integration give

$$Re_x^{-0.2} = 1.057 \left[0.0284 \, Pr^{-0.4} \ln{(1 + B_h)} \, \frac{(1 + B_h)}{B_h} \right]^{0.25} Re_{\Delta_2}^{-0.25}$$

2. Using $St_{x,v_w} \sim Re_x$ relation from previous slide

$$St_{x,v_w} = 0.0125 \, Pr^{-0.5} \, Re_{\Delta_2}^{-0.25} \, (1 + B_h)^{0.25} \left[\frac{\ln{(1 + B_h)}}{B_h} \right]^{1.25}$$

We assume validity of this relation even when B_h, U_∞ and $(T_w - T_\infty)$ vary arbitrarily with x (see next slide)

Effect of B_h, U_∞ and $(T_w - T_\infty)$ - L30($\frac{9}{17}$)

For this case, IEE will read as

$$\frac{d}{dx}\left[\Delta_2\, U_\infty\, (T_w - T_\infty)\right] = 0.0125\, Pr^{-0.5}\, Re_{\Delta_2}^{-0.25}\, U_\infty\, (T_w - T_\infty) \times \left[\frac{(1 + B_h)}{B_h}\ln(1 + B_h)\right]^{1.25}$$

Integration gives

$$St_x = 0.0284\, Pr^{-0.4} \times \frac{\nu^{0.2}\, (T_w - T_\infty)^{0.25}\, (1 + B_h)^{0.25}\, \{\ln(1 + B_h)/B_h\}^{1.25}}{\left[\int_0^x U_\infty\, (T_w - T_\infty)^{1.25}\, \{(1 + B_h)\ln(1 + B_h)/B_h\}^{1.25}\, dx\right]^{0.2}}$$

Crawford and Kays show remarkable good fit to experimental data and predictions using mixing length.

Similarity Method for TBL - L30($\frac{10}{17}$)

1. The governing Eqn for a Temperature TBL is

$$u\,\frac{\partial T}{\partial x} + v\,\frac{\partial T}{\partial y} = +\nu\,\frac{\partial}{\partial y}\,\left[b_{Pr}\,\frac{\partial T}{\partial y}\right]$$

where $b_{Pr} = \alpha/\nu + \alpha_t/\nu = Pr^{-1} + Pr_T^{-1}\,\nu_t^+$, where $Pr_T = \nu_t/\alpha_t$, $\nu^+ = \nu_t/\nu$ and ν_t is given by Prandtl's mixing length.

2. Using similarity variables defined in lecture 29,

$$\frac{d}{d\,\eta}(b_{Pr}\,\theta^{'}) + f\,\theta^{'} + (\frac{2n}{m+1})\,f^{'}\,(1-\theta) = \frac{2x}{m+1}\,(f^{'}\,\frac{d\theta}{dx} - \theta^{'}\,\frac{df}{dx})$$

$$m = (\frac{x}{U_\infty})\,\frac{dU_\infty}{dx},\quad n = (\frac{x}{T_w - T_\infty})\,\frac{d(T_w - T_\infty)}{dx},\quad \theta = \frac{T_w - T}{T_w - T_\infty}$$

Iterative solution is required at each x.
BCs: $\theta(0) = 0$ and $\theta(\infty) = 1$.

Wall law - Pipe Flow - 1 - L30($\frac{11}{17}$)

1. Writing wall-law for Pipe centerline
   $T_{cl}^+ = Pr_T\,(u_{cl}^+ + PF_\infty) = Pr_T\,(\overline{u^+} + 1.5/\kappa + PF_\infty)$ where

$$
\begin{aligned}
T_{cl}^+ &= \frac{T_w - T_b}{q_w} \times (\frac{T_w - T_{cl}}{T_w - T_b}) \times \rho\, C_p\, u_\tau \\
&= (\frac{k}{h\,D}) \times (\frac{\bar{u}\,D}{\alpha}) \times (\frac{u_\tau}{\bar{u}}) \times (\frac{T_w - T_{cl}}{T_w - T_b}) \\
&= \frac{Re\,Pr}{Nu} \times \sqrt{\frac{f}{2}} \times (\frac{T_w - T_{cl}}{T_w - T_b})
\end{aligned}
$$

2. Hence, Equating for T_{cl}^+,

$$
Nu = \frac{Re\,Pr\,\sqrt{f/2}}{Pr_T\,(\sqrt{2/f} + 1.5/\kappa + PF_\infty)}\,(\frac{T_w - T_{cl}}{T_w - T_b})
$$

Wall law - Pipe Flow - 2 - L30($\frac{12}{17}$)

1. To evaluate temperature ratio, we use Power laws

$$\left(\frac{T - T_w}{T_{cl} - T_w}\right) = \left(\frac{y}{R}\right)^{1/7} = \frac{u}{u_{cl}}$$

   Then, using definition of T_b, it can be shown that

$$\left(\frac{T_w - T_{cl}}{T_w - T_b}\right) = \frac{6}{5} \simeq 1 \quad \text{and} \quad \frac{u_{cl}}{\overline{u}} = \frac{60}{49} \simeq 1.22$$

2. The most widely used correlation due to Gnienlenski is

$$Nu = \frac{(Re - 1000)\, Pr \, \sqrt{f/2}}{\sqrt{2/f} + 12.7\,(Pr^{2/3} - 1)}$$

   valid for $0.5 < Pr < 2000$ and $2300 < Re < 5 \times 10^6$

Analogy Method - Pipe Flow - 1 - L30($\frac{13}{17}$)

1. In the FD Pipe flow , dp/dx = const. Hence, the axial momentum eqn and its consequences are

$$\frac{1}{r}\frac{d\,(r\,\tau_{tot})}{dr} = -\frac{dp}{dx} \rightarrow \frac{\tau_{tot}}{\tau_w} = \frac{r}{R} = 1 - \frac{y}{R}$$

$$\text{But } \tau_{tot} = \rho\,(\nu + \nu_t)\,\frac{du}{dr} = -\rho\,(\nu + \nu_t)\,\frac{du}{dy}$$

$$(1 + \frac{\nu_t}{\nu}) = \frac{1 - y^+/R^+}{du^+/dy^+}$$

2. Then form Slide 2,

$$\frac{dT^+}{dy^+} = (1 - \frac{y^+}{R^+})\left[\frac{1}{Pr} + (\frac{1 - y^+/R^+}{du^+/dy^+} - 1)\,\frac{1}{Pr_T}\right]^{-1}$$

Analogy Method - Pipe Flow - 2 - L30($\frac{14}{17}$)

1. Integrating from y = 0 to R^+, and using the 3-layer law for u^+, and hence for (du^+/dy^+) it can be shown that

$$
\begin{aligned}
T^+_{sl} - 0 &= 5\,Pr \quad (y^+_{sl} = 5) \\
T^+_{trl} - T^+_{sl} &= 5\,Pr_T \ln(1 + 5\,\frac{Pr}{Pr_T} \quad (y^+_{trl} = 30) \\
T^+_{cl} - T^+_{trl} &= 2.5\,Pr_T \ln(\frac{R^+}{30}) \quad \text{for Pr} \quad \geq 1^2
\end{aligned}
$$

where

$$T^+_{cl} = \frac{Re\,Pr}{Nu}\sqrt{\frac{f}{2}}\,(\frac{T_w - T_{cl}}{T_w - T_b})$$

Therefore, adding the three equations (see next slide)

[2] For Pr $<<$ 1, closed form soln cannot be obtained

Analogy Method - Pipe Flow - 3 - L30($\frac{15}{17}$)

1. With $R^+ = (Re/2)\sqrt{f/2}$, addition gives

$$Nu = \frac{Re\,Pr\,\sqrt{f/2}\,(T_w - T_{cl})/(T_w - T_b)}{5\,Pr + 5\,Pr_T \ln\left(1 + 5\,\frac{Pr}{Pr_T}\right) + 2.5\,Pr_T\,\ln\left\{\left(\frac{Re}{60}\right)\sqrt{\frac{f}{2}}\right\}}$$

2. Dittus Boelter Correlation - Nu = 0.023 $Re^{0.8}\,Pr^n$, n = 0.4 for heating and n = 0.3 for cooling.
3. Sliecher and Rouse Correlation

$$Nu = 5 + 0.015 Re^a Pr^b, (0.1 < Pr < 10^4), (10^4 < Re < 10^6)$$

$$a = 0.88 - \frac{0.24}{4 + Pr} \quad b = 0.333 + 0.5\,\exp\,(-0.6\,Pr)$$

For Liquid Metals, $Nu = a + b\,Re^{0.85}\,Pr^{0.93}$ where
(a = 6.3 and b = 0.0167 for q_w = const) and
(a = 4.8 and b = 0.0156 for T_w = const)

Comparison of Correlations - L30($\frac{16}{17}$)

Re	3000	10000	50000	10^5	10^6
Pr = 0.5, Temp rat = 1.1, $Pr_T = 0.943$					
Gin	8.13	25.2	84.5	147	883
DB	10.5	27.6	100	174	1100
SR	11.9	23.7	75.6	130	845
Anal	10.3	24.4	81.0	139	880
Pr = 5.0 , Temp rat = 1.1, $Pr_T = 0.887$					
Gin	19.2	70.1	287	524	3750
DB	26.5	69.4	251	438	2760
SR	29.7	74.1	278	498	3520
Anal	28.1	76.5	293	531	3860
Pr = 25.0 , Temp rat = 1.1, $Pr_T = 0.882$					
Gin	33.2	126	545	1020	7780
DB	50.4	132	479	834	5260
SR	52.1	139	552	1010	7540
Anal	40.3	114	455	842	6500

Summary - L30($\frac{17}{17}$)

1. The correlations for Pipe flow can be applied to non-circular ducts by evaluating f, Re and Nu based on hydraulic diameter
2. The easy-to-use Dittus-Boelter correlation overpredicts Nu for $Pr < 1$ and underpredicts Nu for $Pr > 1$
3. For complete description of flow and heat transfer involving complex ducts, strong and changing strain rates due to body forces etc, it is best to adopt CFD techniques with two-eqn or stress-flux eqn models.
4. This completes discussion of Turbulent flow and Heat Transfer. In the remaining lectures, we shall discuss Convective Mass Transfer