ME-662 CONVECTIVE HEAT AND MASS TRANSFER

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LECTURE-36 EVALUATION OF g and N_w

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1. Laminar Boundary Layers
2. Turbulent Boundary Layers
3. Overall Procedure for calculating N_w

Laminar BL - 1 - L36($\frac{1}{10}$)

1. Consider Laminar BL with T_w = const. and with suction/blowing and without viscous dissipation. For this case, Similarity soln for const properties is

$$\frac{Nu_x}{Re_x^{0.5}} = -\theta'(0) = F(m, Pr, B_f)$$

$$Nu_x = \frac{h_x\, x}{k} = \frac{x\,(\partial T/\partial y)_w}{T_\infty - T_w} \quad \text{and} \quad B_f = \frac{V_w}{U_\infty} Re_x^{0.5}$$

2. This corresponds to $\Psi = T$ and $\omega_k = 1$ with constant specific heat in all states. Hence, in terms of mass transfer coeff (g)

$$g = \frac{\Gamma_\Psi\,(\partial \Psi/\partial y)_w}{\Psi_\infty - \Psi_w} \quad \text{or} \quad Sh_x = \frac{g_x\, x}{\Gamma_\Psi} = \frac{x\,(\partial \Psi/\partial y)_w}{\Psi_\infty - \Psi_w} = Nu_x$$

Laminar BL - 2 - L36($\frac{2}{10}$)

1. Similarly, B_f can be interpreted as

$$
\begin{aligned}
B_f &= \frac{V_w}{U_\infty} Re_x^{0.5} = \frac{N_w}{\rho\, U_\infty} Re_x^{0.5} = \frac{g\, B_\Psi}{\rho\, U_\infty} Re_x^{0.5} \\
&= (\frac{g_x\, x}{\Gamma_\Psi}) \times (\frac{\Gamma_\Psi}{\mu}) \times (\frac{\mu}{\rho\, U_\infty\, x}) \times Re_x^{0.5}\, B_\Psi \\
&= Sh_x\, (\frac{\Gamma_\Psi}{\mu})\, Re_x^{-0.5}\, B_\Psi
\end{aligned}
$$

2. This shows that driving force $B_\Psi \propto B_f$. Hence, similarity soln to the Ψ-eqn can be interpreted as

$$\frac{Sh_x}{Re_x^{0.5}} = F(m, \frac{\mu}{\Gamma_\Psi}, B_\Psi)$$

Laminar BL - 3 - L36($\frac{3}{10}$)

1. Using the last relation, the constant property heat transfer solutions (lecture 9 - slide 10) can be converted to mass transfer solutions .
2. Thus, consider case of B_f = - 2, m = 0 and Pr = μ/Γ_h = 1.0.
3. For this case, $Nu_x \, Re_x^{-0.5} = -\,\theta'(0) = 2.1 = Sh_x \, Re_x^{-0.5}$.
4. Hence,

$$B_\Psi = \frac{(\mu/\Gamma_\Psi)}{-\,\theta'(0)} \times B_f = \frac{1}{2.1} \times (-2.0) = -0.9524$$

5. Next slide shows conversions for μ/Γ_Ψ = 0.7 and m = 0

Laminar BL - 4 - L36($\frac{4}{10}$)

Conversions for $Sc = \mu/\Gamma_\Psi$ = 0.7 - m = 0

	m=0			
B_f	-θ' (0)	$B_\Psi = \frac{Sc\, B_f}{-\theta'(0)}$	$\frac{g}{g^*} = \frac{-\theta'(0)}{0.291}$	$\frac{\ln(1+B_\Psi)}{B_\Psi}$
-2.0	1.52	-0.921	5.223	2.756
-1.0	0.872	-0.8027	3.00	2.022
-0.5	0.570	-0.614	1.959	1.55
-0.25	0.429	-0.4079	1.474	1.285
0.0	0.291	0.0	1.0	1.0
0.25	0.166	1.054	0.57	0.683
0.375	0.107	2.453	0.368	0.505
0.5	0.0517	6.77	0.1776	0.303

For $-0.25 < B_\Psi < 0.25$, $(g/g^*) \simeq \ln(1 + B_\Psi)/B_\Psi$. But, for large $|B_\Psi|$, the Reynolds flux model is not at all satisfactory. For these cases, numerical solutions are desirable. These observations also apply to other values of m and Sc.

Laminar BL - 5 - L36($\frac{5}{10}$)

For Arbitrarily varying U_∞, Integral solns (Spalding D B and Chi S W, IJHMT, vol 6, p 363-385 (1963) , show that

$$\text{Stanton}_{\text{mass}} = \frac{g}{\rho\, U_\infty} = \frac{K_1\, \mu^{1.2}\, (\rho\, U_\infty)^{K_2}}{\left[\int_0^x (\rho\, U_\infty)^{K_3}\, dx\,\right]^{0.5}}$$

Sc	B_Ψ	K_1	K_2	K_3
0.7	-0.9	1.85	0.05	1.1
	0.0	0.418	0.435	1.87
	9.0	0.06	1.90	4.8
5.0	-0.9	0.431	0.45	1.9
	0.0	0.117	0.595	2.19
	9.0	0.023	0.90	2.8
> 5	-0.9	$1.037\ Sc^{-0.67}$	0.9	2.8
	0.0	$0.339\ Sc^{-0.67}$	0.9	2.8
	9.0	$0.077\ Sc^{-0.67}$	0.9	2.8

Turbulent BL - L36($\frac{6}{10}$)

1. In Turbulent BLs, the analogy between heat and mass transfer is more perfect because $\Gamma_{eff} = \Gamma_l + \Gamma_t \simeq \mu_l + \mu_t$ with $\Gamma_t >> \Gamma_l$ and $\mu_t >> \mu_l$. That is, for gases $Pr \simeq Sc$ and $Pr_t = Sc_t \simeq 0.9$
2. The turbulent heat transfer correlations for $V_w = 0$ take the form of $St_{x,V_w=0} = C\,Re_x^{-m}\,Pr^{-n}$. Then, from analogy,

$$St_{x,V_w=0} = \frac{g^*}{\rho\,U_\infty} = C\,Re_x^{-m}\,Sc^{-n}$$

$$\frac{g}{\rho\,U_\infty} = \frac{g^*}{\rho\,U_\infty} \times \frac{\ln(1+B_\Psi)}{B_\Psi}$$

$$g^* = \frac{h_{cof,V_w=0}}{c_{pm}}\left(\frac{Pr}{Sc}\right)^{-n} \quad \rightarrow \quad Sc = \frac{\mu}{\Gamma_\Psi}$$

Effect of Property Variations - L36($\frac{7}{10}$)

1. Deviations from (g / g^*) = ln (1 + B) / B at large B_Ψ mainly occur due to property variations through the boundary layer
.
2. For Laminar BLs , the recommended property-correction is

$$\frac{g}{g^*} = \frac{\ln(1 + B_\Psi)}{B_\Psi} \times (\frac{M_w}{M_\infty})^{0.66}$$

3. For Turbulent BLs , the recommended property-correction is

$$\frac{g}{g^*} = \frac{\ln(1 + B_\Psi)}{B_\Psi} \times (\frac{M_w}{M_\infty})^{0.40}$$

where M is molecular weight of the mixture. These relations also apply to internal flows where $B_\Psi = (\Psi_b - \Psi_w)/(\Psi_w - \Psi_T)$ and Ψ_b is the bulk value.

Binary Diffusion Coeffs L36($\frac{8}{10}$)

Binary diffusion coefficient D_{ab} (m^2/s) at 1 atm and T = 300 K.

Pair	$D_{ab} \times 10^6$	Pair	$D_{ab} \times 10^6$
H_2O-air	24.0	CO_2-air	14.0
CO-air	19.0	CO_2-N_2	11.0
H_2-air	78.0	O_2-air	19.0
SO_2-air	13.0	NH_3-air	28.0
CH_3OH-air	14.0	C_2H_5OH-air	11.0
C_6H_6-air	8.0	CH_4-air	16.0
$C_{10}H_{22}$-air	6.0	$C_{10}H_{22}$-N_2	6.4
C_8H_{18}-air	5.0	C_8H_{18}-N_2	7.0
C_8H_{16}-N_2	7.1	C_6H_{14}-N_2	8.0
O_2-H_2	70.0	CO_2-H_2	55.0

Assuming ideal gas behavior, the kinetic theory of gases predicts that $D_{ab} \propto (T^{1.5}/p)$, where T is in Kelvin.

Overall Procedure for N_w L36($\frac{9}{10}$)

1. For the type of mass transfer problem, identify the appropriate conserved property Ψ .
2. Make sure that B_Ψ can be evaluated from Ψ_∞, Ψ_T (usually known) and Ψ_w (usually not known). If not, select linear combinations of Ψs.
3. Sometimes, Ψ_w needs to be established from iterations.
4. Identify the heat transfer situation with $V_w = 0$ corresponding to the mass transfer problem at hand . Hence, evaluate $h_{cof,V_w=0}$ and $g_h^* = h_{cof,V_w=0}/c_{pm}$.
5. Hence, evaluate

$$N_w = g \times B = g_h^* \times (\frac{Pr}{Sc})^n \times (\frac{M_w}{M_\infty})^x \times \ln(1 + B_\Psi)$$

where n and x correspond to the problem at hand.

Summary L36($\frac{10}{10}$)

1. We have thus examined the validity of

$$N_w = g \times B_\Psi = g^* \ln(1 + B_\Psi) \text{ and}$$
$$\frac{g}{g^*} = F(B) = \frac{\ln(1 + B_\Psi)}{B_\Psi}$$

2. It is shown that deviations from this formulas occur when fluid properties vary significantly in the boundary layer at large B_Ψ. Hence, the calculation of N_w is corrected as

$$N_w = g \times B = g_h^* \times (\frac{Pr}{Sc})^n \times (\frac{M_w}{M_\infty})^x \times \ln(1 + B_\Psi)$$

where $g_h^* = h_{cof,V_w=0}/c_{pm}$.

3. In the next 3 lectures, we will demonstrate applications of Stefan-, Couette- and Reynolds-flow models to problems of engineering relevance.