

ME-662 CONVECTIVE HEAT AND MASS TRANSFER

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LECTURE-37 DIFFUSION MASS TRANSFER PROBLEMS

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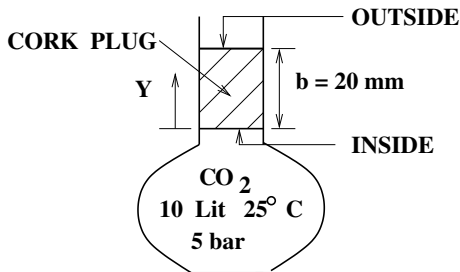
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Simple Diffusion - L37($\frac{1}{12}$)

Prob: A 10 lit vessel contains CO_2 at 25° and 5 bar. The vessel is fitted with a 20 mm cork whose surface area is 300 mm^2 . Determine (a) Initial rate of mass loss of CO_2 and (b) Reduction in vessel pressure after 6 months .

Given

$D_{\text{CO}_2\text{-Cork}} = 1.1 \times 10^{-10} \text{ m}^2/\text{s}$
and Solubility (S) = $0.04015 \text{ kmol}/\text{m}^3\text{-bar}$.



Soln: Here, CO_2 diffuses through stationary cork. Hence, $N_{\text{cork}} = 0$ and using Stefan-flow model

$$N_{\text{CO}_2, y} = \frac{-\rho_m D}{1 - \omega_{\text{CO}_2}} \frac{d\omega_{\text{CO}_2}}{dy} = \text{const}$$

Soln (Contd) - 1 - L37($\frac{2}{12}$)

- ① Integration from $y = 0$ ($\omega_{CO_2,i}$) to $y = b$ ($\omega_{CO_2,o} = 0$), we have

$$N_{CO_2} = -\frac{\rho_m D}{b} \ln(1 - \omega_{CO_2,i}) \simeq \frac{\rho_m \omega_{CO_2,i} D}{b} = \rho_{CO_2,i} \frac{D}{b}$$

because $\omega_{CO_2,i}$ is very small as shown below.

- ② $\rho_{CO_2,i} = S \times p \times M_{CO_2} = 0.04015 \times 5 \times 44 = 8.83 \text{ kg/m}^3$.
But, $\rho_{cork} = 1.93 \times 10^5 \text{ kg/m}^3$. Hence,
 $\omega_{CO_2,i} = 4.575 \times 10^{-5}$.

- ③ Thus, Initial rate of CO_2 loss is

$$\begin{aligned} \dot{m} &= A \times N_{CO_2} = 300 \times 10^{-6} \times \left(\frac{8.83 \times 1.1 \times 10^{-10}}{0.02} \right) \\ &= 1.457 \times 10^{-11} \text{ kg/s} \quad \text{Ans (a)} \end{aligned}$$

Soln (Contd) - 2 - L37($\frac{3}{12}$)

① Now, Instantaneous mass loss is given by

$$\dot{m} = -V \frac{d \rho_{CO_2}}{dt} = -\frac{V M_{CO_2}}{R_u T} \frac{d p_{CO_2}}{dt} = A \frac{D}{b} S M_{CO_2} p_{CO_2}$$

② Canceling M_{CO_2} , we have

$$\frac{1}{p_{CO_2}} \frac{d p_{CO_2}}{dt} = -\frac{A D S R_u T}{b V} \quad \text{or}$$
$$\ln \frac{p_{CO_2}}{p_{CO_2,t=0}} = -\frac{A D S R_u T}{b V} \times t$$

③ Taking $R_u = 8314$, $t = 6 \times 30 \times 24 \times 3600$, $V = 10 \times 10^{-3}$, and $p_{CO_2,t=0} = 5$, we have

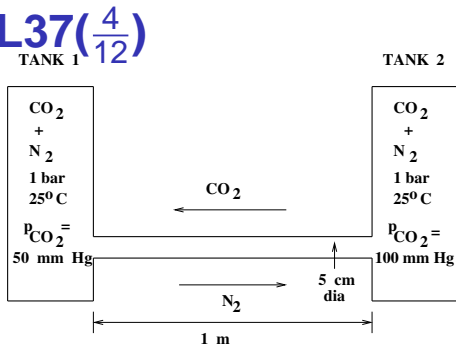
$$p_{CO_2} = 5 \times \exp(-0.2637) = 3.84 \text{ bar}$$

Hence, loss in pressure $(5 - 3.84) = 1.16 \text{ bar} - \text{Ans (b)}$

Counter Diffusion - L37($\frac{4}{12}$)

Prob: Consider two large tanks containing $CO_2 + N_2$ at total pressure 1 bar and $25^\circ C$. The tanks are connected by 1 m long tube (5 cm dia). The partial pressures are $p_{CO_2,2} = 100$ mm Hg and $p_{CO_2,1} = 50$ mm Hg. Calculate mass transfer rate of CO_2 under steady state.

Soln: Here, CO_2 will diffuse from Tank 2 ($p_{CO_2,2} = 100 / 760 = 0.1316$ bar) to Tank 1 ($p_{CO_2,1} = 50 / 760 = 0.06579$ bar) and N_2 will diffuse in opposite direction so that $N_{CO_2} + N_{N_2} = 0$.



Therefore, $N_{CO_2} = (N_{CO_2} + N_{N_2}) \rho_{CO_2} - D \frac{d \rho_{CO_2}}{dx}$ or,

$$N_{CO_2} = -D \frac{d \rho_{CO_2}}{dx} = \text{const}$$

Soln (Contd) - L37($\frac{5}{12}$)

But, $\rho_{CO_2} = p_{CO_2}/(R_{CO_2} T)$ where $R_{CO_2} = 188.95 \text{ J/kg-K}$.

Therefore,

$$N_{CO_2} = - \frac{D}{R_{CO_2} T} \frac{d p_{CO_2}}{dx} = \text{const}$$

where, from Lecture 36 (slide 8)

$D_{CO_2-N_2} = 11 \times 10^{-6} \times (298/300)^{1.5} \times (1.0125/1) = 11.034 \times 10^{-6} \text{ m}^2/\text{s}$. Integration from $x = 0$ to $L = 1 \text{ m}$, gives

$$\begin{aligned} N_{CO_2} &= \frac{-D}{R_{CO_2} T L} (p_{CO_2,2} - p_{CO_2,1}) \\ &= \frac{-11.034 \times 10^{-6}}{188.95 \times 298 \times 1} (0.1325 - 0.06579) \times 10^5 \\ &= -1.3 \times 10^{-6} \left(\frac{\text{kg}}{\text{m}^2 \cdot \text{s}} \right) \end{aligned}$$

or $\dot{m}_{CO_2} = N_{CO_2} \times (\pi/4) (0.05)^2 = -2.55 \times 10^{-9} \text{ kg/s (Ans)}$

Catalysis - L37($\frac{6}{12}$)

Prob: It is proposed to remove NO from the exhaust gases ($T_\infty = 500^\circ\text{C}$, $p = 1 \text{ bar}$, $x_{\text{NO},\infty} = 0.002$ and $M_{\text{mix}} = 30$) of an IC Engine by passing them over a catalyst surface. It is assumed that chemical reactions involving NO are very slow so that NO is neither generated nor destroyed in the gas phase. At the catalyst surface, NO is absorbed with Kinetic rate $N_w = K \rho_m \omega_{\text{NO},w}$ where $K = 0.075 \text{ m/s}$. It is further assumed that NO diffuses to the catalyst surface over a *stagnant* layer of 1 mm with $D_{\text{eff}} = 3 \times D$ where $D = 10^{-4} \text{ m}^2/\text{s}$. Determine (a) steady-state absorption rate and (b) $\omega_{\text{NO},w}$.

Soln: Here, $N_{\text{total}} = N_{\text{NO}} + N_{\text{others}} = N_{\text{NO}}$ because, $N_{\text{others}} = 0$ in the stagnant layer. Let $y = 0$ define catalyst surface. Then, diffusion rate will equal kinetic absorption rate.

$$N_{\text{NO}} = N_{\text{NO}} \omega_{\text{NO}} - \rho_m D \frac{d\omega_{\text{NO}}}{dy} = -K \rho_m \omega_{\text{NO},w}$$

Soln (Contd) - L37($\frac{7}{12}$)

Hence,

$$N_{NO} = \left(\frac{-\rho_m D}{1 - \omega_{NO}} \right) \frac{d\omega_{NO}}{dy} = -K \rho_m \omega_{NO,w}$$

Integration from $y = 0$ to $L = 1$ mm gives

$$\begin{aligned} N_{NO} &= \left(\frac{\rho_m D}{L} \right) \ln \left(\frac{1 - \omega_{NO,\infty}}{1 - \omega_{NO,w}} \right) = g^* \ln \left(1 + \frac{\omega_{NO,\infty} - \omega_{NO,w}}{\omega_{NO,w} - 1} \right) \\ &= -K \rho_m \omega_{NO,w} \end{aligned}$$

where $\omega_{NO,\infty} = x_{NO,\infty} \times (M_{NO}/M_{mix}) = 0.002$, and

$\omega_{NO,w} = 1.144 \times 10^{-3}$ (Ans b) is determined by iteration .

Hence, evaluating $\rho_m = p/(R_{mix} T) = 0.466 \text{ kg/m}^3$,

$N_{NO} = -0.466 \times 0.075 \times 1.144 \times 10^{-3} = 4 \times 10^{-5} \text{ kg/m}^2 - \text{s}$

(Ans a) . Further, $B = -5.608 \times 10^{-4}$ (very small & -ve). Also,

if effect of gas velocity is accounted, g^* must be appropriately evaluated from h_{cof} and $\omega_{NO,w}$ freshly determined .

Evaporation - Large B - L37($\frac{8}{12}$)

Prob: A 50 μm liquid n-hexane (C_6H_{14}) droplet ($\rho_l = 659 \text{ kg} / \text{m}^3$, $h_{fg} = 335 \text{ kJ/kg}$, $T_{bp} = 273 + 69 = 342 \text{ K}$) evaporates in a stationary pure nitrogen environment at 1 atm and 850 K.

Calculate the evaporation time (a) using $T_w = T_{bp}$ and (b) using $T_w = T_l$ and Equilibrium evaporation assumption .

Take $k_m = 0.0478 \text{ W/m-K}$, $c_{pm} = 2.434 \text{ kJ/kg-K}$ and $M_{mix} = 57$.

Soln - Part (a): Since $T_w = T_{bp}$, from lecture 34 (slide 10)

$$B_{mh} = \frac{c_{pm} (T_\infty - T_w)}{h_{fg}} = \frac{2.434 (800 - 342)}{335} = 3.69$$

$$t_{\text{evap}} = \frac{\rho_l D_{wi}^2}{8 (k_m / c_{pm}) \ln (1 + B_{mh})} = 0.0067 \text{ s (Ans)}$$

where $\Gamma_h = k_m / c_{pm} = 19.39 \times 10^{-6} \text{ kg/m-s}$.

Soln (Contd) - 1 - L37($\frac{9}{12}$)

Soln - Part (b): Here $T_w = T_l$ is not known. Hence, we must adopt iterative solution. Thus, with $q_L = 0$

$$\begin{aligned}\dot{m}_w &= 4 \pi r_w \Gamma_h \ln \left\{ 1 + \frac{c_{pm} (T_\infty - T_w)}{h_{fg}} \right\} \\ &= 4 \pi r_w \Gamma_m \ln \left(1 + \frac{\omega_{v,\infty} - \omega_{v,w}}{\omega_{v,w} - 1} \right)\end{aligned}$$

where from lecture 32 (slide 10), $T_m = 0.5 (T_w + T_\infty) = 596$ K,
 $D_{C_6H_{14}-N_2} = 8 \times 10^{-6} \times (596/300)^{1.5} = 22.4 \times 10^{-6}$ kg/m-s.

$$\begin{aligned}\frac{\omega_{v,\infty} - 1}{\omega_{v,w} - 1} &= \left\{ 1 + \frac{c_{pm} (T_\infty - T_w)}{h_{fg}} \right\}^{Le} \quad \text{where} \\ \omega_{v,w} &= \frac{M_v}{M_{mix}} \exp \left\{ - \frac{h_{fg} M_v}{R_u} \left(\frac{1}{T_w} - \frac{1}{T_{bp}} \right) \right\}\end{aligned}$$

Soln (Contd) - 2 - L37($\frac{10}{12}$)

where $M_v = 86 \text{ kg/kmol}$ and $\rho_m = p/(R_{mix} T_m) = 1.165 \text{ kg/m}^3$.
Hence, $Le = \Gamma_h/\Gamma_m = 19.39/(22.4 \times 1.165) = 0.743$. To carry out iterations, we assume T_w and evaluate $\omega_{v,w}$ and properties until the 1st relation on the previous slide is satisfied within a tolerance. The answer is $\omega_{v,w} \simeq 0.7794$ and $T_w = 322.1 \text{ K}$.
Properties are: $c_{pm} = 2009.5$, $k_m = 0.045$, $Le = 0.888$, $M_{mix} = 43.5$. Hence, $B_h = 3.167$ and with $\omega_{v,\infty} = 0$, $B_m = 3.53$. Hence

$$\begin{aligned} t_{\text{evap}} &= \frac{\rho_l D_{wi}^2}{8 (k_m/c_{pm}) \ln(1 + B_h)} \\ &= \frac{659 \times (50 \times 10^{-6})^2}{8 \times 22.4 \times 10^{-6} \times \ln(1 + 3.167)} = 0.00644 \text{ s (Ans)} \end{aligned}$$

This result show that our assumption of $T_w = T_{bp}$ in part (a) is reasonable, although the droplet temperature is now nearly 20°C less than the boiling point.

Liquid Droplet Burning - L37($\frac{11}{12}$)

Prob: A 50 μm liquid n-hexane (C_6H_{14}) droplet ($\rho_l = 659 \text{ kg} / \text{m}^3$, $h_{fg} = 335 \text{ kJ/kg}$, $T_{bp} = 273 + 69 = 342 \text{ K}$) burns in stationary air at 1 atm and 300 K. Assume $T_w = T_{bp}$. Take $k_m = 0.1645 \text{ W/m-K}$, $c_{pm} = 2.839 \text{ kJ/kg-K}$, $\Delta h_c = 45.1 \text{ MJ/kg}$. Calculate (a) initial burn rate and (b) burning time .

Soln: Part (a) In this problem $r_{st} = (6 + 14/4) \times 32/86 = 3.535$. Hence,

$$\begin{aligned} B &= \frac{c_{pm} (T_\infty - T_{bp}) + (\omega_{O_2, \infty} / r_{st}) \Delta H_c}{h_{fg}} \\ &= \frac{2.839 \times (300 - 342) + (0.232/3.535) \times 45.1 \times 10^3}{335} = 8.48 \end{aligned}$$

Notice the large B-value in burning.

Soln (Contd) - L37($\frac{12}{12}$)

Hence, initial burning rate is

$$\begin{aligned} \dot{m}_w &= 4 \times \pi \times \Gamma_h \times r_{w,i} \times \ln(1 + B) \\ &= 4 \times \pi \times 5.83 \times 10^{-5} \times \frac{50}{2} \times 10^{-6} \times \ln(1 + 8.48) \\ &= 4.12 \times 10^{-8} \text{ kg/s. (Ans)} \end{aligned}$$

Further, the burning time is

$$\begin{aligned} t_{\text{burn}} &= \frac{\rho_l D_{wi}^2}{8 (k_m / c_{pm}) \ln(1 + B_h)} \\ &= \frac{659 \times (50 \times 10^{-6})^2}{8 \times 58.3 \times 10^{-6} \times \ln(1 + 8.48)} = 0.00157 \text{ s (Ans)} \end{aligned}$$