CRYOGENIC ENGINEERING

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Lecture No - 10

Earlier Lecture

- For a real gas, J – T coefficient $\left(\frac{\partial T}{\partial p}\right)_h$ depends on $\mathbf{T_{INV}}$.
- The isentropic expansion of a gas always results in cooling irrespective of its initial state.
- J – T expansion is normally used where phase changes are required, while an isentropic expansion is used for single phase fluids.
- The isentropic expansion coefficient is $\left(\frac{\partial T}{\partial p}\right)_s$

Earlier Lecture

- The gases like Air, N_2, show J – T cooling when expanded at room temperature while He, H_2, Neon are required to be precooled to result in J – T cooling.
- In a thermodynamic ideal system, all the gas that is compressed gets liquefied.
- Using the ideal thermodynamic cycle, one can calculate the ideal work requirement for liquefaction of unit mass of a given gas.
- This Ideal Work requirement depends on the initial condition of the gas (p and T).

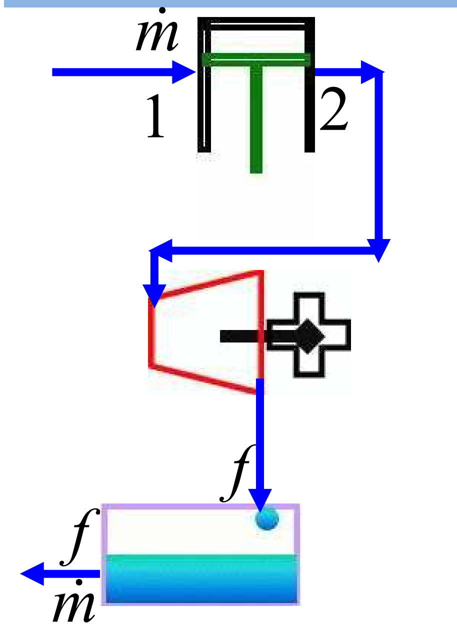
Outline of the Lecture

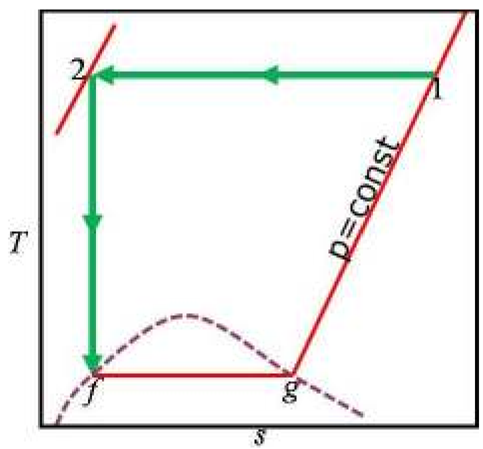
Topic : Gas Liquefaction and Refrigeration Systems (contd)

- Parameters of Gas Liquefaction systems

- Linde – Hampson system
  - Liquid yield
  - Work requirement
  - Optimization of liquid yield

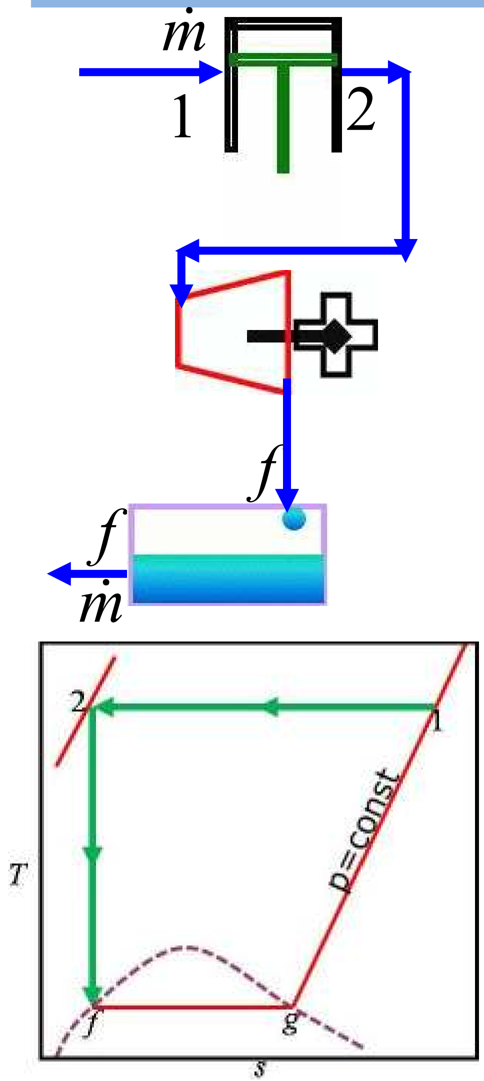
Introduction





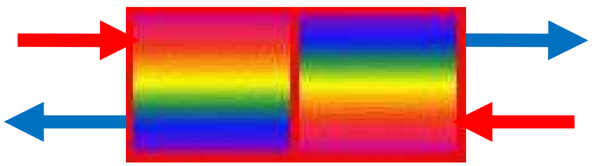
- As seen earlier, the schematic of an Ideal system and its T – s diagram are as shown.
- The processes of compression and expansion are from **1→2** and **2→f** respectively.
- The initial condition **1** of the gas determines the position of point **f**. The point **2** determines the final state of the gas after the compression process.

Introduction

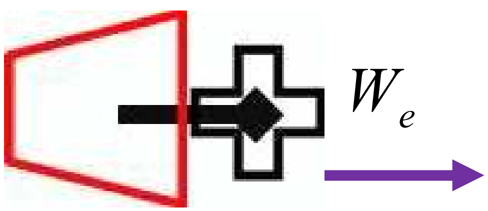


- Lets us take an example of N_2 and the initial condition at point **1** be ambient (1 bar (0.9869 atm), 300 K).
- The required pressure at point **2** to follow an Ideal cycle is more than 70000 bar (690846.3 atm).
- Such high pressures are impractical and hence there is a need to modify the system to lower the maximum pressures.

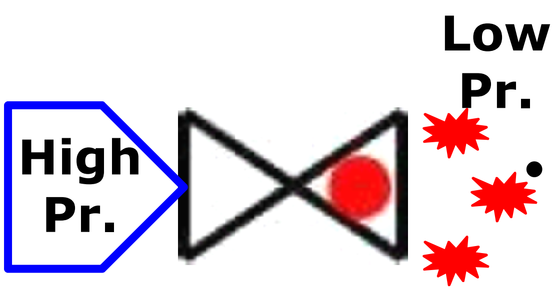
Introduction



- As stated earlier, devices like heat exchangers, J – T valve, turbo expanders can be used to modify the systems.



- The heat exchangers are used to conserve cold and J – T devices are used to achieve lower temperatures.

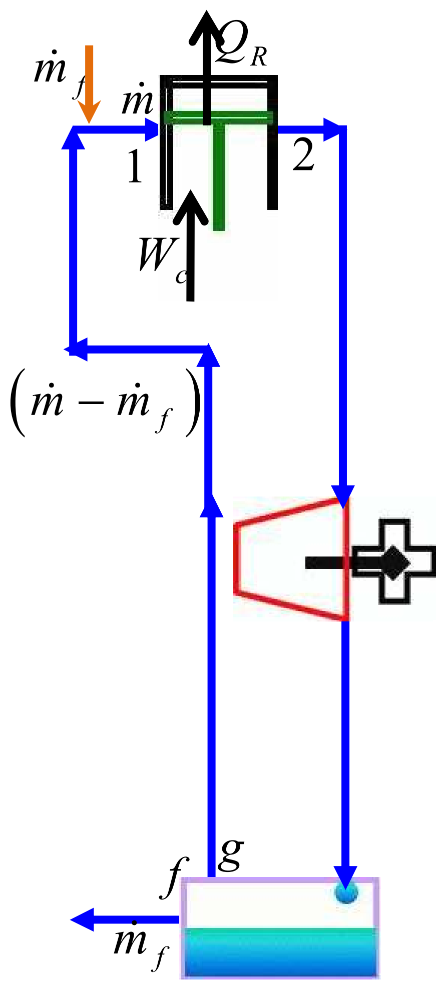


- The following slides explain various cycles that are used for gas liquefaction.

Gas Liquefaction Parameters

- In the refrigeration systems, the Carnot COP is often used as a benchmark to compare the performances.
- On the similar lines, there is a need to compare different liquefaction systems.
- In liquefaction systems, an ideal cycle is used as a benchmark to compare the performances.
- Different ratios and functions are defined to give a qualitative and quantitative information of different liquefaction systems.

Gas Liquefaction Parameters



Performance Parameters	
Work/unit mass of gas compressed	$-\frac{W}{\dot{m}_1}$
Work/unit mass of gas liquefied	$-\frac{W}{\dot{m}_f}$
Compressor isothermal efficiency	$\eta_{c,iso}$
Compressor mechanical efficiency	$\eta_{c,mech}$
Figure of Merit (FOM)	$\frac{W_i}{W}$
Fraction of total gas liquefied	$y = \dot{m}_f / \dot{m}_1$

Fundamentals

Sign Convention

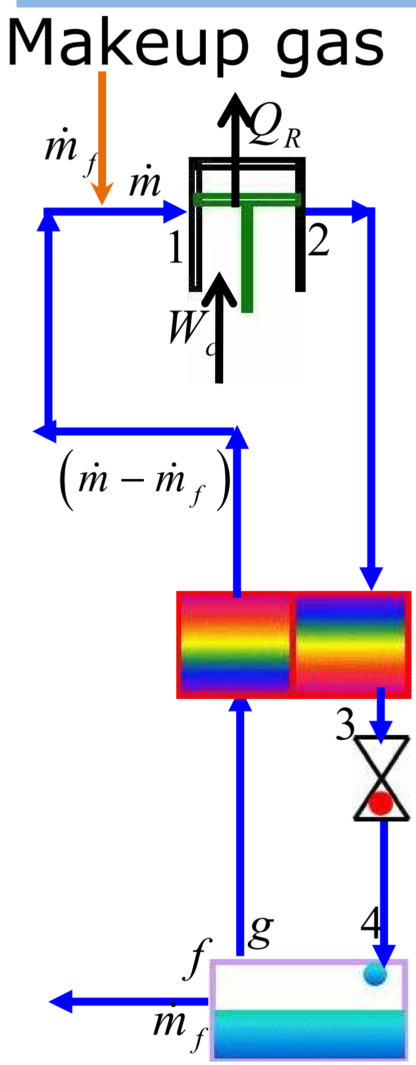
- The work done by the system is taken as positive.
- The heat transferred to the system is taken as positive.

Pressure Measurement

- Bar or Pascal is the S.I. unit. The conversion table is as follows.

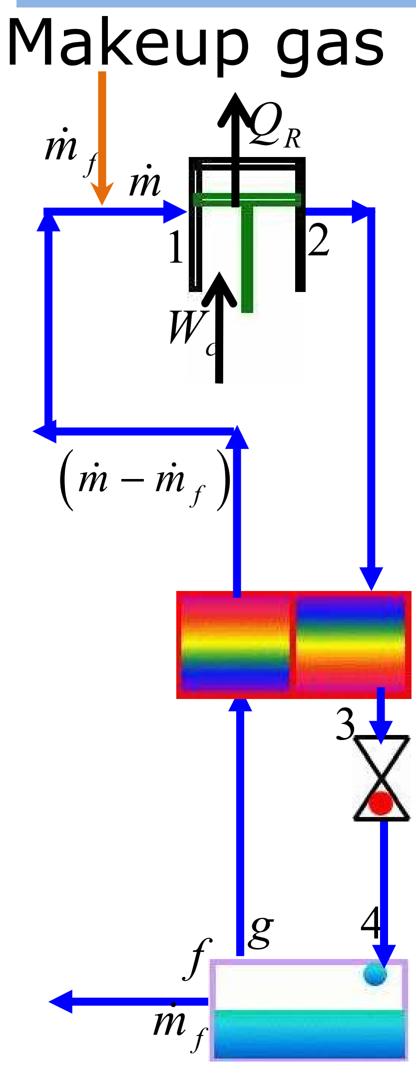
Pressure
1 Pa = 1 N/m^2
1 bar = 10^5 Pa
1 atm = 1.01325 bar

Linde – Hampson System



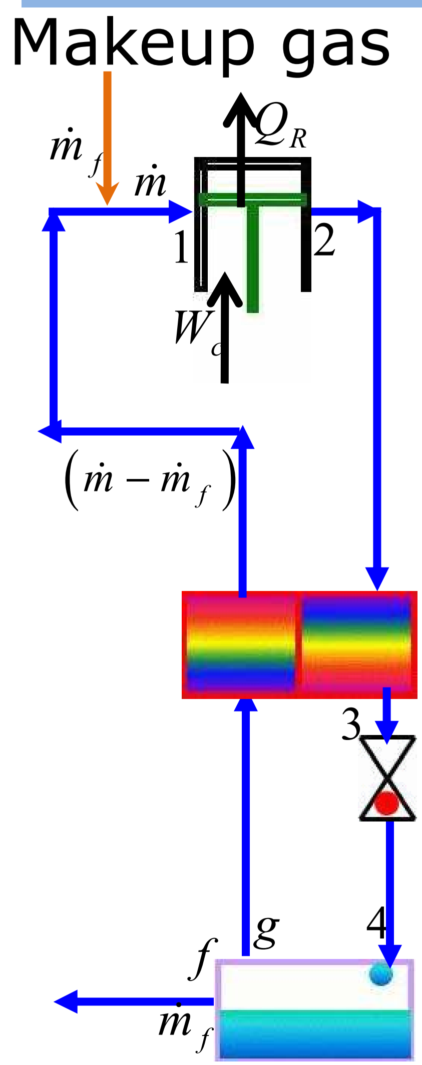
- The salient features of this system are as follows.
- Linde – Hampson cycle consists of compressor, heat exchanger and a J – T expansion device.
- Only a part of the gas that is compressed, gets liquefied.
- Being an open cycle, the mass deficit occurring is replenished by a Makeup Gas connection.

Linde – Hampson System



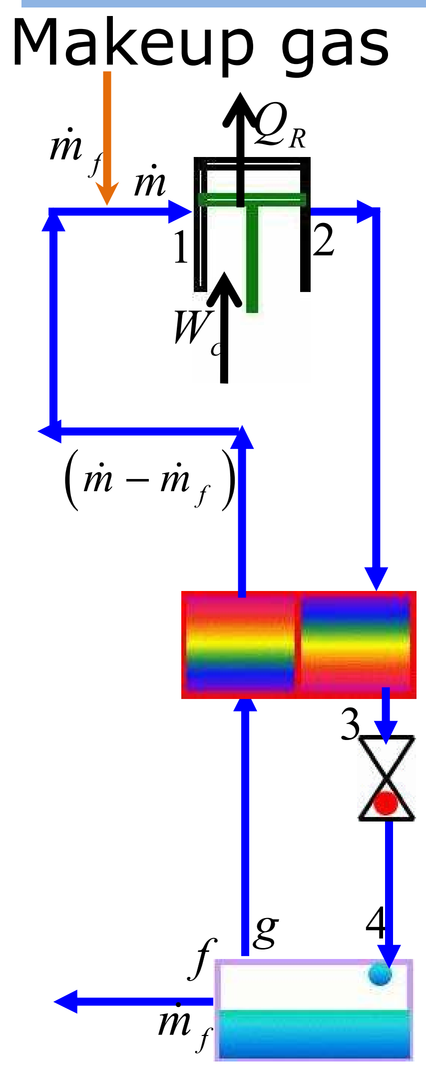
- All the processes are assumed to be ideal in nature and there are no irreversible pressure drops in the system.
- Compression process is isothermal while the J – T expansion is isenthalpic.
- The system incorporates a Two-Fluid heat exchanger which is assumed to be 100% effective.

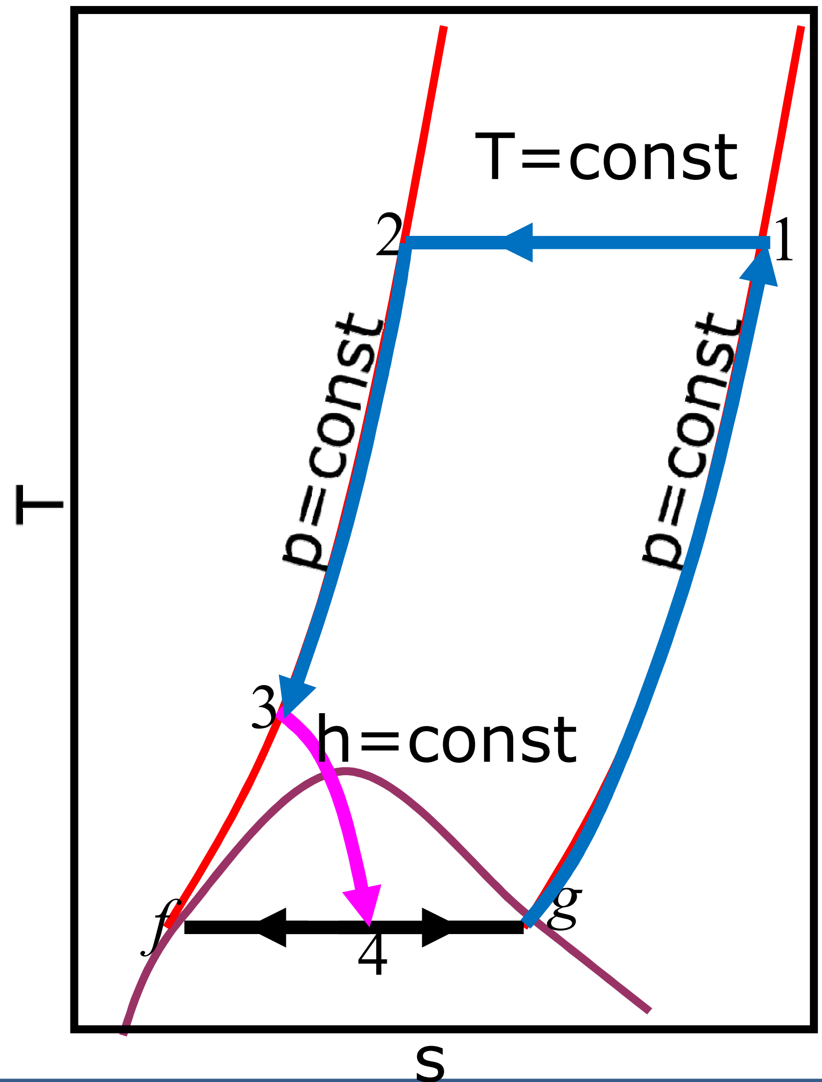
Linde – Hampson System



- The heat exchange process is an isobaric process and it is used to conserve cold in the system.
- That is, the stream of gas (**2→3**) is cooled by the stream of gas (**g→1**).
- The J – T expansion device is used for phase change of gas stream to liquid stream by lowering the temperature.

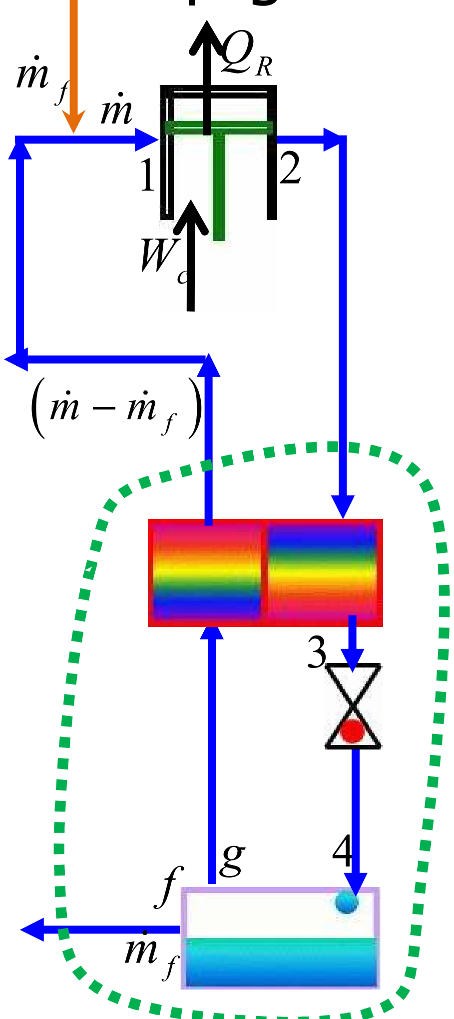
Linde – Hampson System





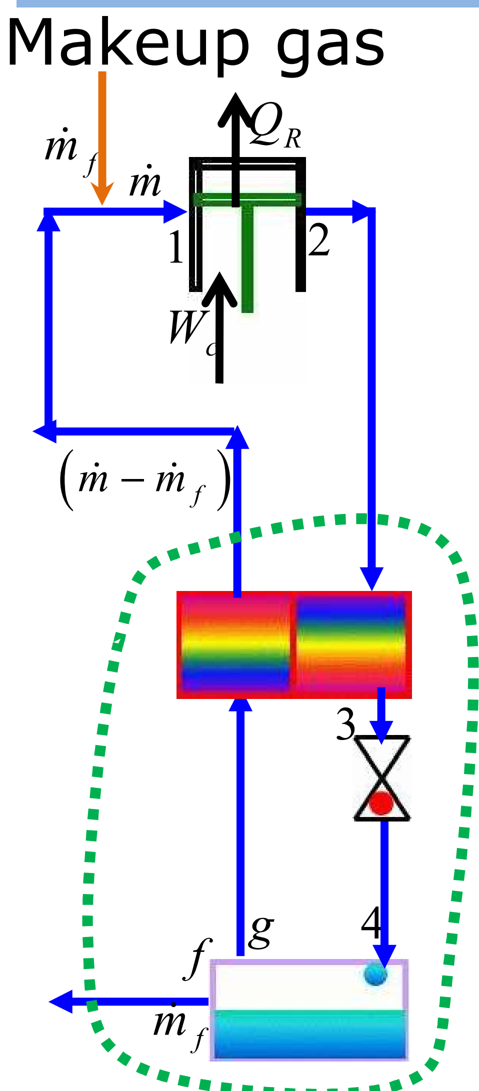
Linde – Hampson System

Makeup gas



- Consider a control volume for this system as shown in the figure.
- It encloses the heat exchanger, J – T device and the liquid container.
- The 1st Law of Thermodynamics is applied to analyse the system.
- The changes in the velocities and datum levels are assumed to be negligible.

Linde – Hampson System



- The quantities entering and leaving this control volume are as given below.

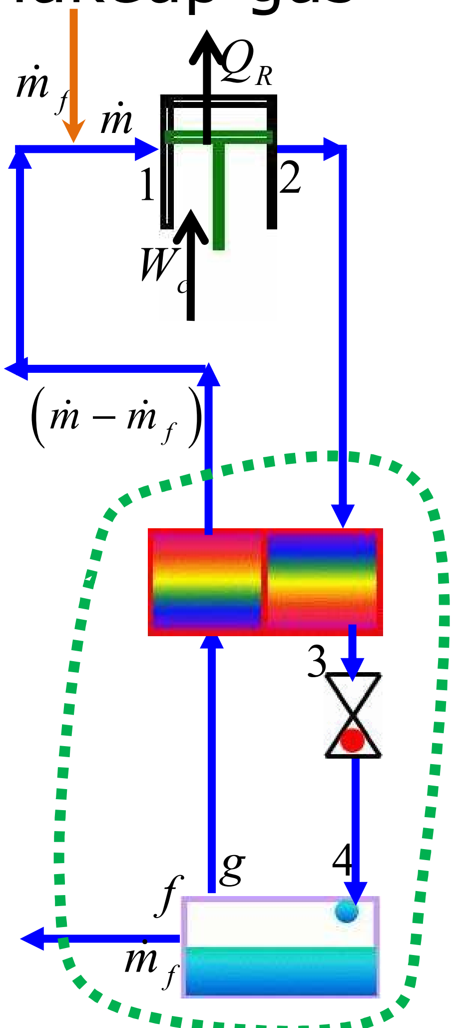
IN	OUT
m @ 2	$(m - m_f)$ @ 1
	m_f @ f

- Using 1st Law , we get

$$E_{in} = E_{out}$$

$$\dot{m}h_2 = \left(\dot{m} - \dot{m}_f\right)h_1 + \dot{m}_f h_f$$

Linde – Hampson System



- Rearranging the terms, we have

$$\frac{\dot{m}_f}{\dot{m}} = \left(\frac{h_1 - h_2}{h_1 - h_f} \right)$$

- The fraction of gas liquefied or liquid yield is defined as

$$\frac{\dot{m}_f}{\dot{m}} = y = \left(\frac{h_1 - h_2}{h_1 - h_f} \right)$$

- **y** depends on the initial conditions and the compression pressure.

Linde – Hampson System

$$y = \left(\frac{h_1 - h_2}{h_1 - h_f} \right)$$

- The values of $\mathbf{h_1}$ and $\mathbf{h_f}$ are governed by the initial conditions, which are often ambient.
- In order to maximize **y**, the value of $\mathbf{h_2}$ should be as small as possible.
- To have a minimum $\mathbf{h_2}$, the change in enthalpy for a given change in pressure should be zero at temperature $\mathbf{T_1}$.

Linde – Hampson System

- Mathematically, $$\left(\frac{\partial h}{\partial p}\right)_{T_1=T_2} = 0$$
- Using calculus, for variables enthalpy (h), pressure (p) and temperature (T), we have seen earlier that

$$\left(\frac{\partial h}{\partial p}\right)_T \left(\frac{\partial p}{\partial T}\right)_h \left(\frac{\partial T}{\partial h}\right)_p = -1$$

- Substituting the J – T coefficient and the C_p for the second and third terms respectively, we have

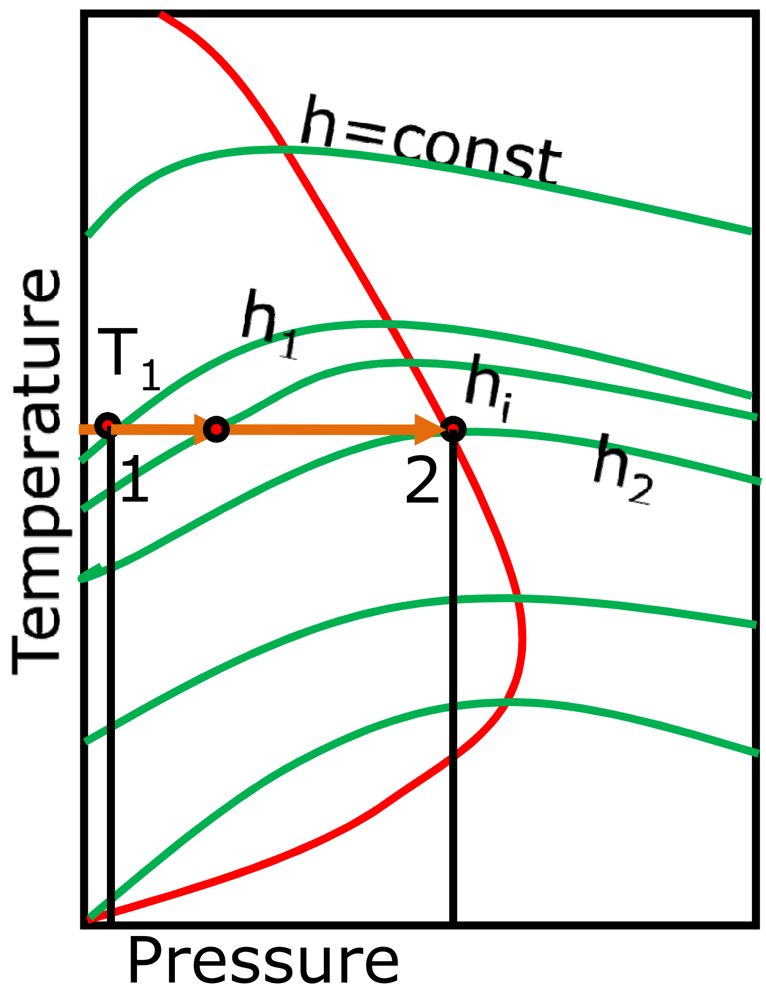
$$\mu_{JT} C_p = 0$$

Linde – Hampson System

$$\mu_{JT} C_p = 0$$

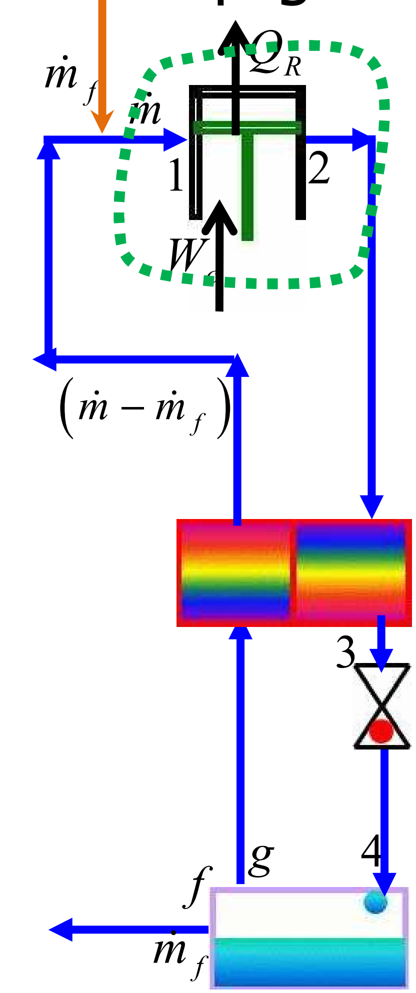
- C_p is a positive quantity and hence cannot be zero.
- Therefore, $\mu_{JT} = 0$
- It implies that, in order to maximize **y**, the state **2** should lie on the inversion curve for a particular gas at the temperature of compression process.

Linde – Hampson System



- Consider three constant enthalpy lines on the T – P chart as shown in the figure.
- Here, $h_1 > h_i > h_2$.
- From the figure, it is clear that (**h_1 - h_2**) is maximum, when the point **2** lies on the inversion curve, so that the **y** is maximum.

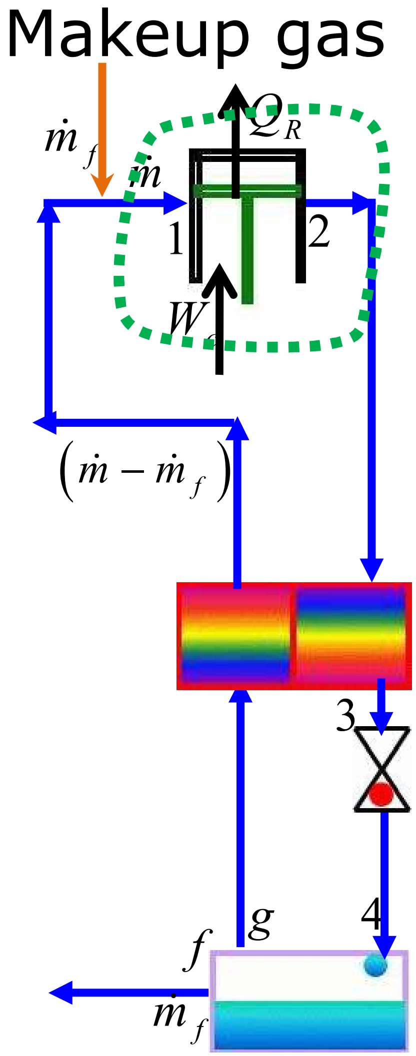
Linde – Hampson System



- The work requirement for a Linde – Hampson system can be derived by considering a control volume enclosing the compressor.
- The quantities entering and leaving this control volume are as given below.

IN	OUT
m @ 1	m @ 2
$-W_c$	$-Q_R$

Linde – Hampson System



- Using 1st Law for the following table, we get

IN	OUT
m @ 1	m @ 2
$-W_c$	$-Q_R$

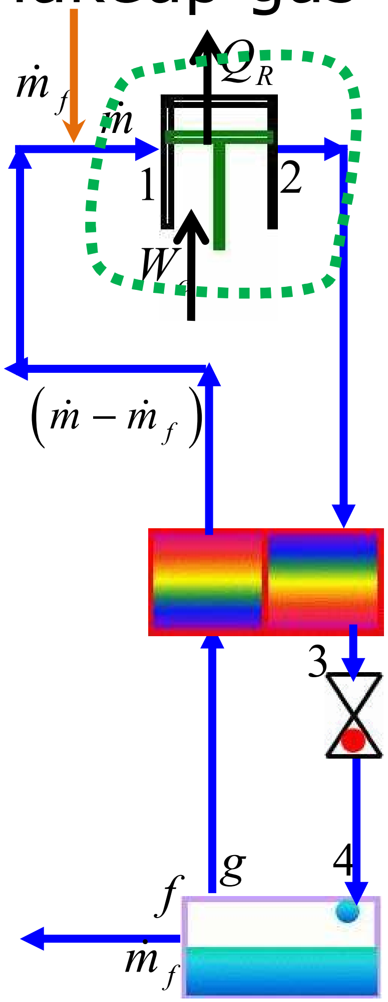
$$E_{in} = E_{out}$$

$$\dot{m}h_1 - W_c = \dot{m}h_2 - Q_R$$

- Rearranging the terms, we have

$$Q_R - W_c = \dot{m}(h_2 - h_1)$$

Linde – Hampson System



$$Q_R - W_c = \dot{m}(h_2 - h_1)$$

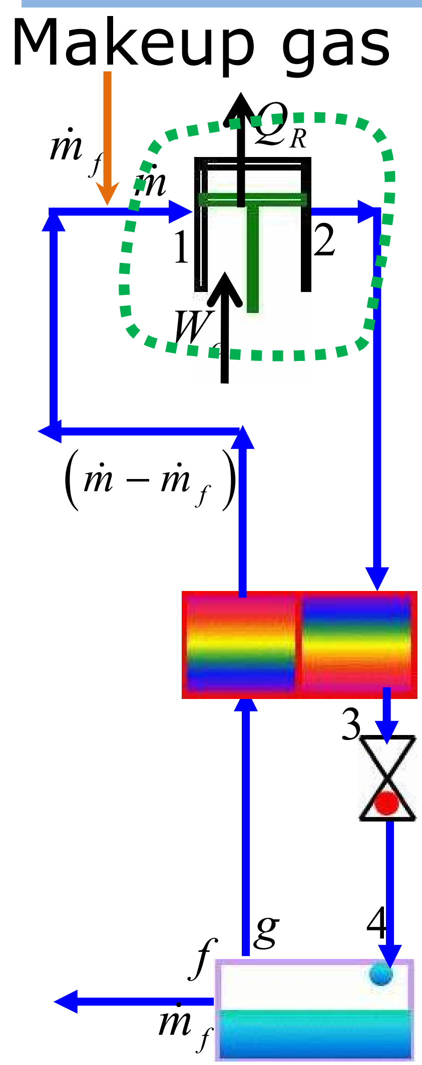
- The expression for Q_R can be obtained by using 2nd Law for an isothermal compression. It is given by,

$$Q_R = \dot{m}T_1(s_2 - s_1)$$

- Combining the above equations, the work required for a unit mass of gas compressed is

$$-\frac{W_c}{\dot{m}} = T_1(s_1 - s_2) - (h_1 - h_2)$$

Linde – Hampson System



$$-\frac{W_c}{\dot{m}} = T_1(s_1 - s_2) - (h_1 - h_2)$$

- The liquid yield **y** is given by

$$y = \frac{\dot{m}_f}{\dot{m}} = \left(\frac{h_1 - h_2}{h_1 - h_f}\right)$$

- Combining the above equations the work required for a unit mass of gas liquefied is

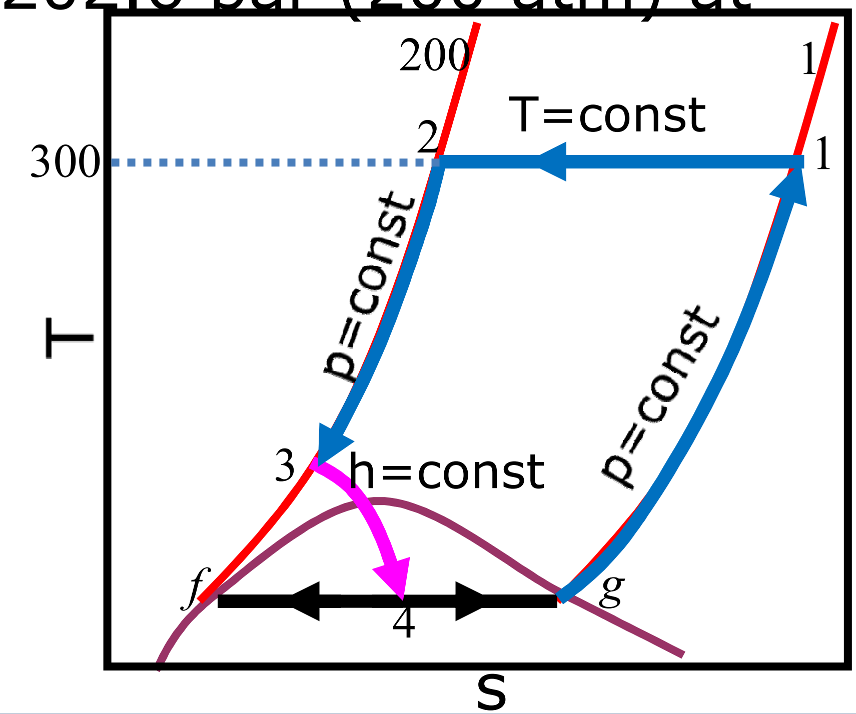
$$-\frac{W_c}{\dot{m}_f} = \frac{-\frac{W_c}{\dot{m}}}{y} = -\frac{W_c}{y\dot{m}}$$

Tutorial – 1

- Determine the liquid yield, the work per unit mass compressed and work for unit mass liquefied for a Linde – Hampson cycle with air as working fluid. The system is operated between 1.013 bar (1 atm) and 202.6 bar (200 atm) at 300 K.

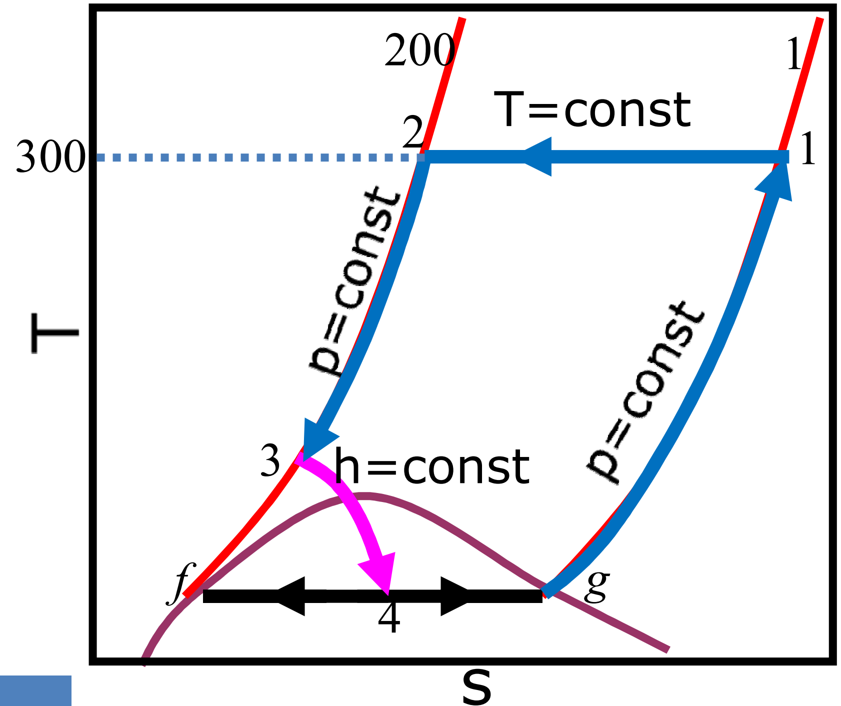
- **Step 1**

- The T – s diagram for a Linde – Hampson Cycle is as shown.



Tutorial – 1

- **Step 2**
- The state properties at different points are as given below.
- The properties are taken from NIST.



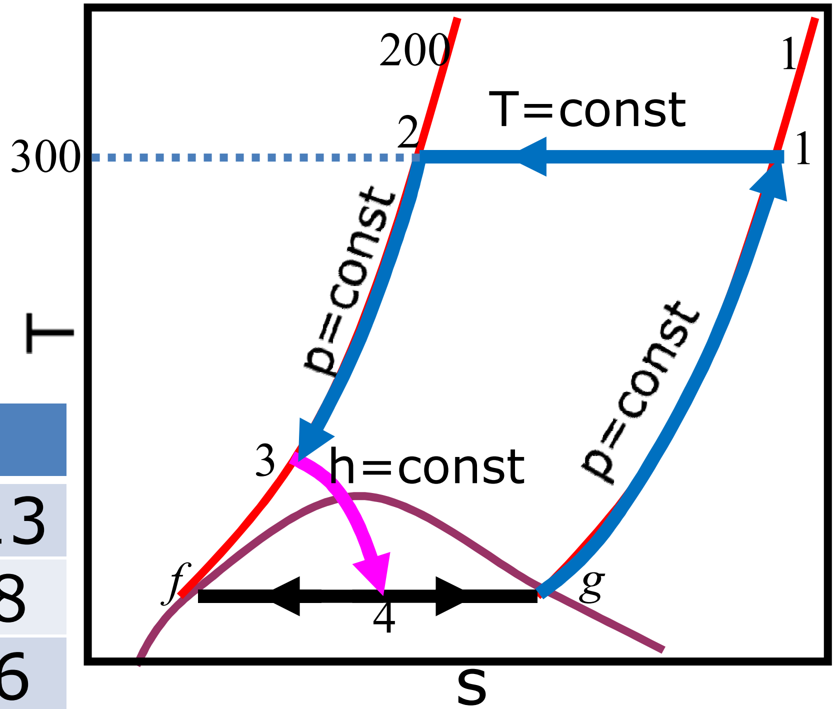
	1	2	f
p (bar)	1.013	202.6	1.013
T (K)	300	300	78.8
h (J/g)	28.47	-8.37	-406
s (J/gK)	0.10	-1.5	-3.9

Tutorial – 1

- **Liquid yield**

$$y = \left(\frac{h_1 - h_2}{h_1 - h_f} \right)$$

	1	2	f
p (bar)	1.013	202.6	1.013
T (K)	300	300	78.8
h (J/g)	28.47	-8.37	-406
s (J/gK)	0.10	-1.5	-3.9



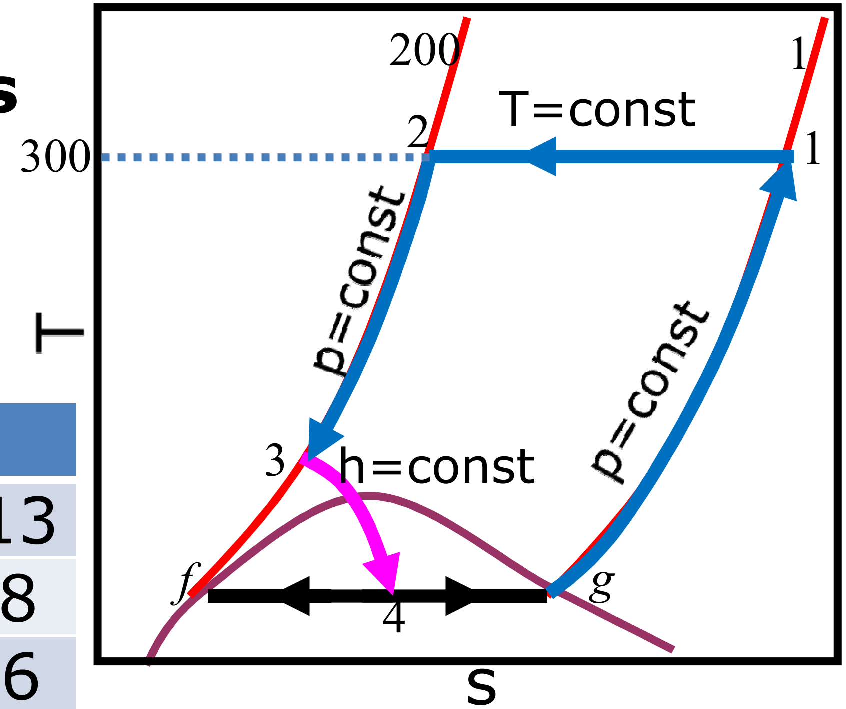
$$y = \left(\frac{h_1 - h_2}{h_1 - h_f} \right) = \left(\frac{28.47 + 8.37}{28.47 + 406} \right) = \left(\frac{36.84}{434.47} \right) = 0.085$$

Tutorial – 1

- **Work/unit mass of gas compressed**

$$-\frac{W_c}{\dot{m}} = T_1\left(s_1 - s_2\right) - \left(h_1 - h_2\right)$$

	1	2	f
p (bar)	1.013	202.6	1.013
T (K)	300	300	78.8
h (J/g)	28.47	-8.37	-406
s (J/gK)	0.10	-1.5	-3.9



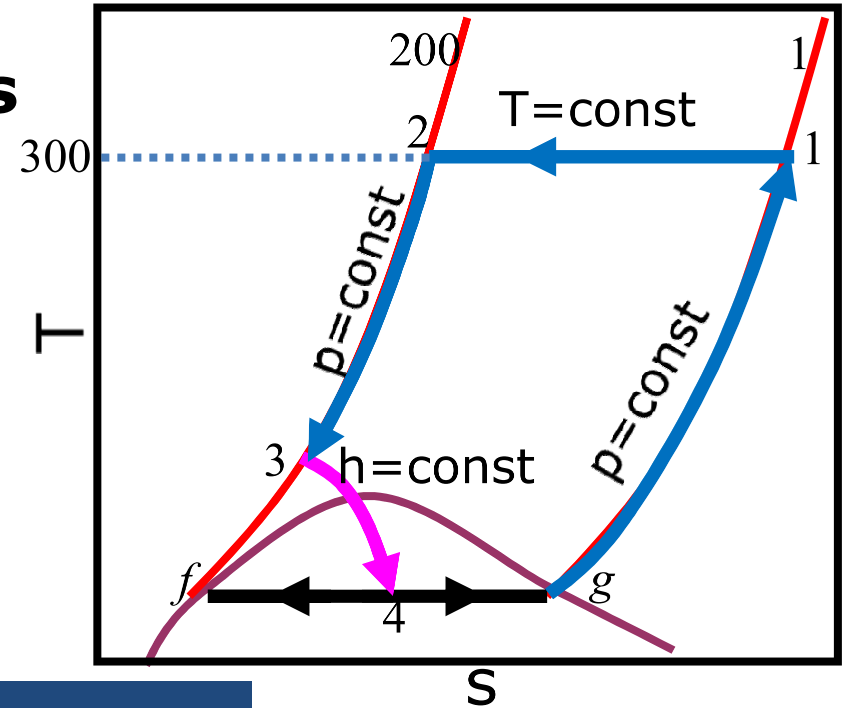
$$-\frac{W_c}{\dot{m}} = 300\left(0.1 + 1.5\right) - \left(28.47 + 8.37\right) = 443.16\ J/g$$

Tutorial – 1

- **Work/unit mass of gas liquefied**

$$-\frac{W_c}{\dot{m}} = 443.16$$

$$y = 0.085$$

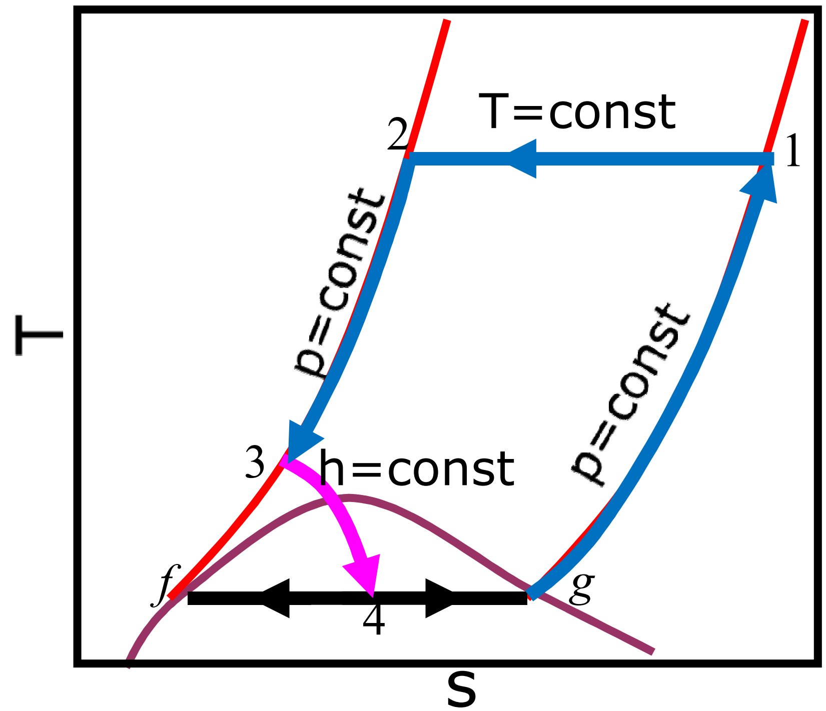


$$-\frac{W_c}{\dot{m}_f} = -\frac{W_c}{y\dot{m}} \qquad = \frac{443.16}{0.085} = 5213.64\ J/g$$

Tutorial – 2

- Determine the liquid yield for a Linde – Hampson cycle with Nitrogen as working fluid for the following operating conditions. Comment on the results.

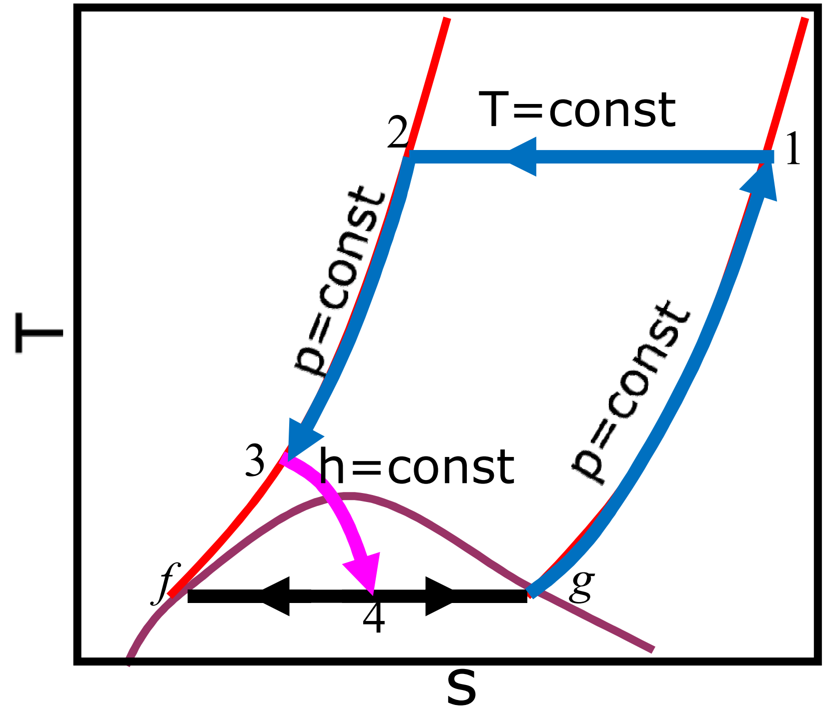
	Point 1	Point 2
I	300 K, 1 bar	300 K, 50 bar
II	200 K, 1 bar	200 K, 50 bar
III	300 K, 1 bar	300 K, 100 bar
IV	200 K, 1 bar	200 K, 100 bar



Tutorial – 2

Step : 1

- Assuming the heat exchange process to 100% effective, the T – s diagram for a Linde – Hampson Cycle is as shown.

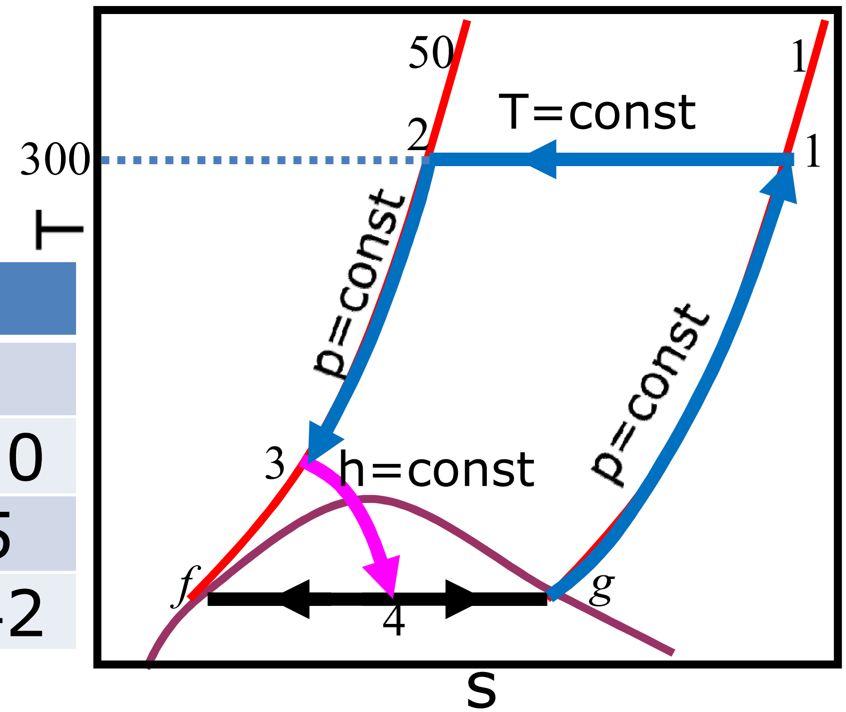


- In this tutorial , we assume that 1 atm = 1 bar.

Tutorial – 2

	Point 1	Point 2
I	300 K, 1 bar	300 K, 50 bar

	1	2	f
p (bar)	1	50	1
T (K)	300	300	77.0
h (J/g)	464	454	35
s (J/gK)	4.35	3.25	0.42

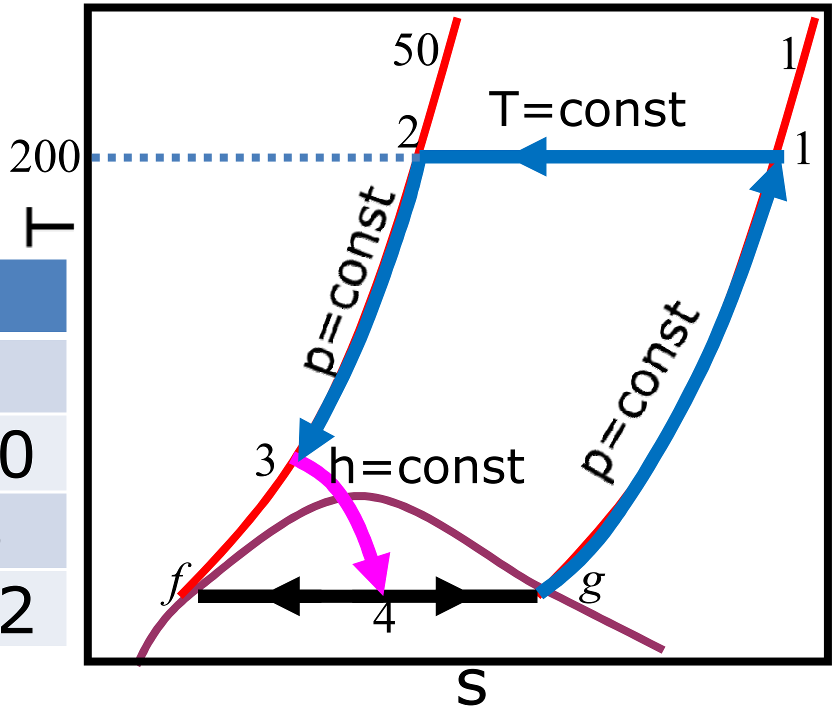


$$y_I = \left(\frac{h_1 - h_2}{h_1 - h_f} \right) = \left(\frac{464 - 454}{464 - 35} \right) = \left(\frac{10}{429} \right) = 0.023$$

Tutorial – 2

	Point 1	Point 2
II	200 K, 1 bar	200 K, 50 bar

	1	2	f
p (bar)	1	50	1
T (K)	200	200	77.0
h (J/g)	357	332	35
s (J/gK)	4.0	2.8	0.42

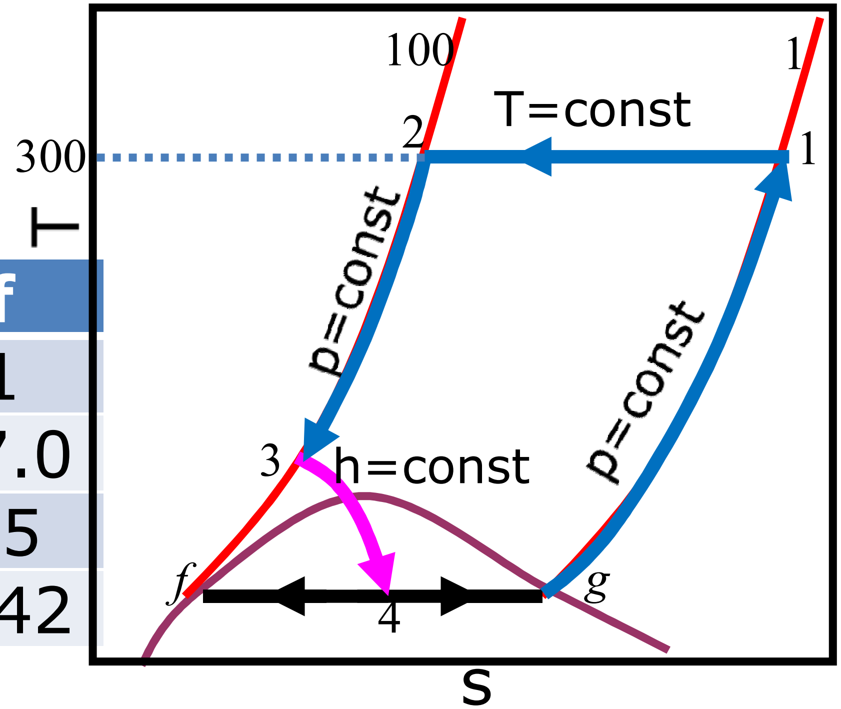


$$y_{II} = \left(\frac{h_1 - h_2}{h_1 - h_f}\right) = \left(\frac{357-332}{357-35}\right) = \left(\frac{25}{325}\right) = 0.076$$

Tutorial – 2

	Point 1	Point 2
III	300 K, 1 bar	300 K, 100 bar

	1	2	f
p (bar)	1	100	1
T (K)	300	300	77.0
h (J/g)	464	445	35
s (J/gK)	4.35	3.1	0.42

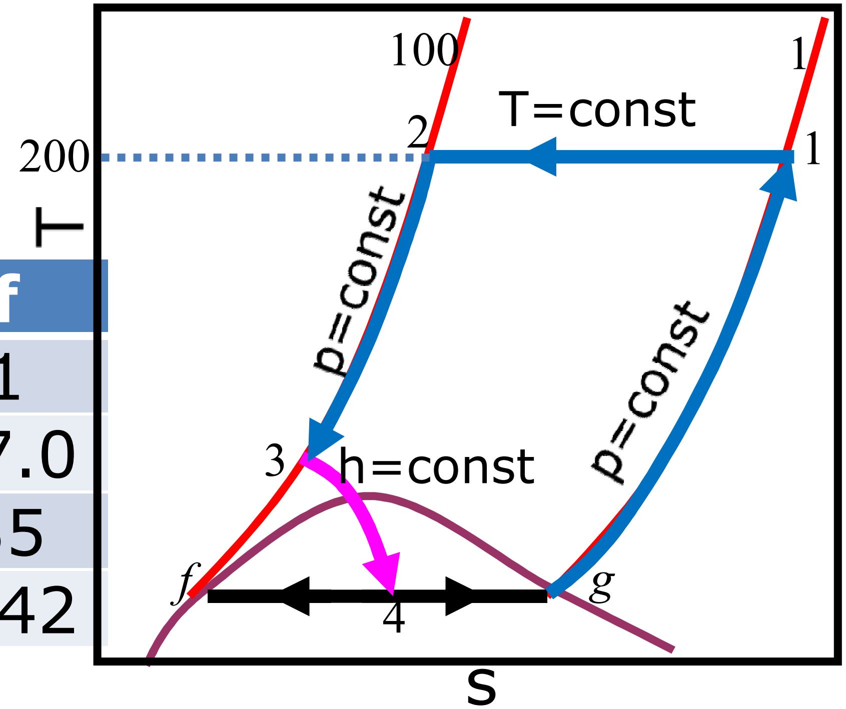


$$y_{III} = \left(\frac{h_1 - h_2}{h_1 - h_f}\right) = \left(\frac{464 - 445}{464 - 35}\right) = \left(\frac{19}{429}\right) = 0.044$$

Tutorial – 2

	Point 1	Point 2
IV	200 K, 1 bar	200 K, 100 bar

	1	2	f
p (bar)	1	100	1
T (K)	200	200	77.0
h (J/g)	357	312	35
s (J/gK)	4.0	2.5	0.42



$$y_{IV} = \left(\frac{h_1 - h_2}{h_1 - h_f}\right) = \left(\frac{357 - 312}{357 - 35}\right) = \left(\frac{45}{322}\right) = 0.14$$

Tutorial – 2

	Point 1	Point 2	y
I	300 K, 1 bar	300 K, 50 bar	0.023
II	200 K, 1 bar	200 K, 50 bar	0.076
III	300 K, 1 bar	300 K, 100 bar	0.044
IV	200 K, 1 bar	200 K, 100 bar	0.14

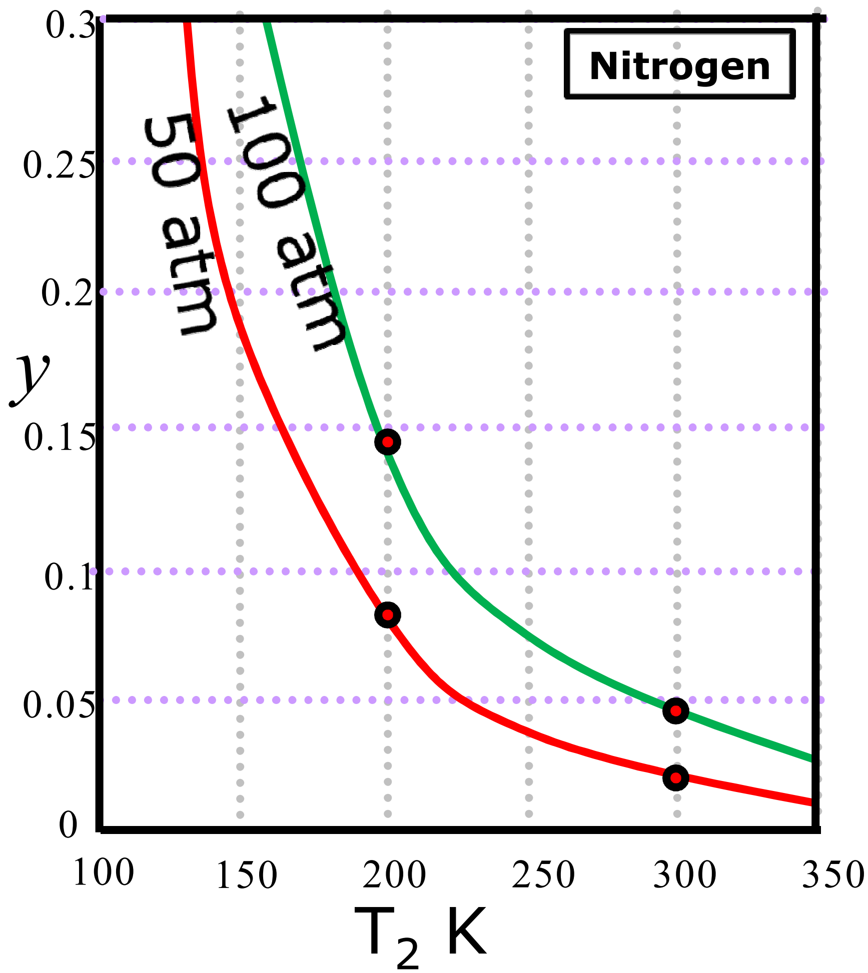
- As the compression pressure increases, the liquid yield **y** increases at a given compression temperature.

Tutorial – 2

	Point 1	Point 2	y
I	300 K, 1 bar	300 K, 50 bar	0.023
II	200 K, 1 bar	200 K, 50 bar	0.076
III	300 K, 1 bar	300 K, 100 bar	0.044
IV	200 K, 1 bar	200 K, 100 bar	0.14

- As the compression temperature decreases, the liquid yield **y** increases at a given compression pressure.

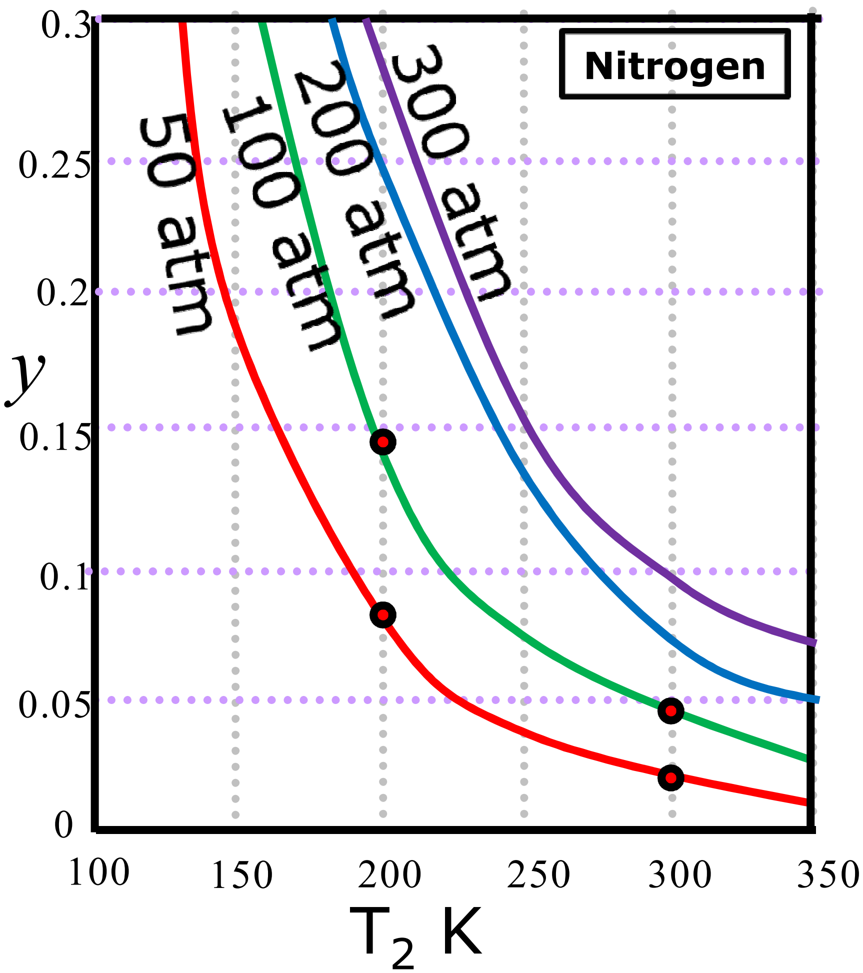
Liquid Yield : Nitrogen



- Plotting the values for the following conditions, we have

	Point 2	y
I	300 K, 50 bar	0.023
II	200 K, 50 bar	0.076
III	300 K, 100 bar	0.044
IV	200 K, 100 bar	0.14

Liquid Yield : Nitrogen



- Summarizing, we have
- As the compression pressure increases, the liquid yield **y** increases at a given compression temperature.
- As the compression temperature decreases, the liquid yield **y** increases at a given compression pressure.

Assignment

1. Consider a Linde – Hampson cycle with Nitrogen as working fluid. The system is operated between 1.013 bar (1 atm) and 101.3 bar (100 atm) at 300 K. Determine

    1. Liquid yield
    2. Work per unit mass compressed
    3. Work for unit mass liquefied.

2. Calculate the above parameters if the same cycle is operated with air as working fluid.

Assignment

3. Determine the liquid yield for a Linde – Hampson cycle with Nitrogen as working fluid for the following operating conditions. Comment on the results.

	Point 1	Point 2
I	250 K, 1 bar	250 K, 50 bar
II	300 K, 1 bar	300 K, 200 bar

Verify your answers from the yield versus temperature chart for Nitrogen.

Summary

- The Ideal cycle demands very high pressure which is impractical and hence modified cycles are proposed to lower the maximum pressure.
- An ideal cycle is used as a benchmark to compare the performances of different liquefaction systems.
- In a Linde – Hampson system, only a part of the gas that is compressed, gets liquefied.

Summary

- A heat exchanger is used in a Linde – Hampson system to conserve cold in the system. This process is an isobaric process and is assumed to be 100% effective.

- In order to maximize **y** for a Linde – Hampson system, the state **2** should lie on the inversion curve at the temperature of compression process.

- The work required for a unit mass of gas compressed for a Linde – Hampson system is

$$-\frac{W_c}{\dot{m}} = T_1\left(s_1 - s_2\right) - \left(h_1 - h_2\right)$$

Summary

- For a Linde – Hampson system following hold true.
  - As the compression pressure increases, the liquid yield **y** increases at a given compression temperature.
  - As the compression temperature decreases, the liquid yield **y** increases at a given compression pressure.

Thank You!