CRYOGENIC ENGINEERING
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Lecture No - 11

Earlier Lecture

- The Ideal thermodynamic cycle for gas liquefaction is impractical and hence modified cycles are proposed.
- An Ideal cycle is used as a benchmark and in effect, different ratios and functions are defined to compare various liquefaction systems.
- A Linde – Hampson cycle consists of a compressor, heat exchanger and a J – T expansion device. In this system, only a part of the gas that is compressed, gets liquefied.

Earlier Lecture

- The isobaric heat exchange process occurring in the heat exchanger is used to conserve cold and J – T expansion device is used for producing lower temperatures.

- The work required for a unit mass of gas compressed for a Linde – Hampson system is

$$-\frac{W_c}{\dot{m}} = T_1\left(s_1 - s_2\right) - \left(h_1 - h_2\right)$$

- The yield **y** is maximum when the state **2** (after compression) lies on the inversion curve at the temperature of the compression process.

Earlier Lecture

- For a Linde – Hampson system following hold true.

  - As the compression pressure increases, the liquid yield **y** increases for a given compression temperature.

  - As the compression temperature decreases, the liquid yield **y** increases for a given compression pressure.

Outline of the Lecture

Topic : Gas Liquefaction and Refrigeration Systems (contd)

- Basics of Heat Exchangers
- Effect of heat exchanger effectiveness on Linde – Hampson system
- Figure of Merit (FOM)

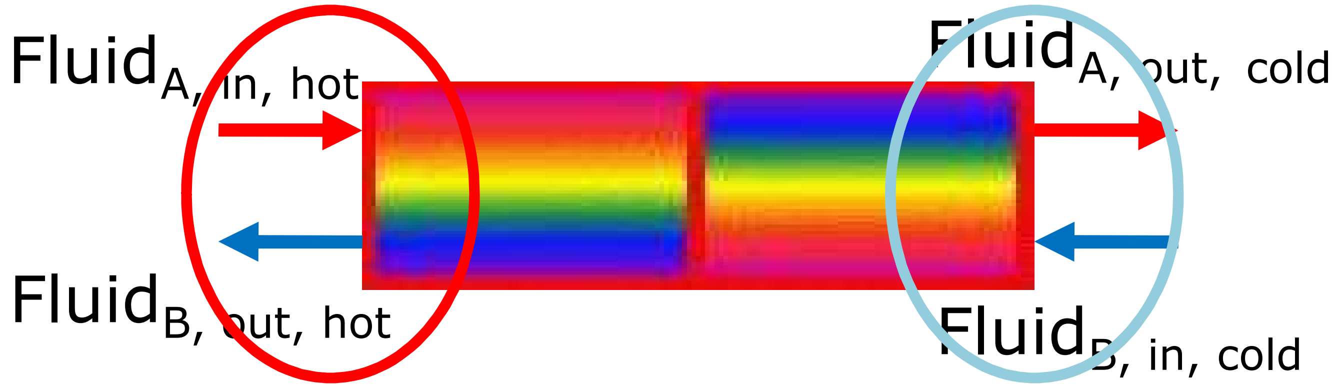
Heat Exchanger

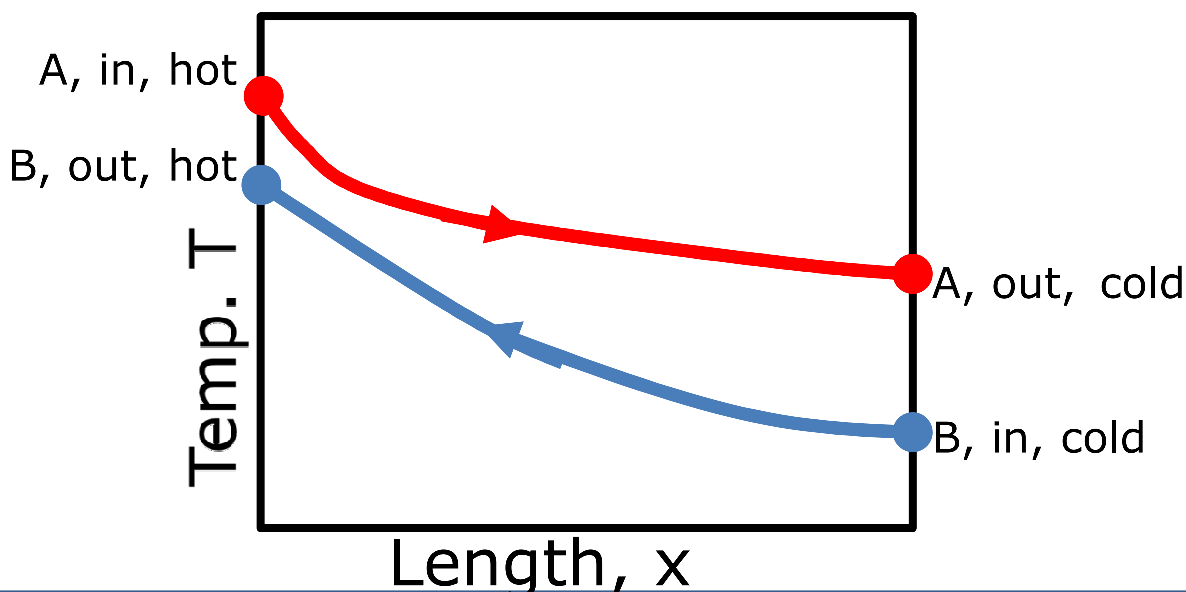
- A heat exchanger is a device in which the cooling effect from cold fluid is transferred to precool the hot fluid.

- It can either be a two – fluid type or a three – fluid type depending upon the number of inlets and outlets attached to the heat exchanger.

- The process of heat exchange occurs at a constant pressure and hence, it is an isobaric process.

Heat Exchanger





Heat Exchanger

- In order to specify the performance of the heat exchanger in terms of actual heat exchange occurring, a term "ε" is defined which is called as Heat Exchanger Effectiveness.

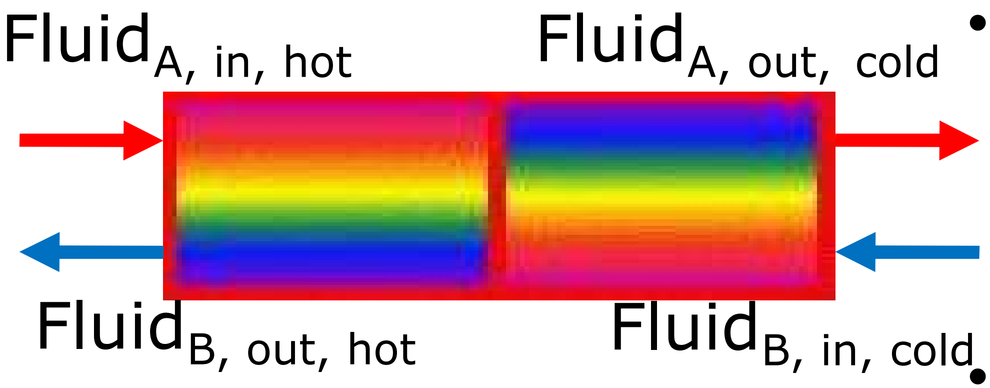
- It is defined as a ratio of actual heat transfer that is occurring to the maximum possible heat transfer that can occur theoretically.

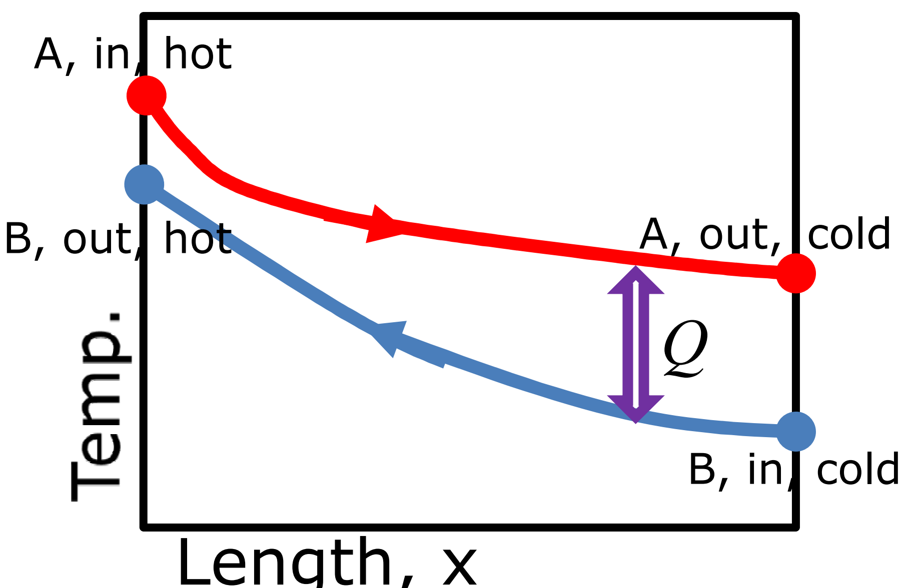
- Mathematically,

$$\varepsilon = \frac{Q_{act}}{Q_{max}}$$

- ε is a dimensionless number between 0 and 1.

Heat Exchanger Effectiveness





- The actual heat exchange is

$$Q_{act} = \dot{m}_B C_{PB} \left(T_{B,in} - T_{B,out} \right) = \dot{m}_A C_{PA} \left(T_{A,in} - T_{A,out} \right)$$

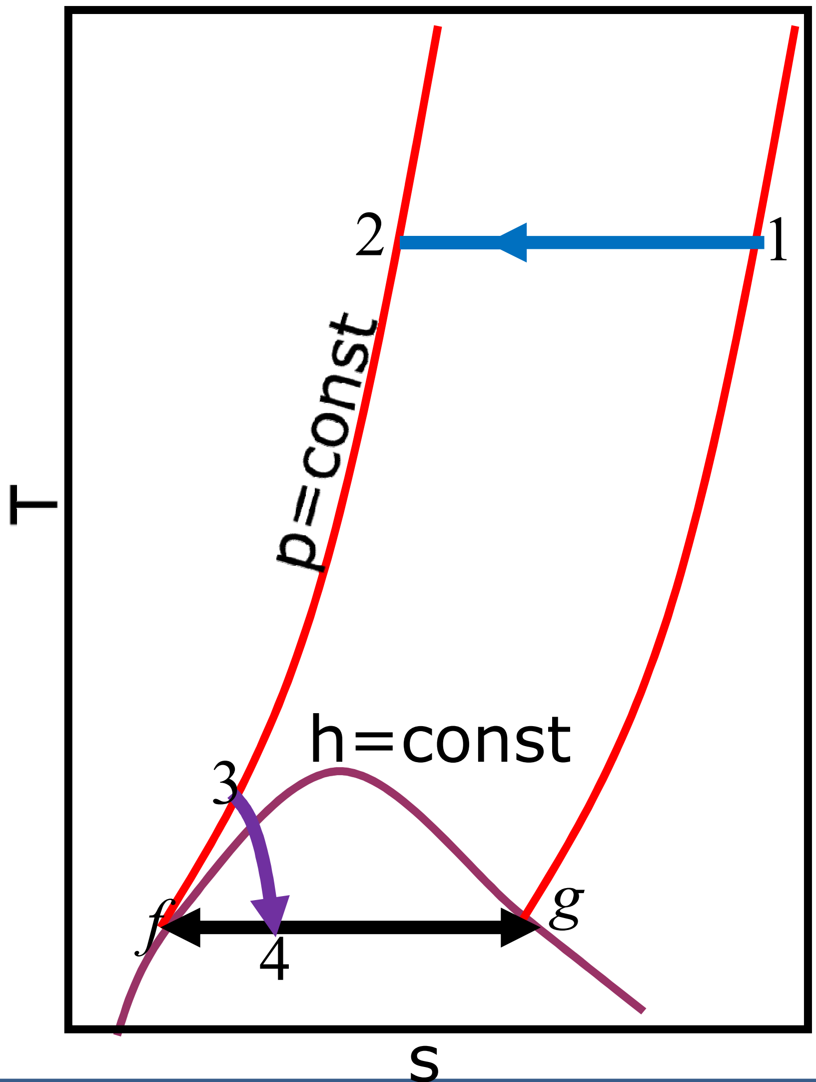
- The maximum possible heat exchange is

$$Q_{max} = \left(\dot{m} C_P \right)_{\min} \left(T_{B,in} - T_{A,in} \right)$$

- Effectiveness is

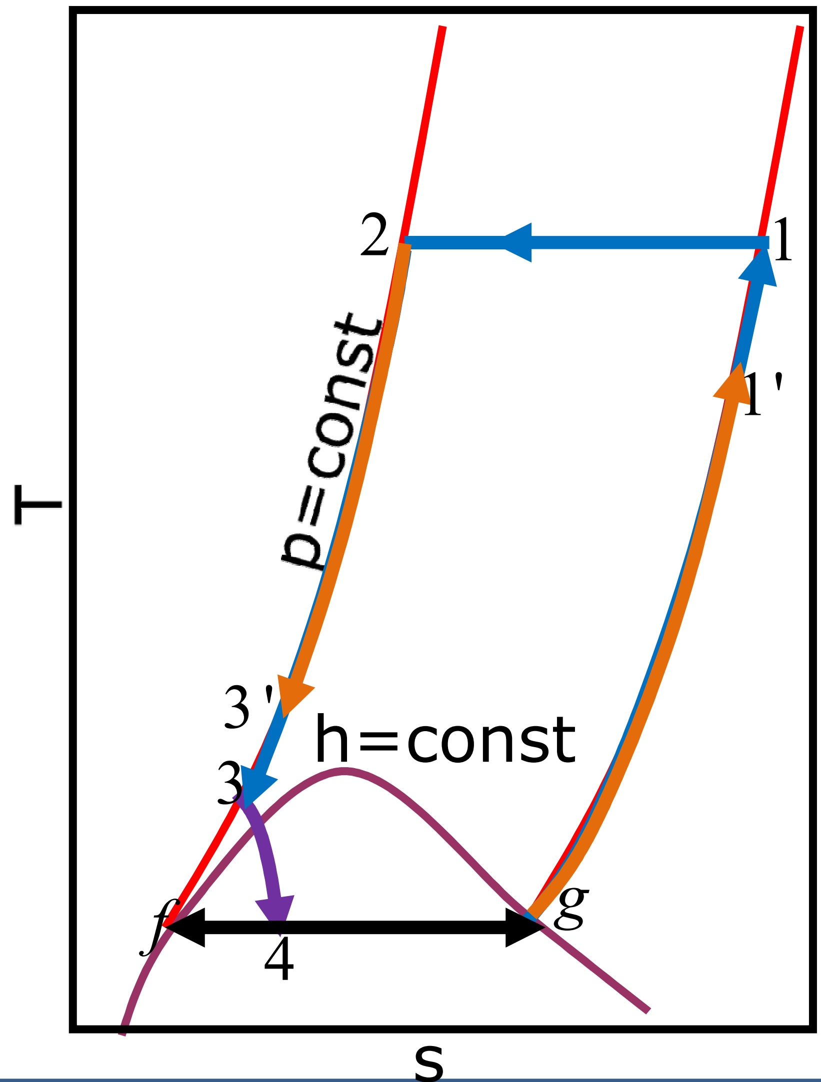
$$\varepsilon = Q_{act} / Q_{max}$$

Linde – Hampson System



- The isothermal compression process for a Linde – Hampson system on T – s diagram is from **1 → 2**.
- The Isenthalpic J – T expansion is from **3 → 4**.
- The gas and liquid states are given by **g** and **f**.

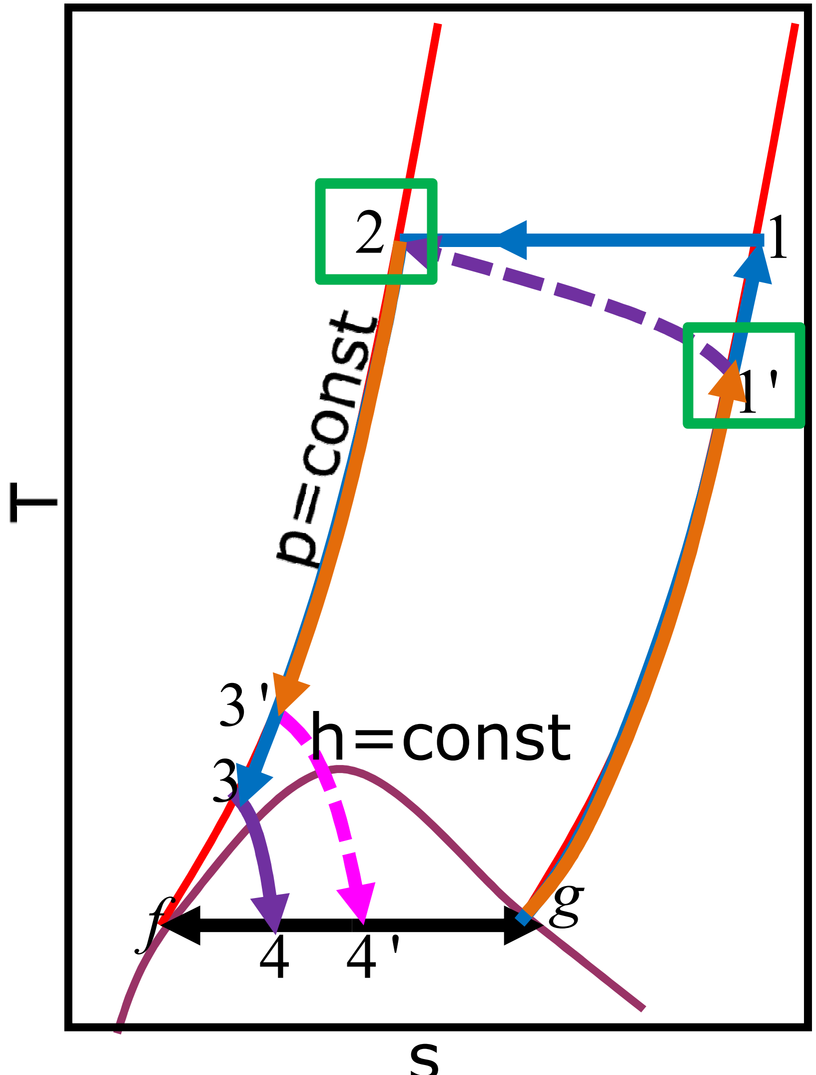
Linde – Hampson System



- If the ε is assumed to be 100%, the isobaric heat exchange would be
  - **2 → 3**
  - **g → 1**

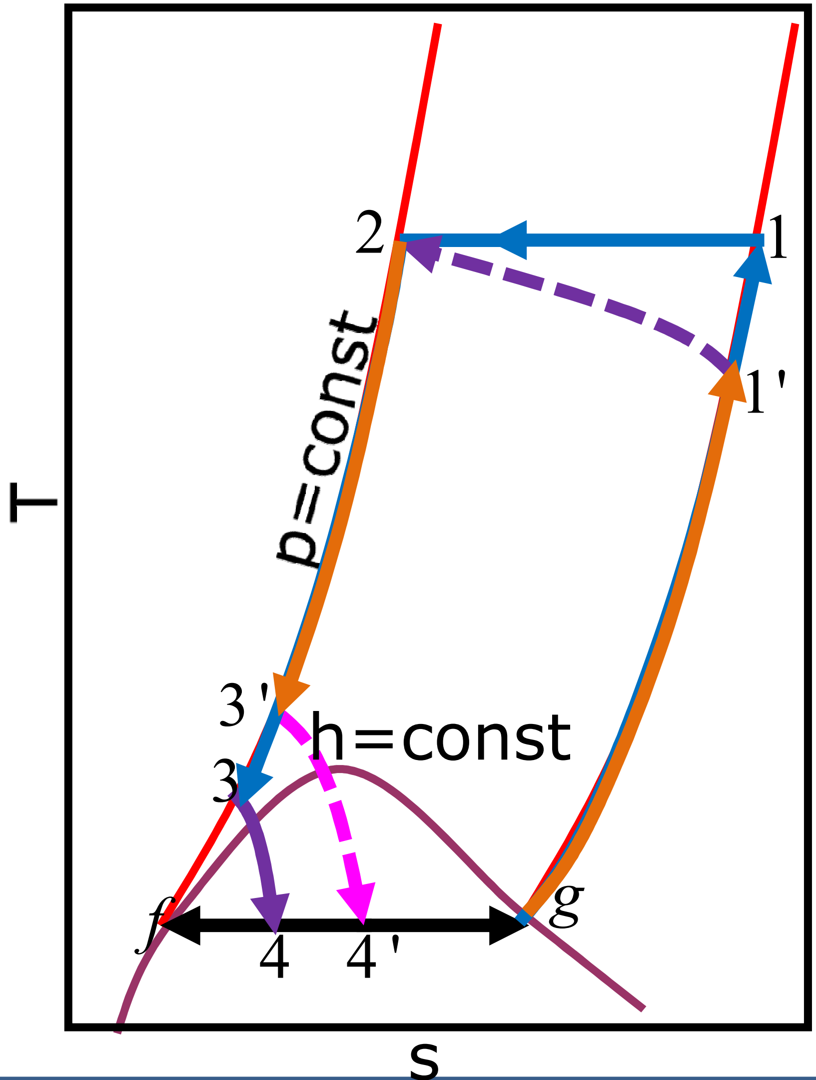
- In actual, the heat exchange is not perfect and hence, these processes are from
  - **2 → 3'**
  - **g → 1'**

Linde – Hampson System



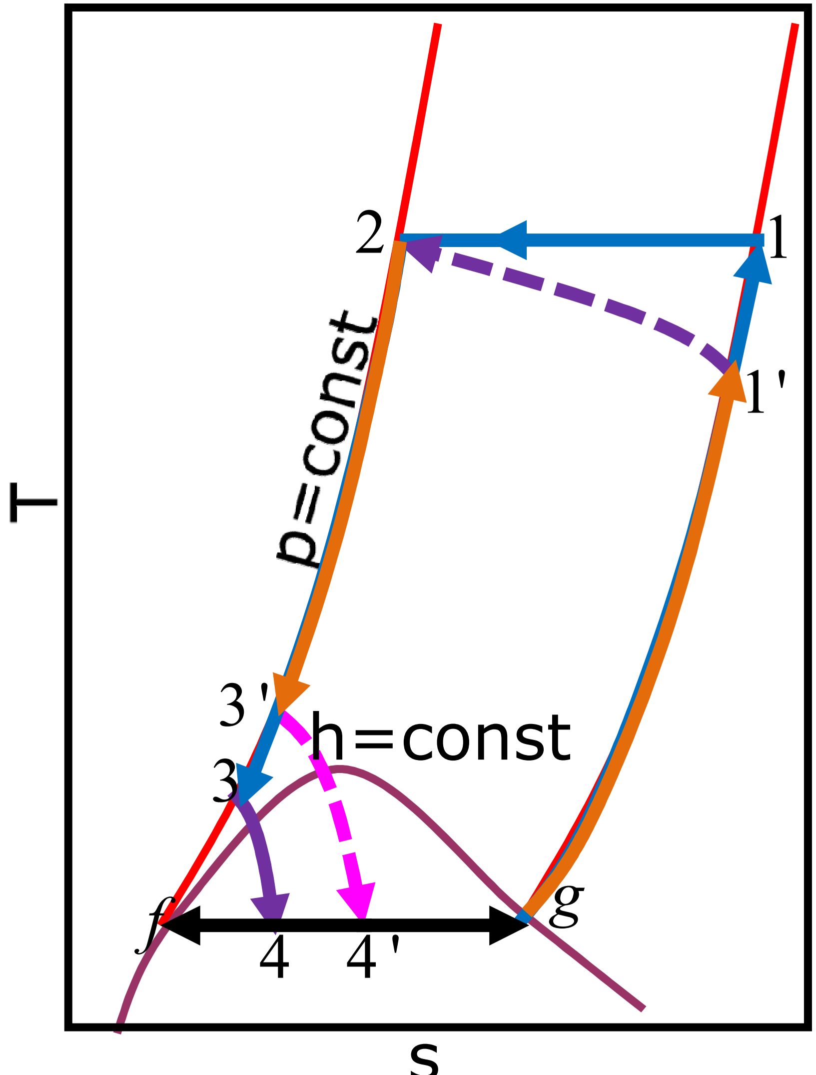
- In such a case, the gas is always compressed from the state **1'** to state **2**.
- The J – T expansion occurs from **3' → 4'** as shown in the figure.

Linde – Hampson System



- It is clear that the actual heat exchange is from **g → 1'**, where as the maximum possible heat exchange is from **g → 1**.
- On the other pressure line, the actual heat exchange is from **2 → 3'**, where as the maximum possible heat exchange is from **2 → 3**.

Linde – Hampson System

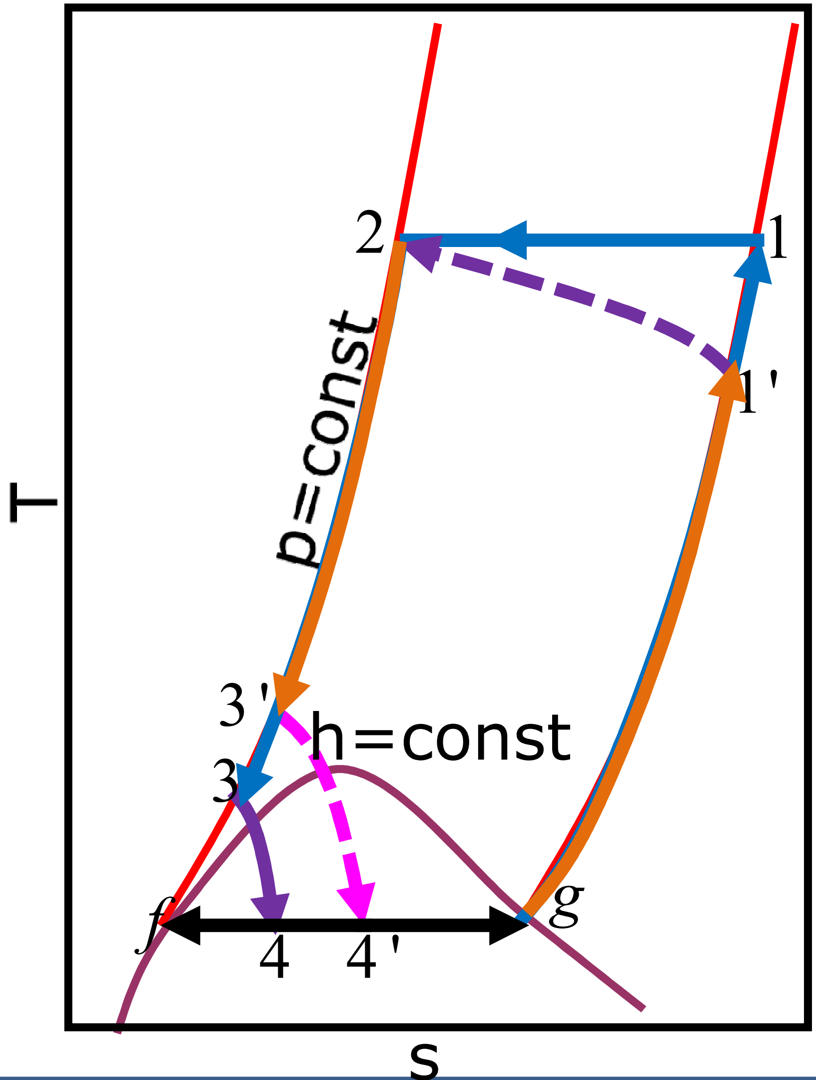


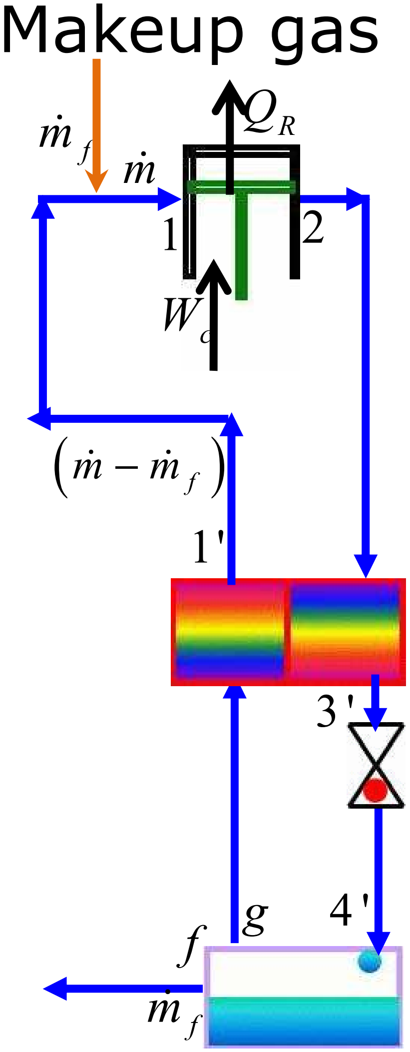
- In an ideal system, change in enthalpy on these two isobaric lines is equal.
- Therefore, the effectiveness is given as
- $$\varepsilon = \frac{h_{1'} - h_g}{h_1 - h_g}$$

OR

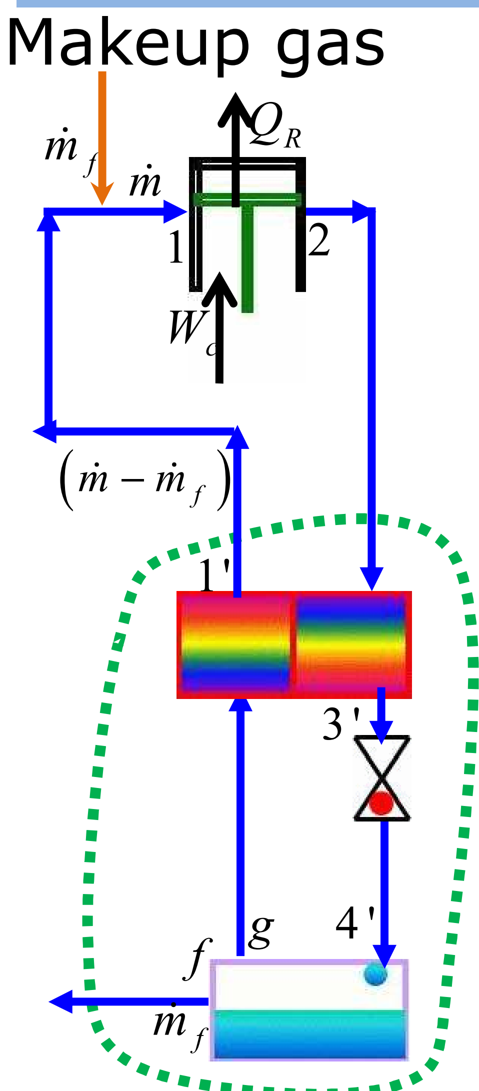
$$\varepsilon = \frac{h_{3'} - h_2}{h_3 - h_2}$$

Linde – Hampson System





Linde – Hampson System



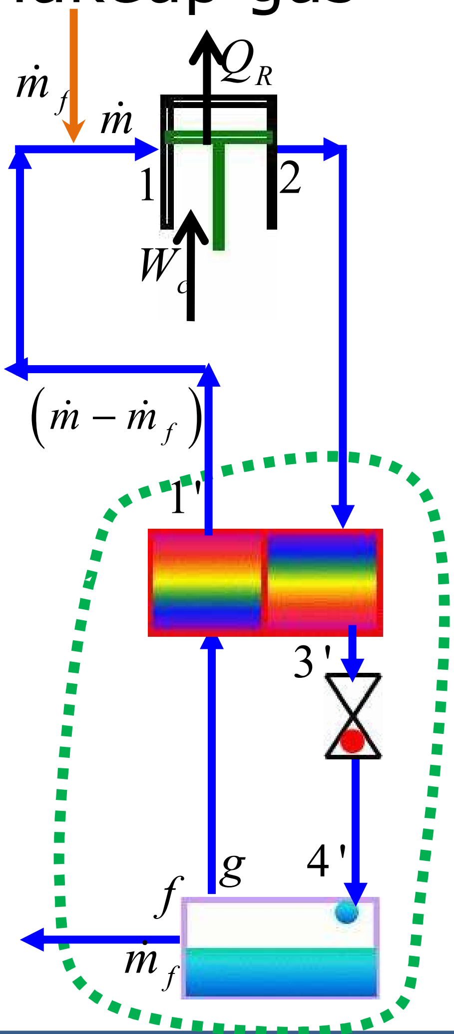
- Consider a control volume for this system as shown in the figure.
- The quantities entering and leaving this control volume are as given below.

IN	OUT
m @ 2	$(m - m_f)$ @ 1′
	m_f @ f

- Using the 1st Law, we have

$$\dot{m}h_2 = \left(\dot{m} - \dot{m}_f\right)h_{1'} + \dot{m}_f h_f$$

Linde – Hampson System



- Rearranging the terms, we have liquid yield **y** as

$$\frac{\dot{m}_f}{\dot{m}} = y = \left(\frac{h_{1'} - h_2}{h_{1'} - h_f} \right)$$

- As seen earlier, the effectiveness is

$$\varepsilon = \frac{h_{1'} - h_g}{h_1 - h_g}$$

- Rearranging the terms, we have

$$h_{1'} = \varepsilon \left(h_1 - h_g \right) + h_g$$

Linde – Hampson System

$$y = \frac{h_{1'} - h_2}{h_{1'} - h_f}$$

$$h_{1'} = \varepsilon \left(h_1 - h_g \right) + h_g$$

- From the above two equations, substituting one into another, we have **y** as

$$y = \frac{\left(h_1 - h_2 \right) - \left(1 - \varepsilon \right)\left(h_1 - h_g \right)}{\left(h_1 - h_f \right) - \left(1 - \varepsilon \right)\left(h_1 - h_g \right)}$$

Linde – Hampson System

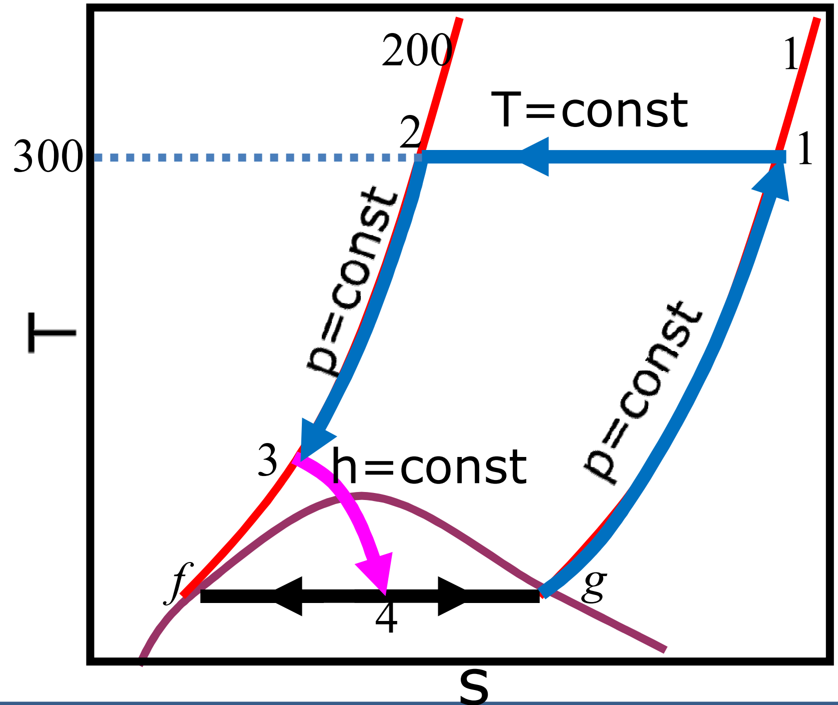
$$y = \frac{(h_1 - h_2) - (1 - \varepsilon)(h_1 - h_g)}{(h_1 - h_f) - (1 - \varepsilon)(h_1 - h_g)}$$

- The second term being negative, it should be minimum to maximize the yield **y**.
- All other parameters being constant for a given cycle, the effectiveness ε should be very close to 1.
- The next tutorial depicts the effect of the heat exchanger effectiveness on the liquid yield.

Tutorial – 1

- Determine the liquid yield for a Linde – Hampson cycle with air as working fluid when the system is operated between 1.013 bar (1 atm) and 202.6 bar (200 atm) at 300 K. The effectiveness of HX is 100%, 95%, 90% and 85%. Comment on the results.

- The T – s diagram of Linde – Hampson system if assumed the heat exchanger to be 100% effective is as shown.



Tutorial – 1

$$y = \frac{(h_1 - h_2) - (1-\varepsilon)(h_1 - h_g)}{(h_1 - h_f) - (1-\varepsilon)(h_1 - h_g)}$$

$$\varepsilon_1 = 1$$

	1	2	f	g
p (bar)	1.013	202.6	1.013	1.013
T (K)	300	300	78.8	78.8
h (J/g)	28.47	-8.37	-406	-199
s (J/gK)	0.10	-1.5	-3.9	-1.29

$$y_1 = \frac{(28.47+8.37) - (1-1)(28.47+199)}{(28.47+406) - (1-1)(28.47+199)} = 0.085$$

$$y_1 = 0.085$$

Tutorial – 1

$$y=\frac{(h_1-h_2)-(1-\varepsilon)(h_1-h_g)}{(h_1-h_f)-(1-\varepsilon)(h_1-h_g)}$$

$$\varepsilon_2=0.95$$

	1	2	f	g
p (bar)	1.013	202.6	1.013	1.013
T (K)	300	300	78.8	78.8
h (J/g)	28.47	-8.37	-406	-199
s (J/gK)	0.10	-1.5	-3.9	-1.29

$$y_2=\frac{(28.47+8.37)-(1-0.95)(28.47+199)}{(28.47+406)-(1-0.95)(28.47+199)}=\frac{25.466}{423.1}=0.060$$

$$y_2=0.060$$

Tutorial – 1

$$y = \frac{(h_1 - h_2) - (1-\varepsilon)(h_1 - h_g)}{(h_1 - h_f) - (1-\varepsilon)(h_1 - h_g)}$$

$$\varepsilon_3 = 0.90$$

	1	2	f	g
p (bar)	1.013	202.6	1.013	1.013
T (K)	300	300	78.8	78.8
h (J/g)	28.47	-8.37	-406	-199
s (J/gK)	0.10	-1.5	-3.9	-1.29

$$y_3 = \frac{(28.47+8.37)-(1-0.90)(28.47+199)}{(28.47+406)-(1-0.90)(28.47+199)} = \frac{14.093}{411.7} = 0.034$$

$$y_3 = 0.034$$

Tutorial – 1

$$y = \frac{(h_1 - h_2) - (1 - \varepsilon)(h_1 - h_g)}{(h_1 - h_f) - (1 - \varepsilon)(h_1 - h_g)}$$

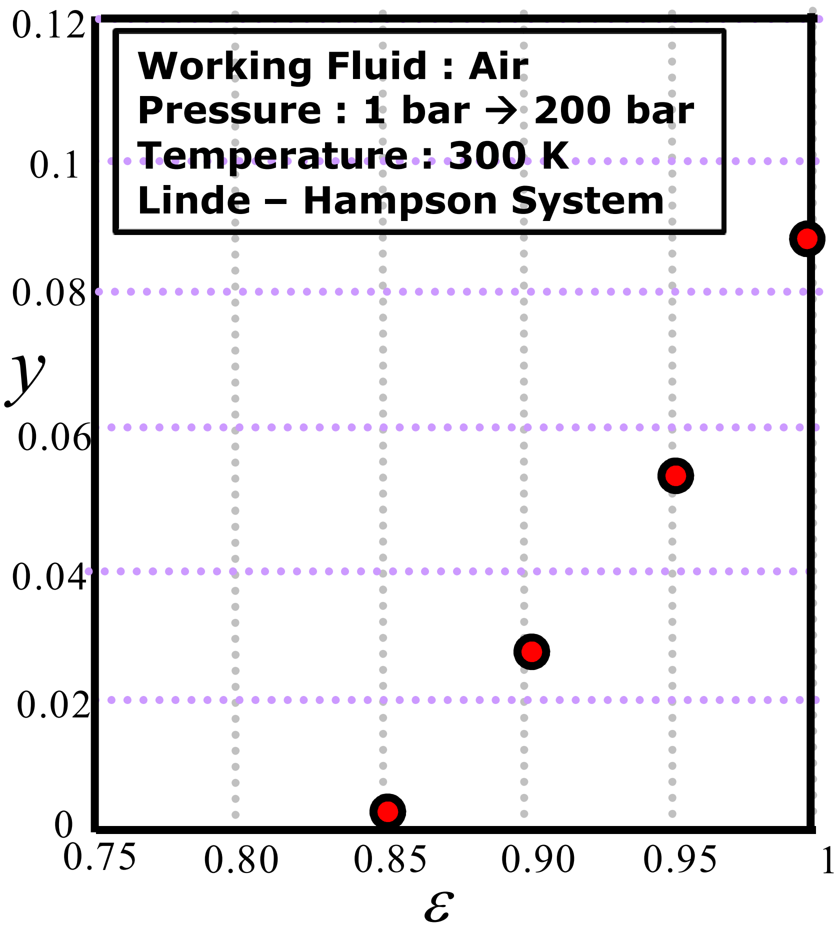
$$\varepsilon_4 = 0.85$$

	1	2	f	g
p (bar)	1.013	202.6	1.013	1.013
T (K)	300	300	78.8	78.8
h (J/g)	28.47	-8.37	-406	-199
s (J/gK)	0.10	-1.5	-3.9	-1.29

$$y_4 = \frac{(28.47 + 8.37) - (1 - 0.85)(28.47 + 199)}{(28.47 + 406) - (1 - 0.85)(28.47 + 199)} = \frac{2.7195}{400.3} = 0.006$$

$$y_4 = 0.006$$

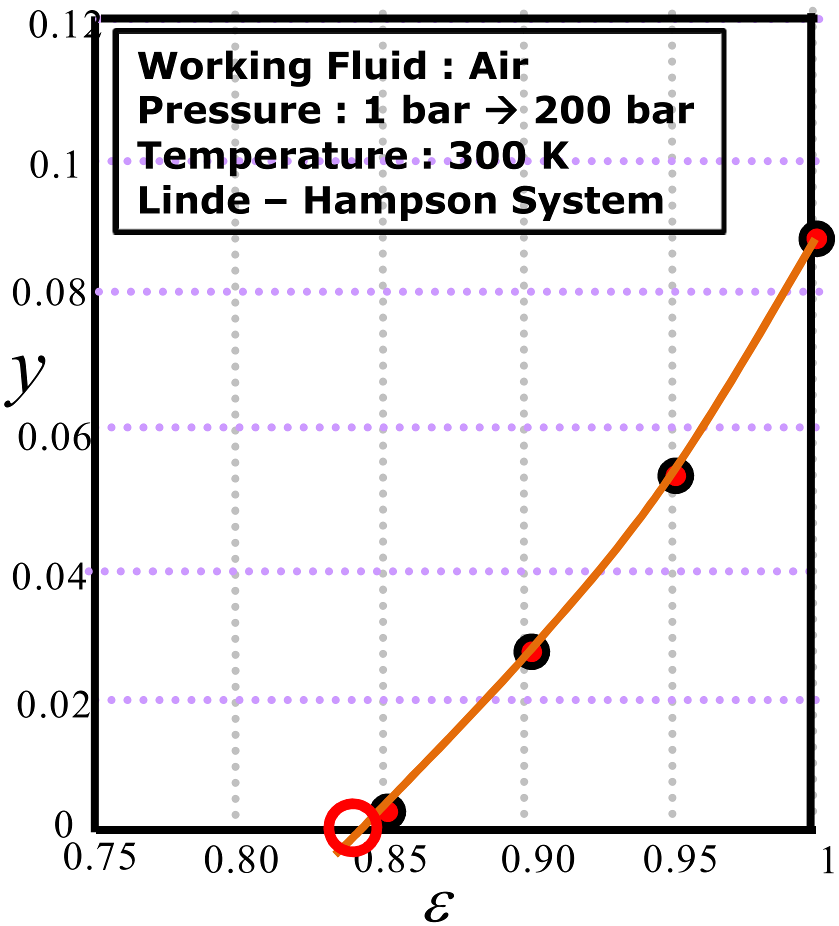
Tutorial – 1



	ε	y
1	1	0.085
2	0.95	0.060
3	0.90	0.034
4	0.85	0.006

- Plotting the values for the above conditions, we have the trend as shown.

Tutorial – 1



- Joining the points, we have the trend as shown in the figure.
- It is clear that as the effectiveness decreases, the yield **y** decreases drastically.
- Furthermore, the effectiveness should be more than 85% in order to have a liquid yield.

Figure of Merit (FOM)

- In the earlier lecture, we have seen that the ideal cycle is used as a benchmark to compare various liquefaction systems.
- In effect, a parameter called Figure of Merit (FOM) is defined.
- It is the ratio of the ideal work input to the actual work input of the system per unit mass of gas liquefied.
- Mathematically, $\frac{W_i}{W}$

Tutorial – 2

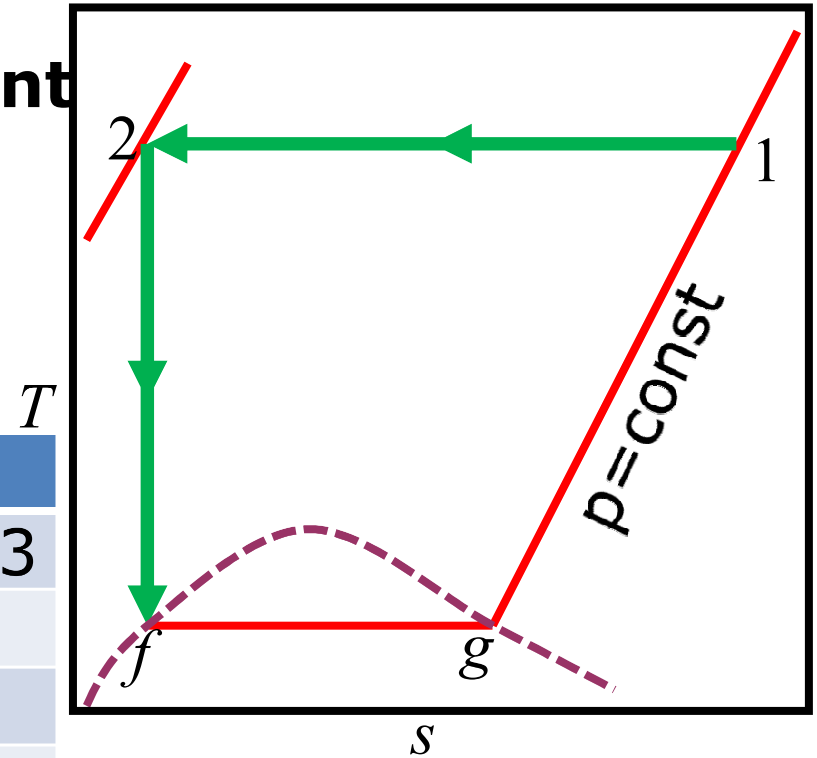
- Determine the following for a Linde – Hampson system with Nitrogen as working fluid when the system is operated between 1.013 bar (1 atm) and 202.6 bar (200 atm) at 300 K. The effectiveness of HX is 100%.

- Ideal Work requirement
- Liquid yield
- Work/unit mass compressed
- Work/unit mass liquefied
- FOM

Tutorial – 2

- **Ideal Work Requirement**

$$-\frac{\dot{W}_i}{\dot{m}} = T_1\left(s_1 - s_f\right) - \left(h_1 - h_f\right)$$



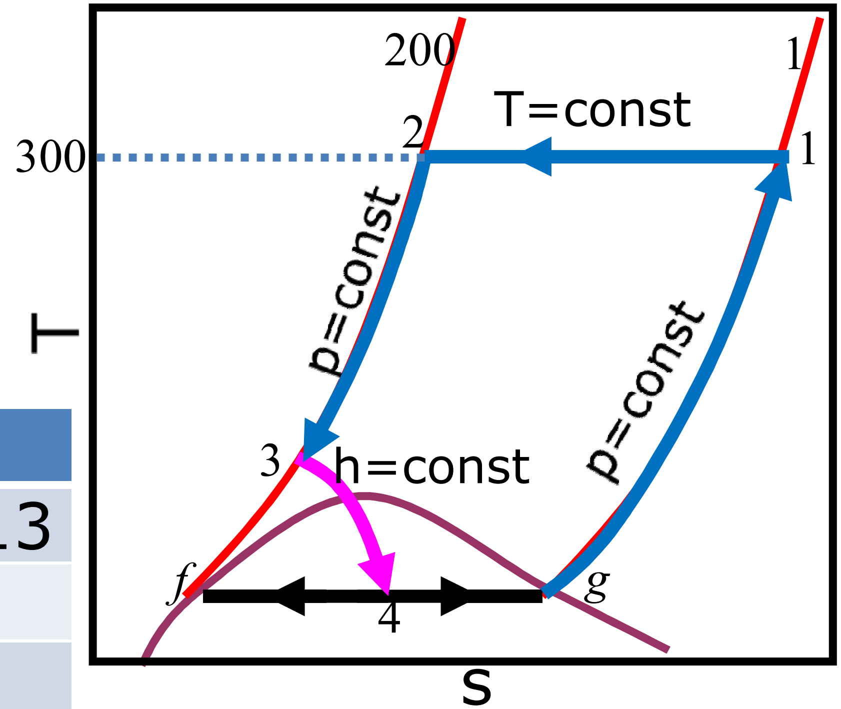
	1	2	f
p (bar)	1.013	202.6	1.013
T (K)	300	300	77
h (J/g)	462	430	29
s (J/gK)	4.4	2.75	0.42

$$-\frac{W_c}{\dot{m}} = 300\left(4.4 - 0.42\right) - \left(462 - 29\right) = 761\ J/g$$

Tutorial – 2

- **Liquid yield**

$$y = \left(\frac{h_1 - h_2}{h_1 - h_f} \right)$$



	1	2	f
p (bar)	1.013	202.6	1.013
T (K)	300	300	77
h (J/g)	462	430	29
s (J/gK)	4.4	2.75	0.42

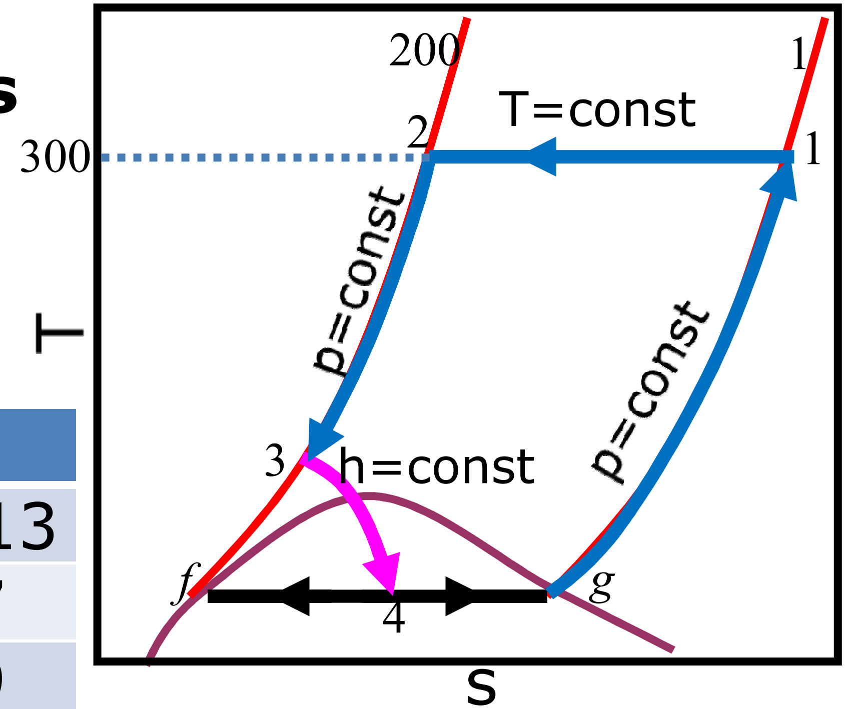
$$y = \left(\frac{h_1 - h_2}{h_1 - h_f} \right) = \left(\frac{462 - 430}{462 - 29} \right) = \left(\frac{32}{433} \right) = 0.074$$

Tutorial – 2

- **Work/unit mass of gas compressed**

$$-\frac{W_c}{\dot{m}} = T_1\left(s_1 - s_2\right) - \left(h_1 - h_2\right)$$

	1	2	f
p (bar)	1.013	202.6	1.013
T (K)	300	300	77
h (J/g)	462	430	29
s (J/gK)	4.4	2.75	0.42



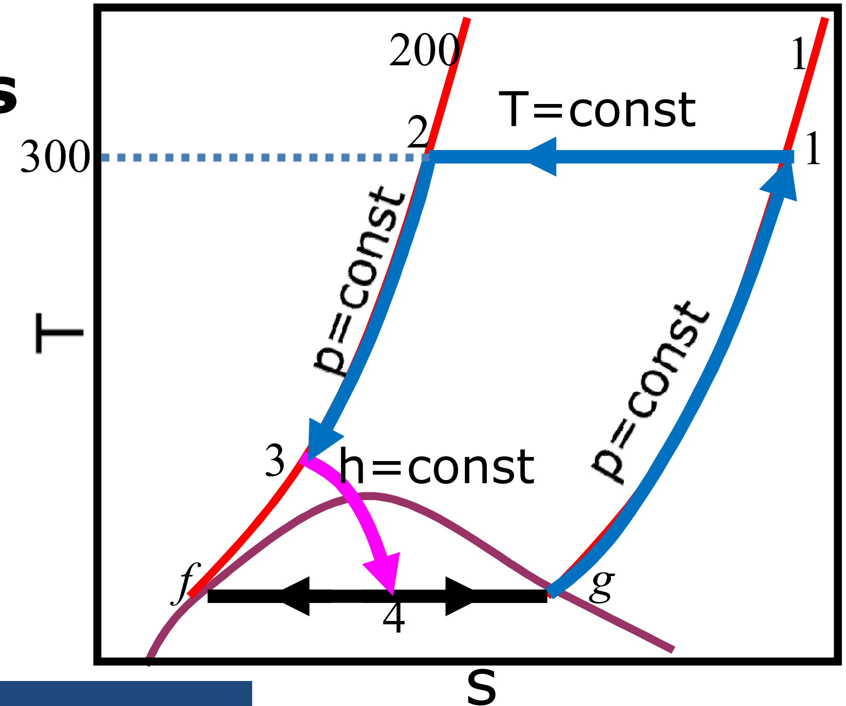
$$-\frac{W_c}{\dot{m}} = 300\left(4.4 - 2.75\right) - \left(462 - 430\right) = 463\ J/g$$

Tutorial – 2

- **Work/unit mass of gas liquefied**

$$-\frac{W_c}{\dot{m}} = 463$$

$$y = 0.074$$



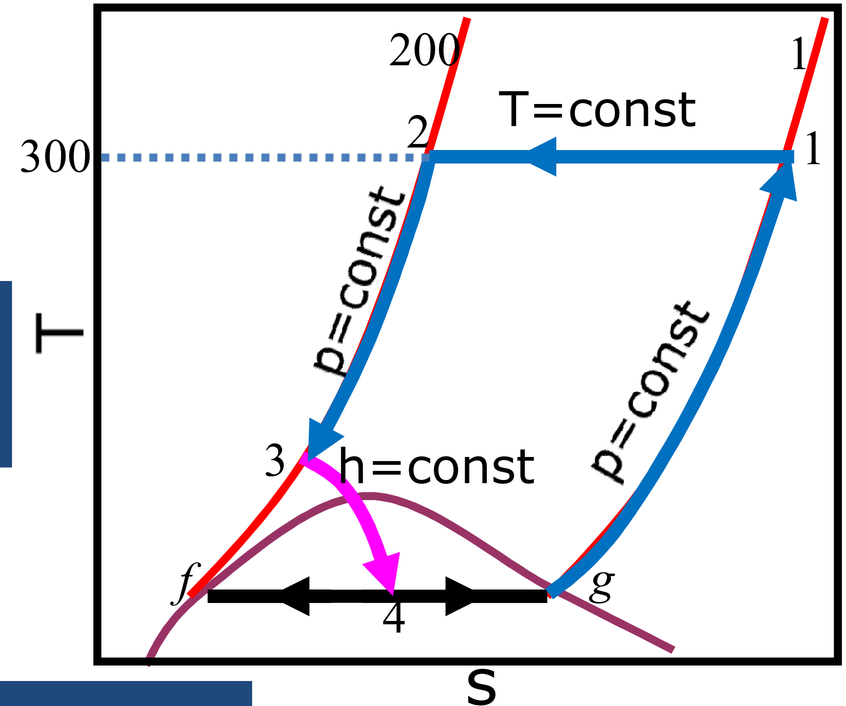
$$-\frac{W_c}{\dot{m}_f} = -\frac{W_c}{y\dot{m}} = \frac{463}{0.074} = 6265.22\ J/g$$

Tutorial – 2

- **Figure of Merit (FOM)**

$$-\frac{W_c}{\dot{m}_f} = 6265.22$$

$$-\frac{W_i}{\dot{m}_f} = 767$$



$$FOM = \frac{W_i}{\dot{m}_f} \Big/ \frac{W_c}{\dot{m}_f} = \frac{767}{6265.22} = 0.1225$$

Tutorial – 3

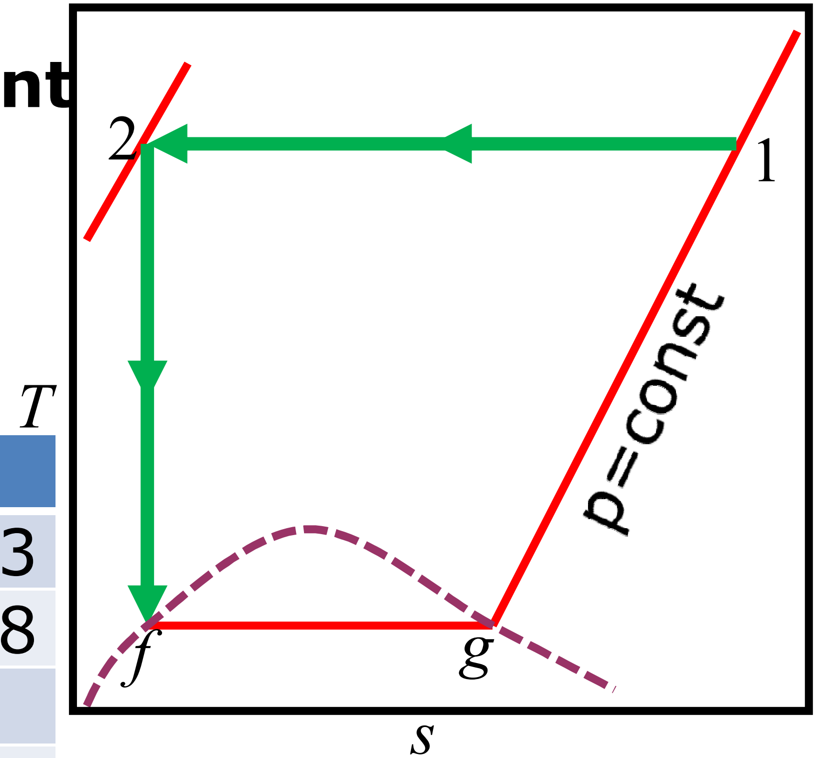
- Determine the following for a Linde – Hampson system with Argon as working fluid when the system is operated between 1.013 bar (1 atm) and 202.6 bar (200 atm) at 300 K. The effectiveness of HX is 100%.

- Ideal Work requirement
- Liquid yield
- Work/unit mass compressed
- Work/unit mass liquefied
- FOM

Tutorial – 3

- **Ideal Work Requirement**

$$-\frac{\dot{W}_i}{\dot{m}} = T_1\left(s_1 - s_f\right) - \left(h_1 - h_f\right)$$



	1	2	f
p (bar)	1.013	202.6	1.013
T (K)	300	300	87.28
h (J/g)	349	315	60
s (J/gK)	3.9	2.7	1.35

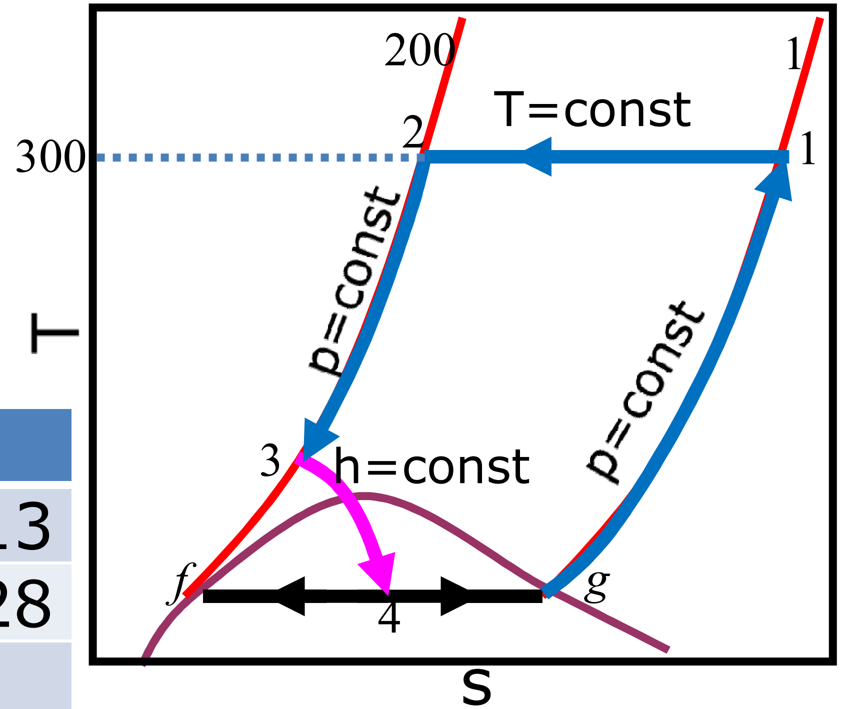
$$-\frac{W_c}{\dot{m}} = 300\left(3.9 - 1.35\right) - \left(349 - 60\right) \quad = 476\ J/g$$

Tutorial – 3

- **Liquid yield**

$$y = \left(\frac{h_1 - h_2}{h_1 - h_f} \right)$$

	1	2	f
p (bar)	1.013	202.6	1.013
T (K)	300	300	87.28
h (J/g)	349	315	60
s (J/gK)	3.9	2.7	1.35



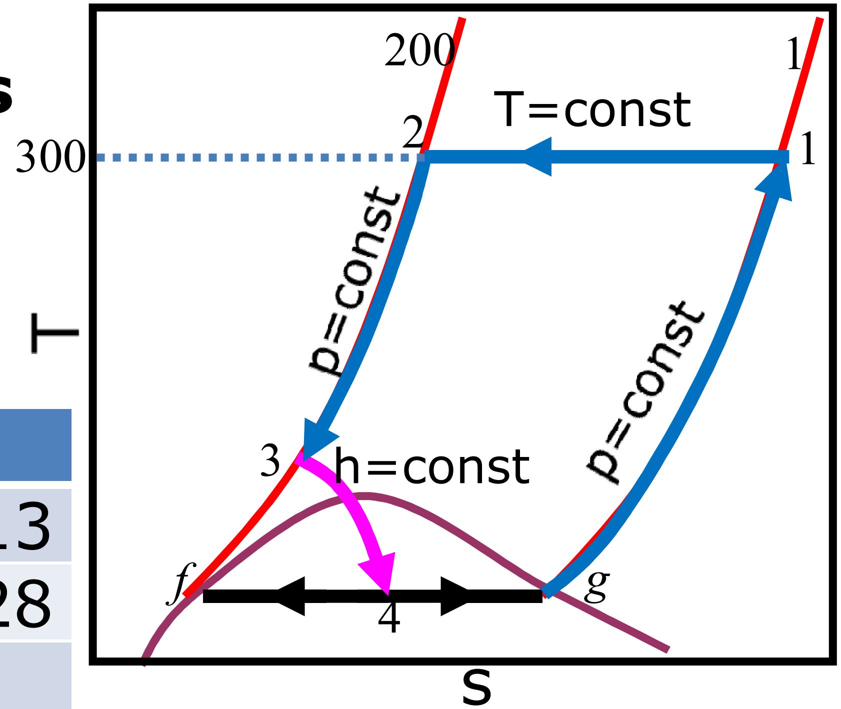
$$y = \left(\frac{h_1 - h_2}{h_1 - h_f} \right) = \left(\frac{349 - 315}{349 - 60} \right) = \left(\frac{34}{289} \right) = 0.1176$$

Tutorial – 3

- **Work/unit mass of gas compressed**

$$-\frac{W_c}{\dot{m}} = T_1\left(s_1 - s_2\right) - \left(h_1 - h_2\right)$$

	1	2	f
p (bar)	1.013	202.6	1.013
T (K)	300	300	87.28
h (J/g)	349	315	60
s (J/gK)	3.9	2.7	1.35



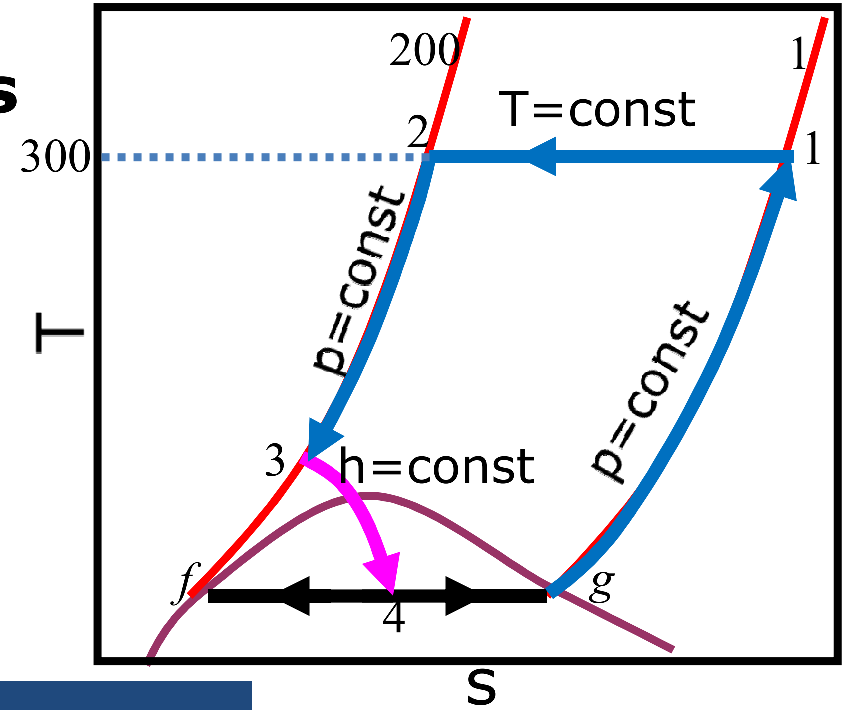
$$-\frac{W_c}{\dot{m}} = 300\left(3.9 - 2.7\right) - \left(349 - 315\right) \quad = 326\ J/g$$

Tutorial – 3

- **Work/unit mass of gas liquefied**

$$-\frac{W_c}{\dot{m}} = 326$$

$$y = 0.1176$$



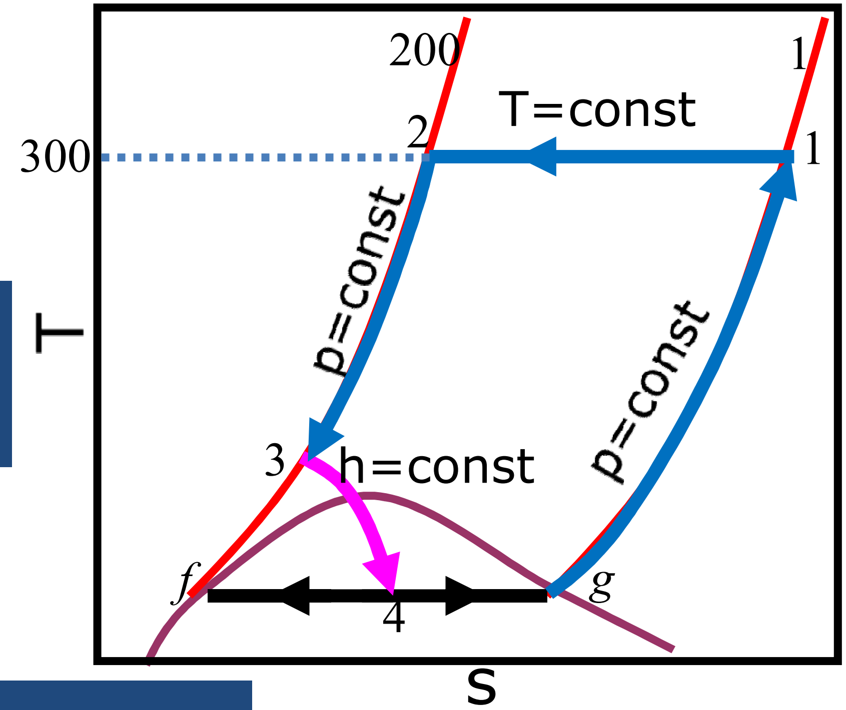
$$-\frac{W_c}{\dot{m}_f} = -\frac{W_c}{y\dot{m}} \quad = \frac{326}{0.1176} \quad = 2772.1\ J/g$$

Tutorial – 3

- **Figure of Merit (FOM)**

$$-\frac{W_c}{\dot{m}_f} = 2772.1$$

$$-\frac{W_i}{\dot{m}_f} = 476.0$$



$$FOM = \frac{W_i}{\dot{m}_f} \bigg/ \frac{W_c}{\dot{m}_f} = \frac{476.0}{2772.1} = 0.1717$$

Performance of L – H System

Fluid	Boil. Pt	y	$-\frac{W_c}{\dot{m}}$	$-\frac{W_c}{\dot{m}_f}$	FOM
N_2	77.3	0.074	463	6265.2	0.122
Ar	87.2	0.117	326	2772.1	0.171
Air	78.8	0.081	454	5621.0	0.131
O_2	90.1	0.106	405	3804.0	0.167

- The above table is for a Linde – Hampson system when the pressures are from 1 bar to 200 bar at 300K.
- The heat exchanger effectiveness is 100%.

Tutorial – 4

- For the conditions specified in Tutorial 2 for the Linde – Hampson system with Nitrogen as working fluid, calculate the following when the effectiveness of HX is 90%.

- Liquid yield
- Work/unit mass compressed
- Work/unit mass liquefied
- FOM

- Comment on the results

Tutorial – 4

- **Liquid yield**

$$y = \frac{(h_1 - h_2) - (1 - \varepsilon)(h_1 - h_g)}{(h_1 - h_f) - (1 - \varepsilon)(h_1 - h_g)}$$

$$\varepsilon = 0.90$$

	1	2	f	g
p (bar)	1.013	202.6	1.013	1.013
T (K)	300	300	77	77
h (J/g)	462	430	29	230
s (J/gK)	4.4	2.75	0.42	3.2

$$y = \frac{(462-430)-(1-0.90)(462-230)}{(462-29)-(1-0.90)(462-230)} = 0.021$$

Tutorial – 4

- **Location of $h_{1'}$ and Additional Work**

$$\varepsilon = \frac{h_{1'} - h_g}{h_1 - h_g}$$

$$h_{1'} = \varepsilon\left(h_1 - h_g\right) + h_g$$

	1	2	f	g
p (bar)	1.013	202.6	1.013	1.013
T (K)	300	300	77	77
h (J/g)	462	430	29	230
s (J/gK)	4.4	2.75	0.42	3.2

$$h_{1'} = 0.9\left(462 - 230\right) + 230 \quad = 438.8\ J/g$$

$$\left(-\frac{W_c}{\dot{m}}\right)_{add} = h_1 - h_{1'} \quad = 462 - 438.8 \quad = 23.2\ \ J/g$$

Tutorial – 4

- **Work/unit mass of gas compressed**

$$-\frac{W_c}{\dot{m}} = T_1\left(s_1 - s_2\right) - \left(h_1 - h_2\right)$$

	1	2	f	g
p (bar)	1.013	202.6	1.013	1.013
T (K)	300	300	77	77
h (J/g)	462	430	29	230
s (J/gK)	4.4	2.75	0.42	3.2

$$-\frac{W_c}{\dot{m}} = 300\left(4.4 - 2.75\right) - \left(462 - 430\right) = 463\ J/g$$

$$\left(-\frac{W_c}{\dot{m}}\right)_{total} = \left(-\frac{W_c}{\dot{m}}\right)_{h_1 \rightarrow h_1} + \left(-\frac{W_c}{\dot{m}}\right)_{add} = 463 + 23.2 = 486.2\ J/g$$

Tutorial – 4

- **Work/unit mass of gas liquefied**

$$-\frac{W_c}{\dot{m}} = 486.2$$

$$y = 0.0215$$

$$-\frac{W_c}{\dot{m}_f} = -\frac{W_c}{y\dot{m}} = \frac{486.2}{0.0215} = 22613.95\ J/g$$

- **Figure of Merit (FOM)**

$$-\frac{W_c}{\dot{m}_f} = 22613.95$$

$$-\frac{W_i}{\dot{m}_f} = 761$$

$$FOM = \frac{W_i}{\dot{m}_f} / \frac{W_c}{\dot{m}_f} = \frac{761}{22613.95} = 0.0336$$

Tutorial – 4

	$\varepsilon = 1$	$\varepsilon = 0.9$	% change
y	0.074	0.021	71.62
$-\frac{W_c}{\dot{m}}$	463	486.2	-5.01
$-\frac{W_c}{\dot{m}_f}$	6265.2	22614	-261
FOM	0.1225	0.0336	72.57

- The above table highlights the significance of heat exchanger effectiveness for a Linde – Hampson system

Assignment

1. Determine the following for a Linde – Hampson system with Air as working fluid when the system is operated between 1.013 bar (1 atm) and 202.6 bar (200 atm) at 300 K. The effectiveness of HX is 100%.

- Ideal Work requirement
- Liquid yield
- Work/unit mass compressed
- Work/unit mass liquefied
- FOM

Assignment

2. Determine the following for a Linde – Hampson system with Oxygen as working fluid when the system is operated between 1.013 bar (1 atm) and 202.6 bar (200 atm) at 300 K. The effectiveness of HX is 100%.

- Ideal Work requirement
- Liquid yield
- Work/unit mass compressed
- Work/unit mass liquefied
- FOM

Assignment

3. Repeat the Problem 1 and Problem 2 the Linde – Hampson system when the heat exchanger effectiveness is 90%. Compare and comment on the results.

Summary

- A heat exchanger is a device in which the cold is transferred from cold fluid to hot fluid.

- Effectiveness ε is defined as a ratio of actual heat transfer that is occurring to the maximum possible heat transfer that can occur theoretically.

- It is a dimensionless number between 0 and 1.

- In a Linde – Hampson cycle, the heat exchanger effectiveness ε is $\varepsilon = \dfrac{h_{1'} - h_g}{h_1 - h_g}$ or $\varepsilon = \dfrac{h_{3'} - h_2}{h_3 - h_2}$

Summary

- The liquid yield **y** for a Linde – Hampson system is given by

$$y = \frac{(h_1 - h_2) - (1-\varepsilon)(h_1 - h_g)}{(h_1 - h_f) - (1-\varepsilon)(h_1 - h_g)}$$

- As the effectiveness decreases, the yield **y** decreases drastically.

- Furthermore, the effectiveness should be more than 85% in order to have a liquid yield in Linde – Hampson cycle.

Thank You!