CRYOGENIC ENGINEERING

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Lecture No - 12

Earlier Lecture

- We studied the effect of the heat exchanger effectiveness ε on the performance of a Linde – Hampson system.

- Mathematically, $$\varepsilon = \frac{Q_{act}}{Q_{\max}}$$

- In a Linde – Hampson cycle, the heat exchanger effectiveness ε is $$\varepsilon = \frac{h_{1'} - h_g}{h_1 - h_g}$$ or $$\varepsilon = \frac{h_{3'} - h_2}{h_3 - h_2}$$

Earlier Lecture

- The liquid yield **y** for a Linde – Hampson system is given by

$$y = \frac{(h_1 - h_2) - (1-\varepsilon)(h_1 - h_g)}{(h_1 - h_f) - (1-\varepsilon)(h_1 - h_g)}$$

- The effectiveness should be more than 85% in order to have a liquid yield in the Linde – Hampson cycle.

Outline of the Lecture

Topic : Gas Liquefaction and Refrigeration Systems (contd)

- Precooled Linde – Hampson system
  - Liquid yield
  - Work requirement
  - Maximum liquid yield

- Comparison between the Simple and Precooled Linde – Hampson systems

Introduction

- We have seen earlier that, as the compression temperature decreases, the yield **y** increases for a Linde – Hampson system.

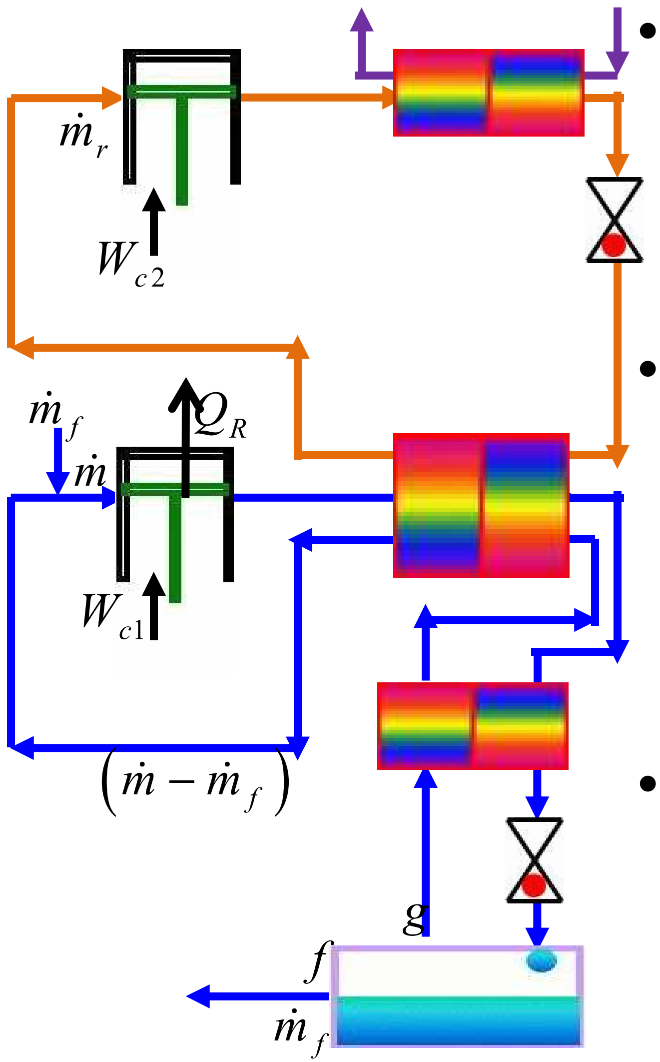
- The method of cooling the gas after the compression or before the entrance to the heat exchanger is called as precooling.

Introduction

- The Linde – Hampson cycle with a precooling arrangement is called as Precooled Linde – Hampson cycle.

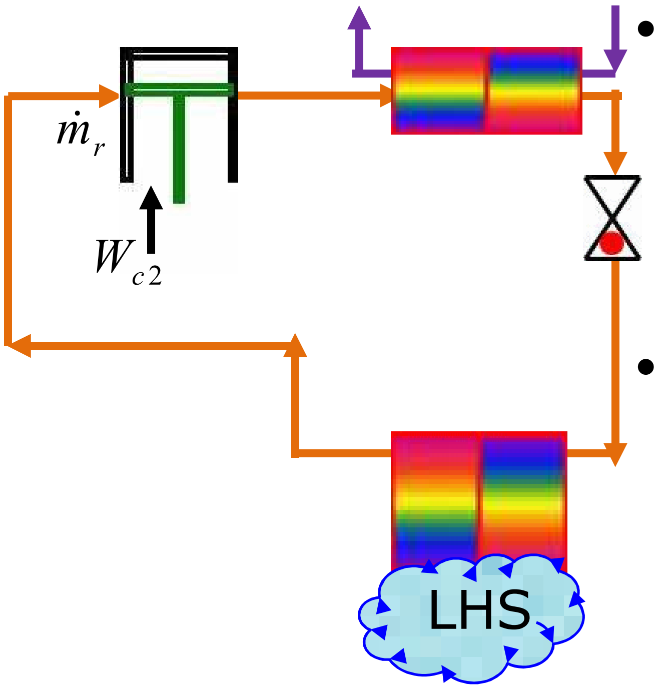
- Here after, we refer these two cycles as Simple Linde – Hampson system and Precooled Linde – Hampson system respectively.

Precooled L – H Cycle



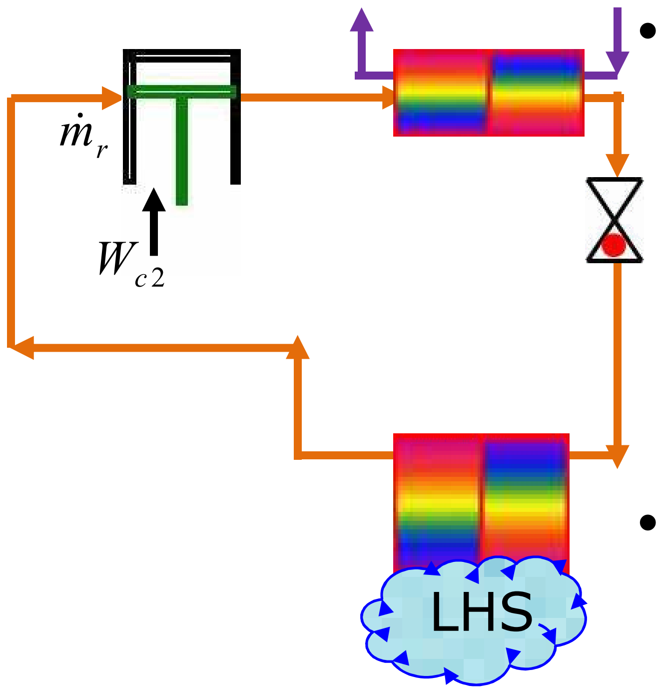
- The Simple Linde – Hampson system is as shown in the figure.
- A 3 fluid heat exchanger is used to thermally couple the precooling and the Linde – Hampson systems.
- Hence, the temperature is lowered after compression or before the entry to the heat exchanger.

Precooled L – H Cycle



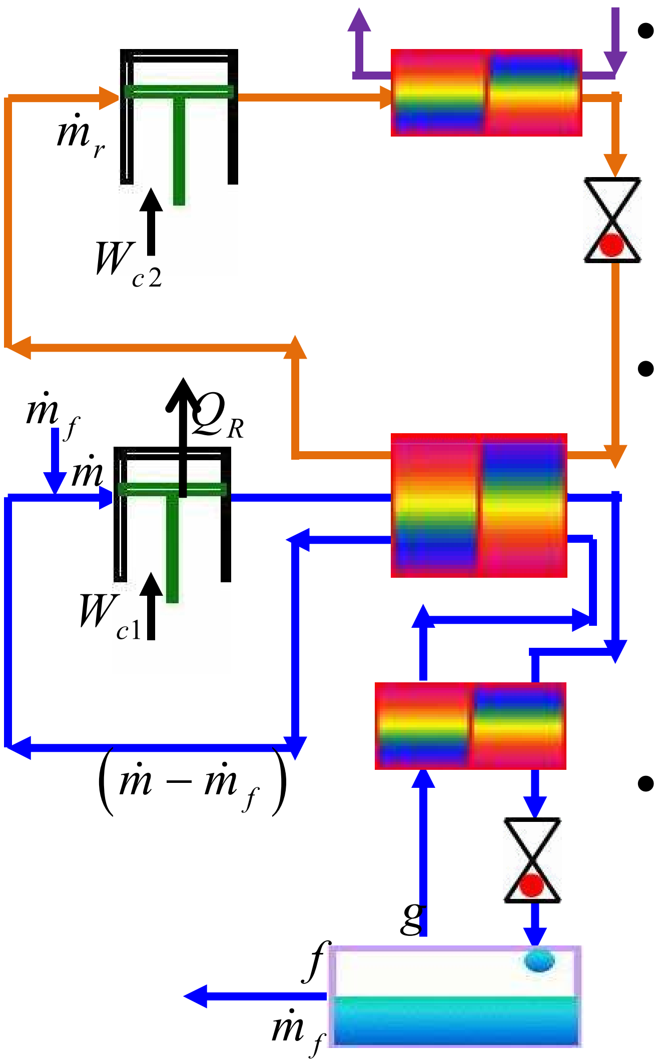
- The features of the precooling system are as follows.
- It is a closed cycle refrigerator with the cold heat exchanger thermally coupled to the simple Linde – Hampson system.
- In other words, the cooling object for this refrigerator is the Linde – Hampson cycle.

Precooled L – H Cycle



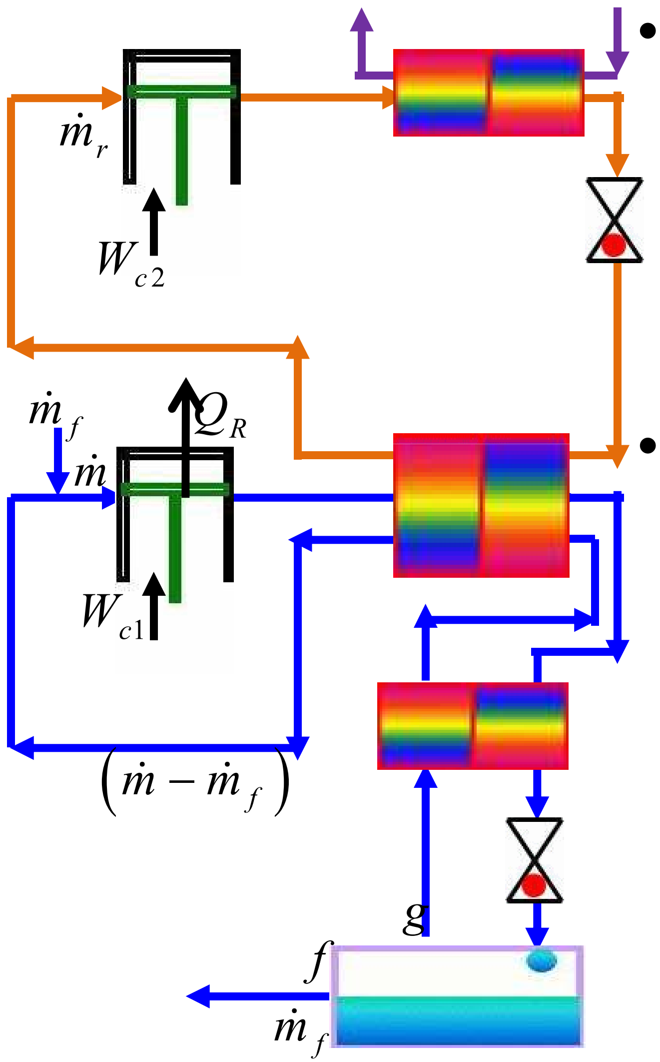
- The heat exchanger of precooling system is cooled by water and J – T device is used to attain lower temperature.
- The process of compression is assumed to be adiabatic. Hence, $Q_R = 0$.
- R134a, NH_3, CO_2 are the common refrigerants in the precooling systems.

Precooled L – H Cycle



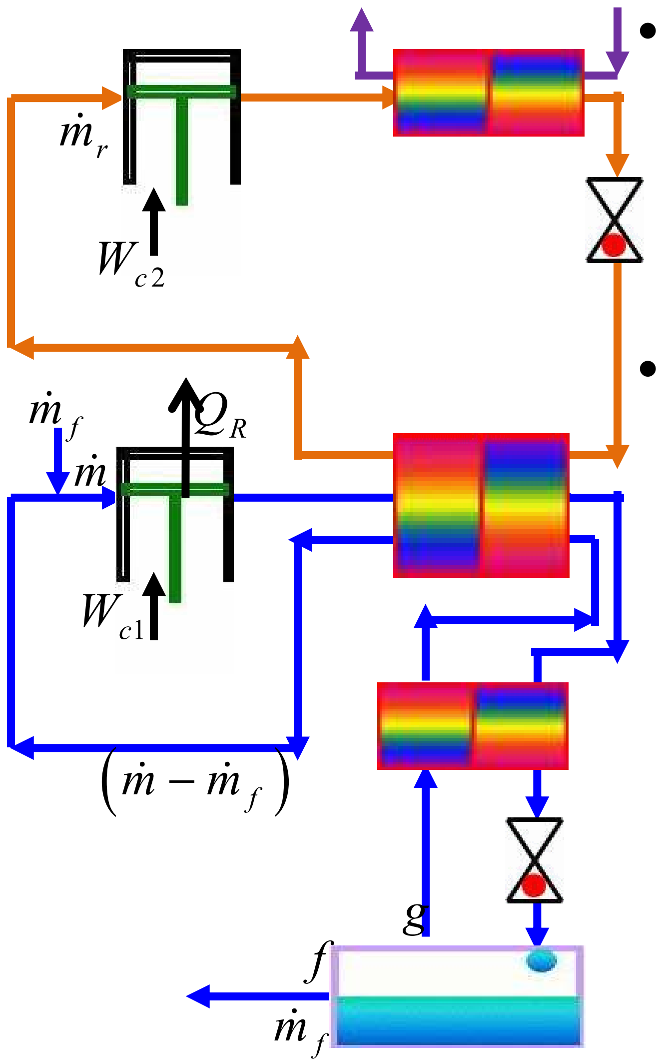
- The salient features of a Precooled Linde – Hampson system are as follows.
- The system consists of a compressor, heat exchangers (2 and 3 – fluid) and a J – T expansion device.
- Compression process is isothermal (adiabatic in precooling system) while the J – T expansion is isenthalpic.

Precooled L – H Cycle



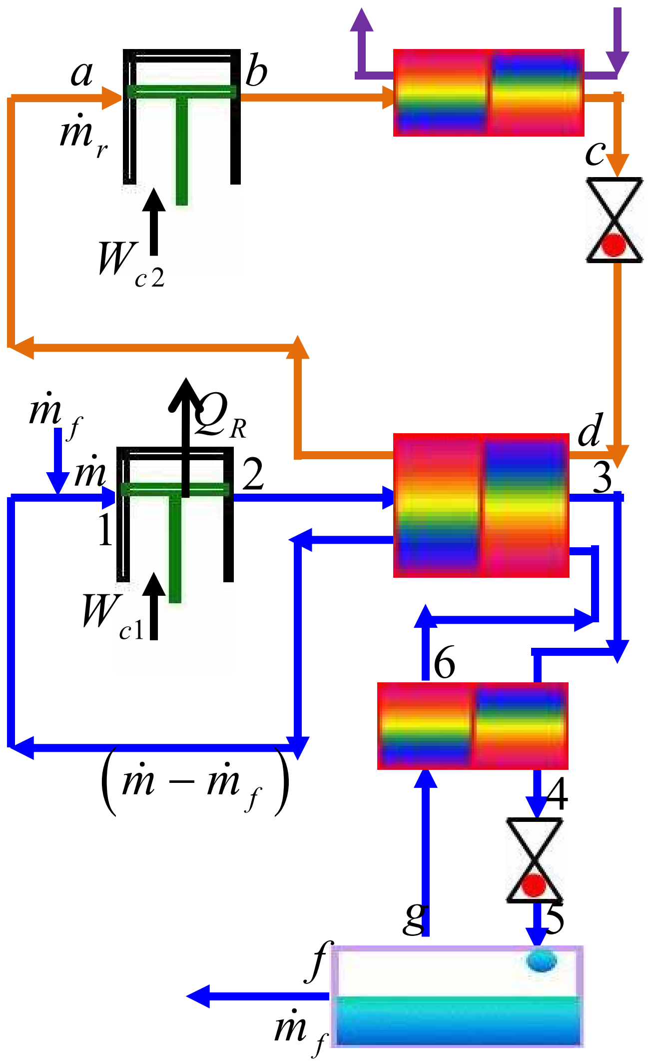
- All the processes are assumed to be ideal in nature and there are no pressure drops in the system.
- The heat exchangers are assumed to be 100% effective and the processes are isobaric in nature.

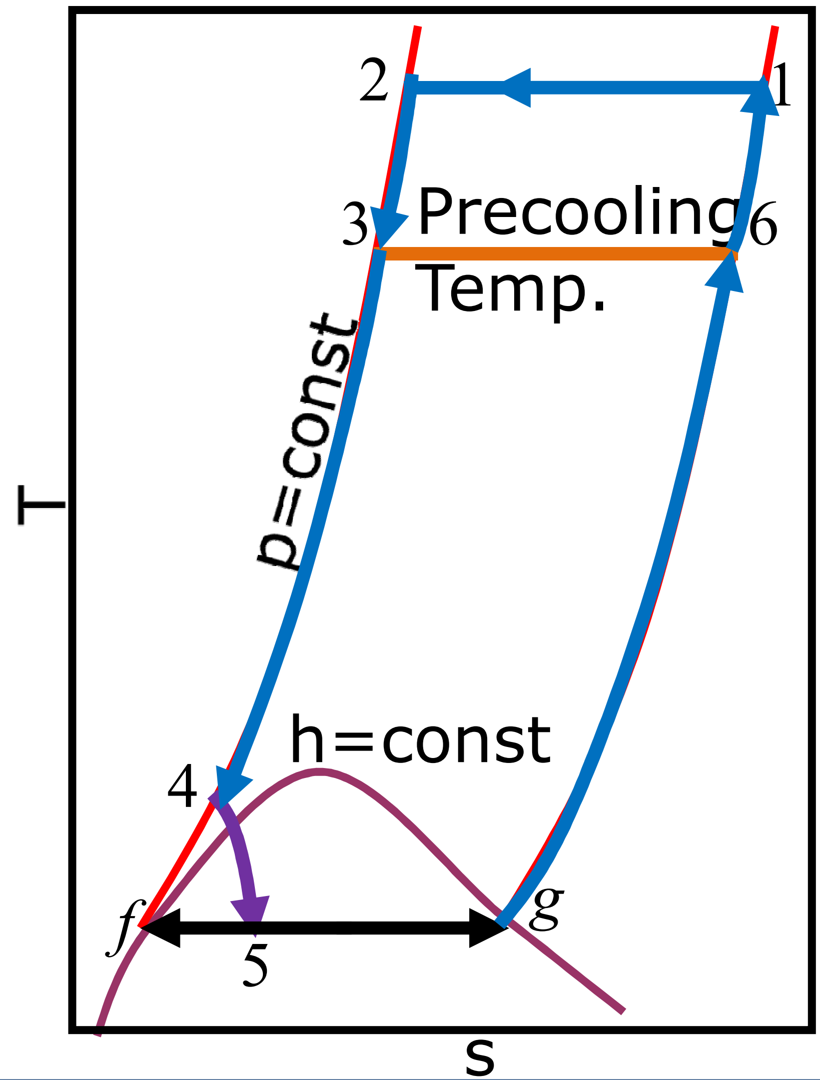
Precooled L – H Cycle



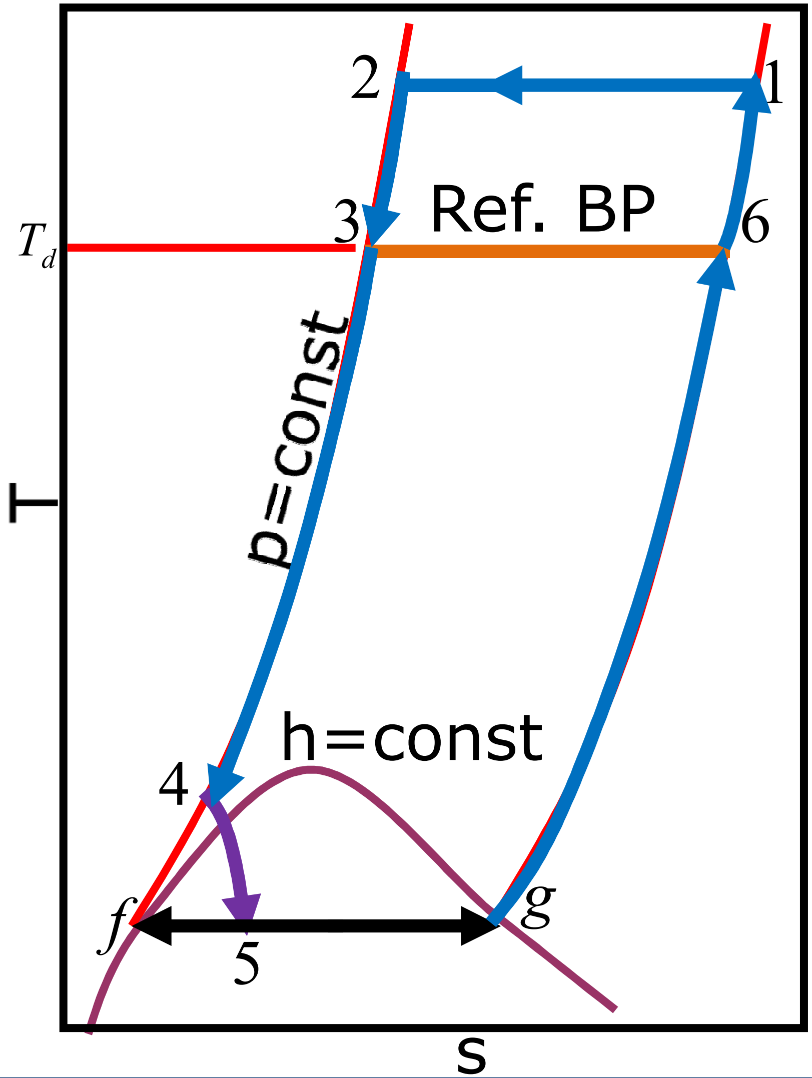
- The gas to be liquefied by the liquefaction system is called as Primary Fluid.
- Whereas, the refrigerant in the precooling system is called as Secondary Fluid.

Precooled L – H Cycle





Precooled L – H Cycle

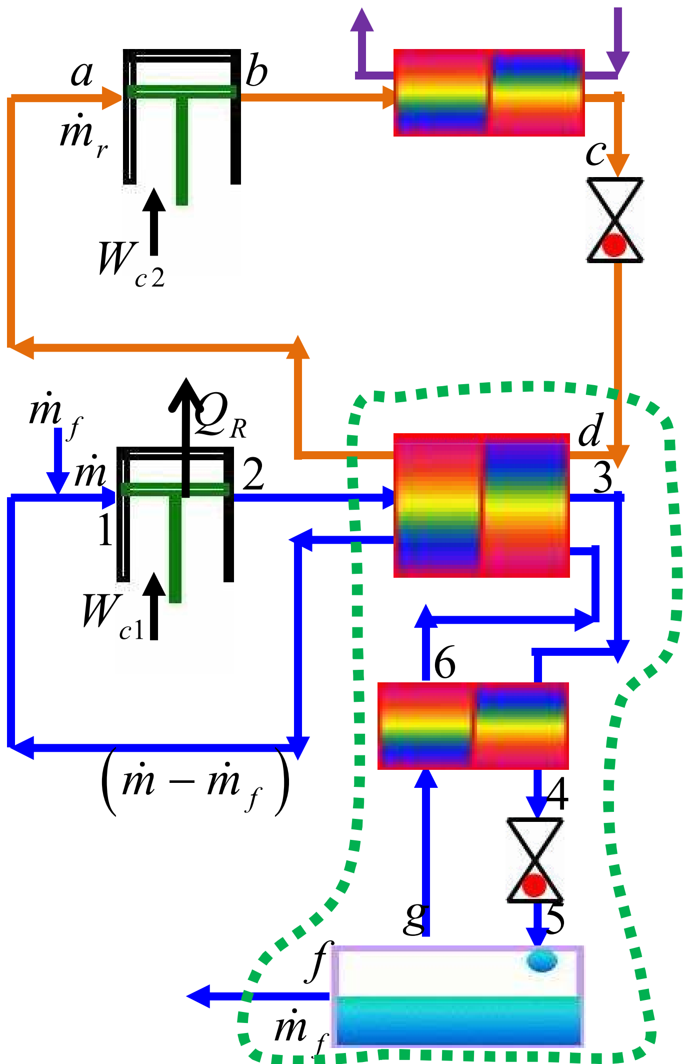


- The precooling limit of the precooling cycle is governed by the boiling point of refrigerant at its suction pressure.

- Boiling point of the common refrigerants at 1 bar are

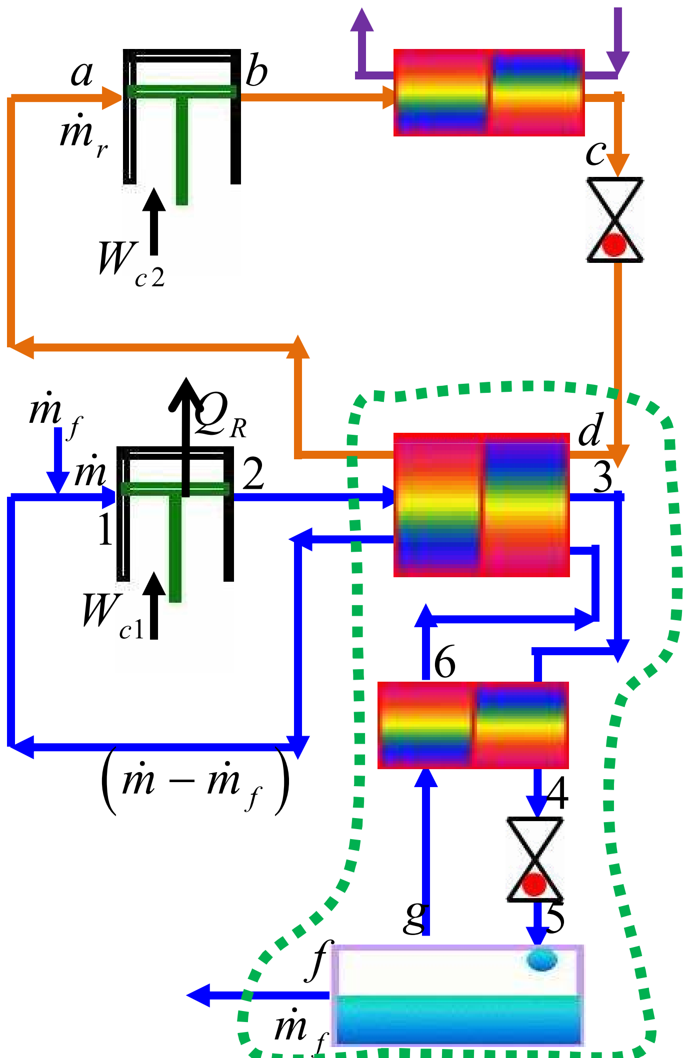
Fluid	Boiling Pt.
CO_2	216.6 K
NH_3	240 K
R134a	247 K

Precooled L – H Cycle



- Consider a control volume for this system as shown in the figure.
- It encloses the 3 fluid heat exchanger, J – T device and the liquid container.
- The 1st Law is applied to analyse the system. The changes in the velocities and datum levels are assumed to be negligible.

Precooled L – H Cycle



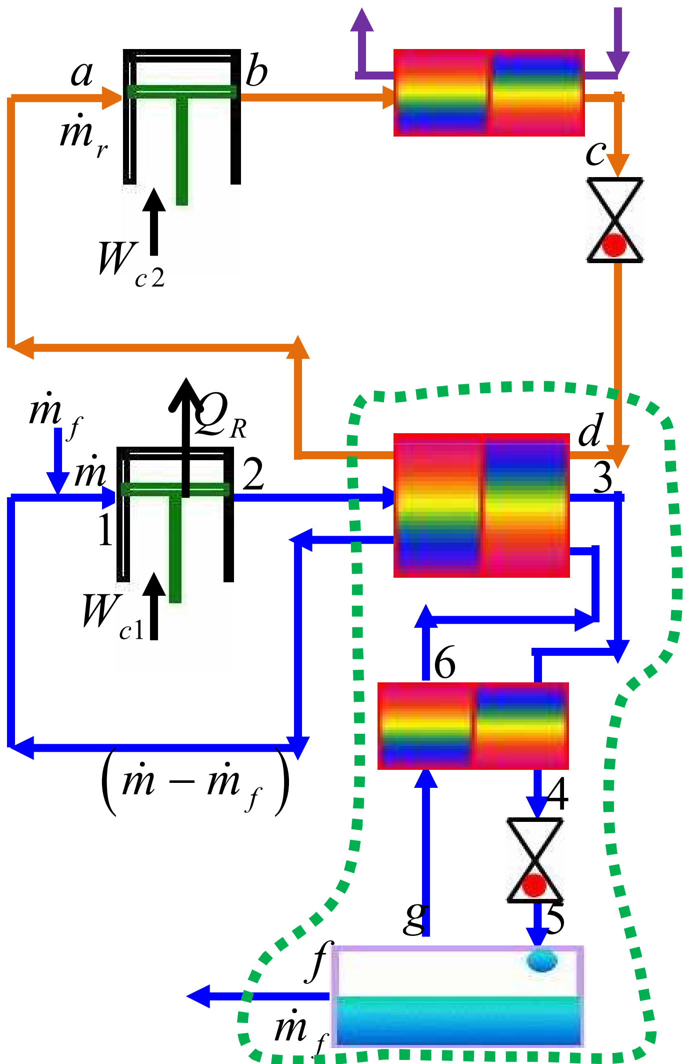
- The quantities entering and leaving the control volume are as follows.

IN	OUT
m_r @ d	m_r @ a
m @ 2	$m - m_f$ @ 1
	m_f @ f

- Applying the 1st law, we have

$$\dot{m}_r h_{d,r} + \dot{m} h_2 = \dot{m}_r h_{a,r} + \left(\dot{m} - \dot{m}_f\right) h_1 + \dot{m}_f h_f$$

Precooled L – H Cycle



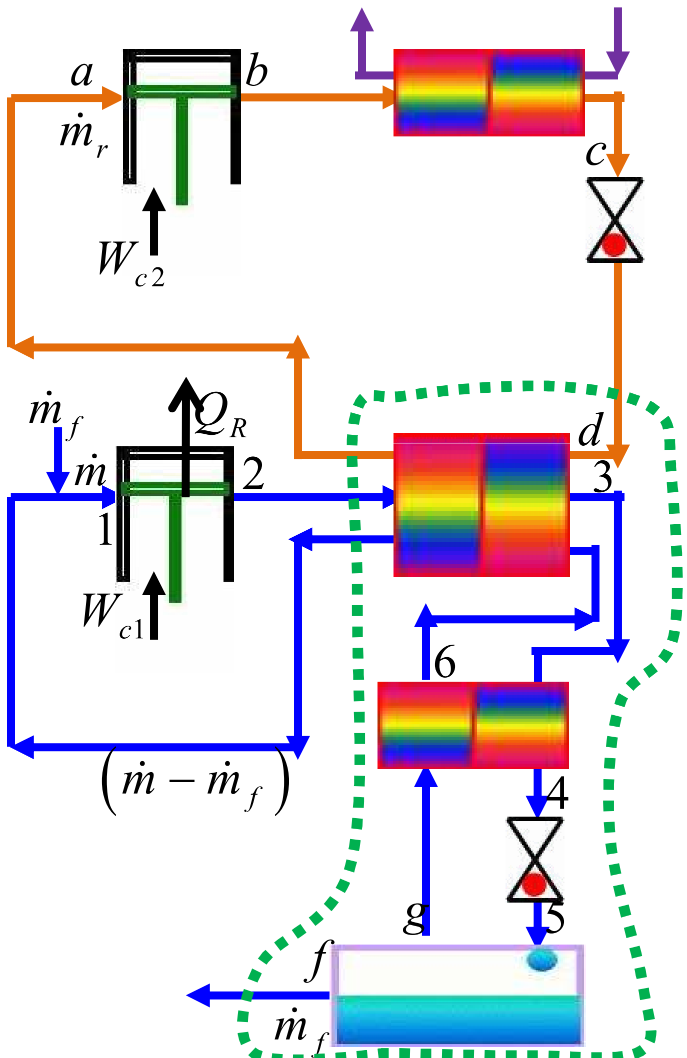
$$\dot{m}_r h_{d,r} + \dot{m} h_2 = \dot{m}_r h_{a,r} + \left(\dot{m}-\dot{m}_f\right) h_1 + \dot{m}_f h_f$$

- Rearranging the terms, we have

$$\frac{\dot{m}_f}{\dot{m}} = \left(\frac{h_1 - h_2}{h_1 - h_f}\right) + \frac{\dot{m}_r}{\dot{m}}\left(\frac{h_{a,r} - h_{d,r}}{h_1 - h_f}\right)$$

- Denoting the ratio $\frac{\dot{m}_r}{\dot{m}} = r$

Precooled L – H Cycle



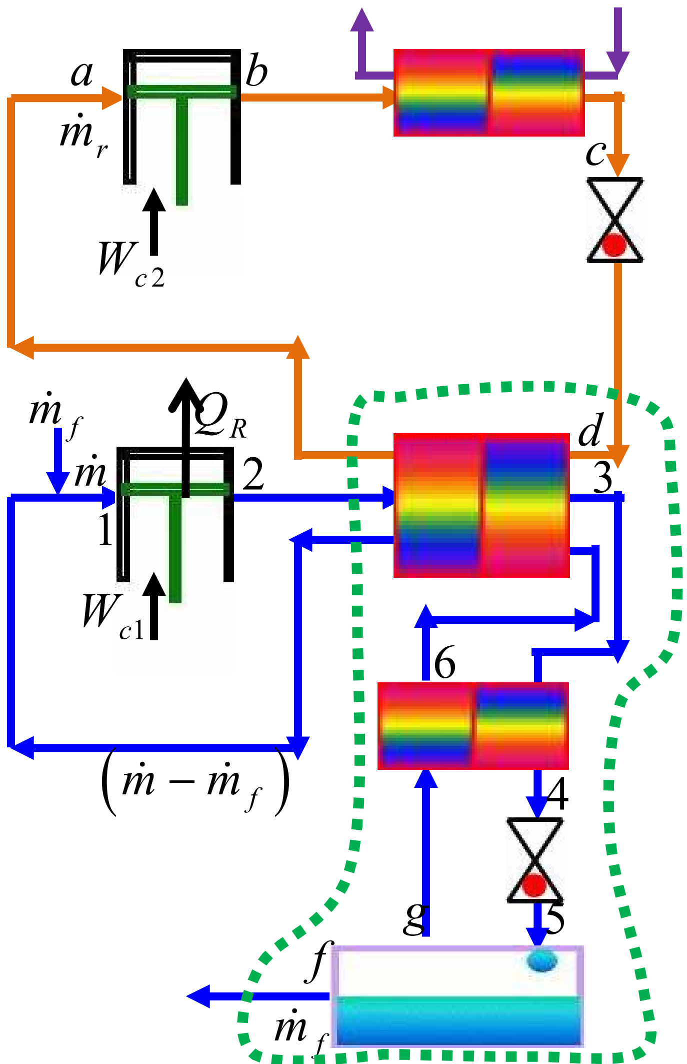
- We have,

$$y = \frac{\dot{m}_f}{\dot{m}} = \frac{h_1 - h_2}{h_1 - h_f} + r\left(\frac{h_{a,r} - h_{d,r}}{h_1 - h_f}\right)$$

- The first term in the above expression is the yield for a simple Linde – Hampson system.

- The second term is the additional yield occurring due to the precooling of the Simple system.

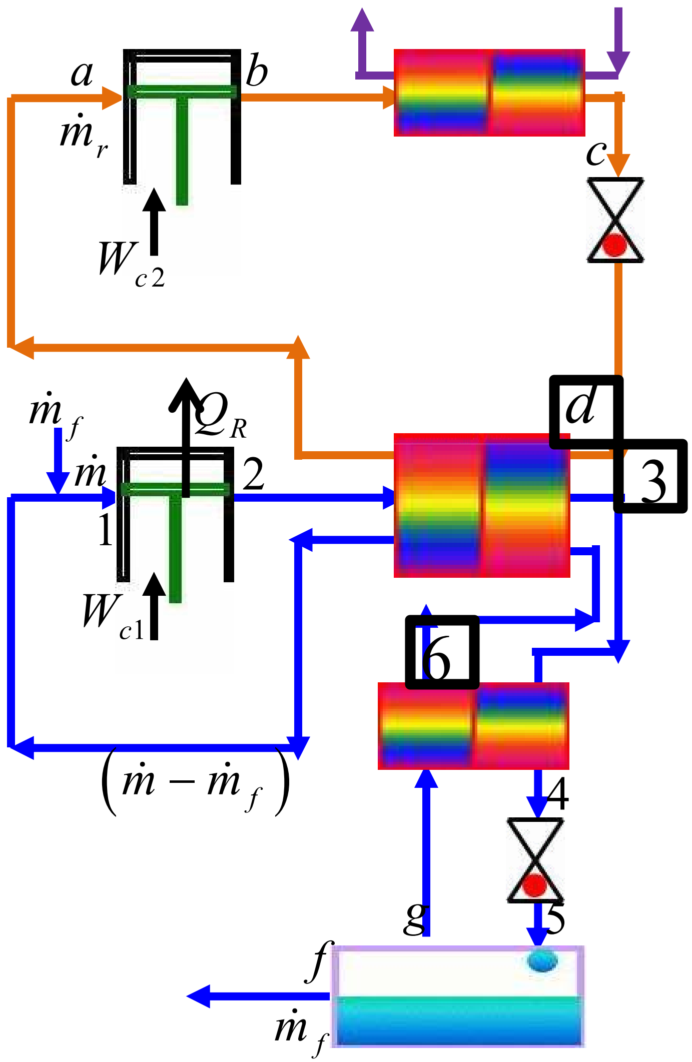
Precooled L – H Cycle



$$y = \frac{\dot{m}_f}{\dot{m}} = \frac{h_1 - h_2}{h_1 - h_f} + r\left(\frac{h_{a,r} - h_{d,r}}{h_1 - h_f}\right)$$

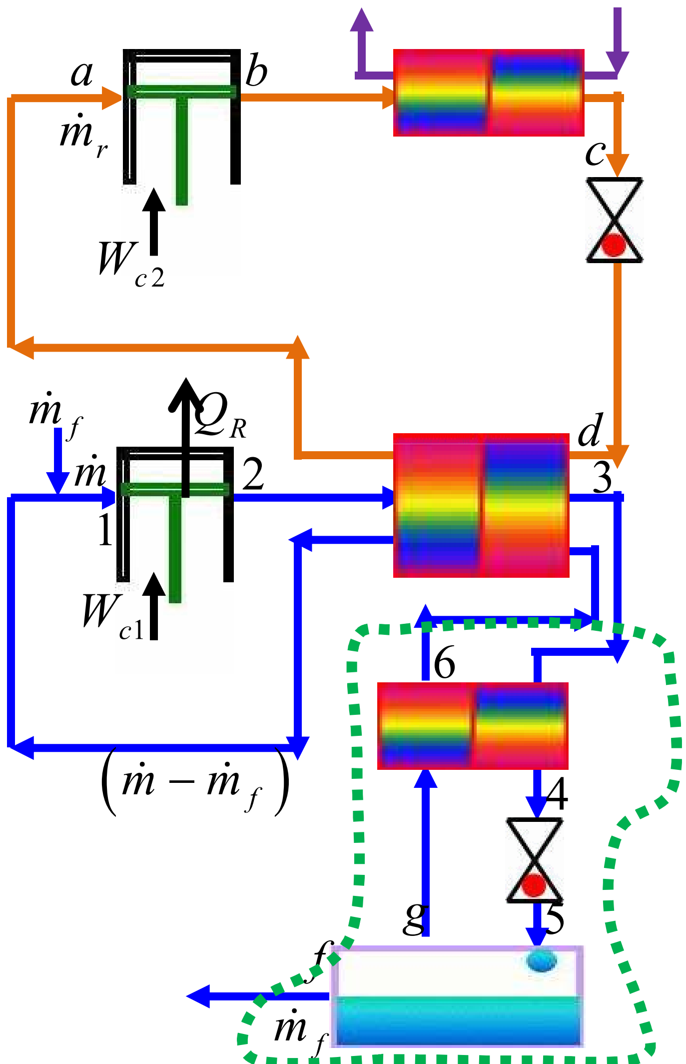
- This increment in the yield is dependent on the
  - The change in enthalpy values from (**h_d → h_a**) of the refrigerant.
  - Refrigerant flow rate (m_r).

Precooled L – H Cycle



- Since, the 3 – fluid heat exchanger is assumed to be 100% effective, the following conditions hold true.
- The minimum value of T_3 would be equal to T_d, which is the boiling point of the refrigerant.
- The maximum value of T_6 would be equal to T_d, which is the boiling point of the refrigerant.

Precooled L – H Cycle

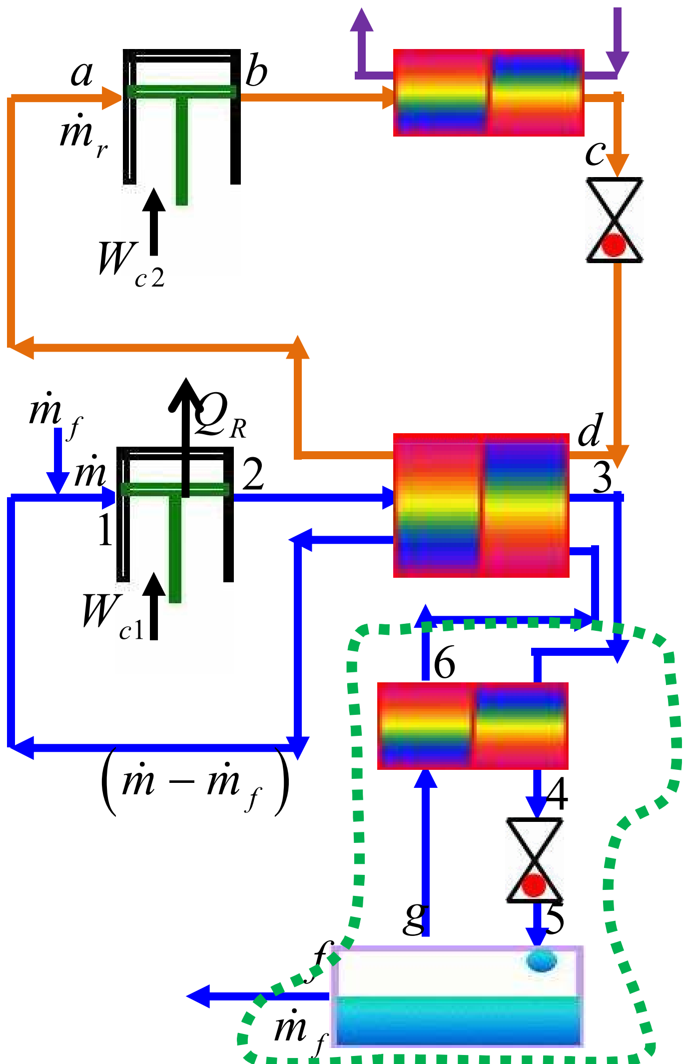


- At this condition, the system produces the maximum yield for a given refrigerant.

- Mathematically,
- $y = y_{max}$ for $T_3 = T_6 = T_d$

- Consider a control volume enclosing the heat exchanger, J – T device and the liquid container as shown in the figure.

Precooled L – H Cycle



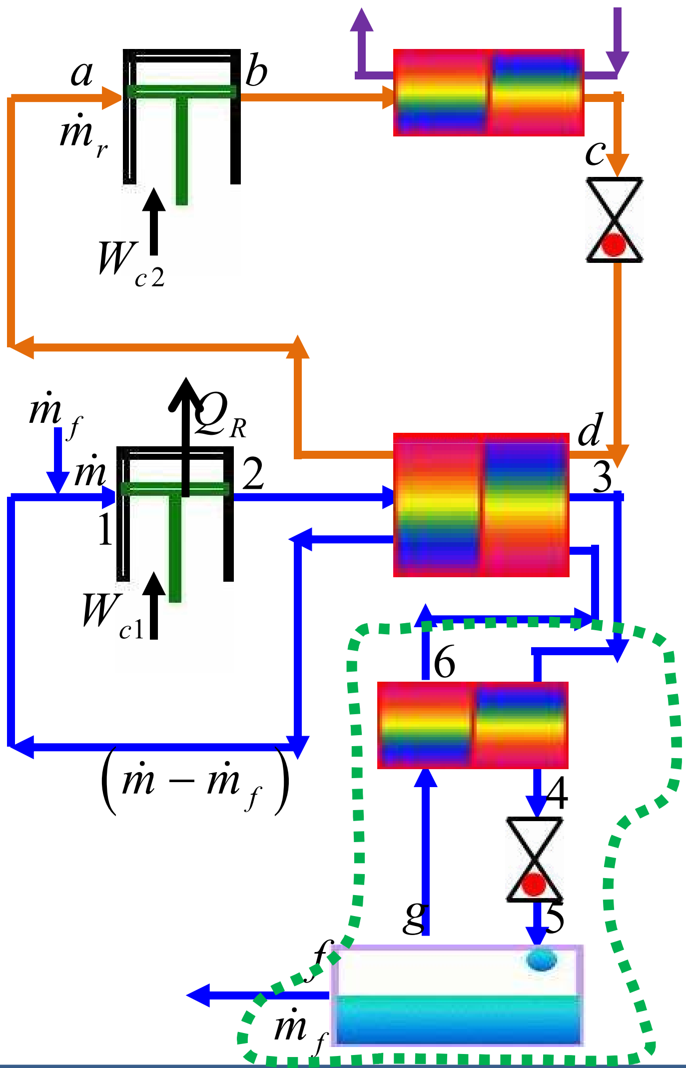
- The quantities entering and leaving the control volume are as follows.

IN	OUT
m @ 3	m_f @ f
	$m - m_f$ @ 6

- Applying the 1st law, we have

$$\dot{m}h_3 = \dot{m}_f h_f + \left(\dot{m} - \dot{m}_f\right) h_6$$

Precooled L – H Cycle



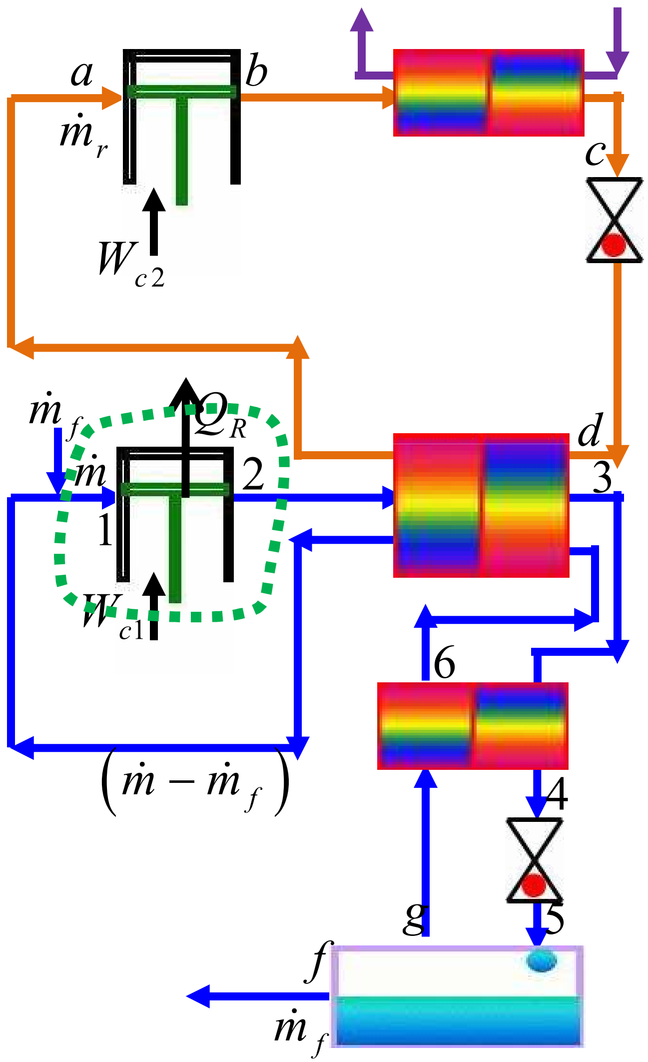
- Rearranging the terms, we have

$$\dot{m}_f\left(h_6-h_f\right)=\dot{m}\left(h_6-h_3\right)$$

$$y_{max}=\frac{\dot{m}_f}{\dot{m}}=\frac{h_6-h_3}{h_6-h_f}$$

- The quantities h_3 and h_6 are evaluated at the boiling point of the refrigerant (T_d).

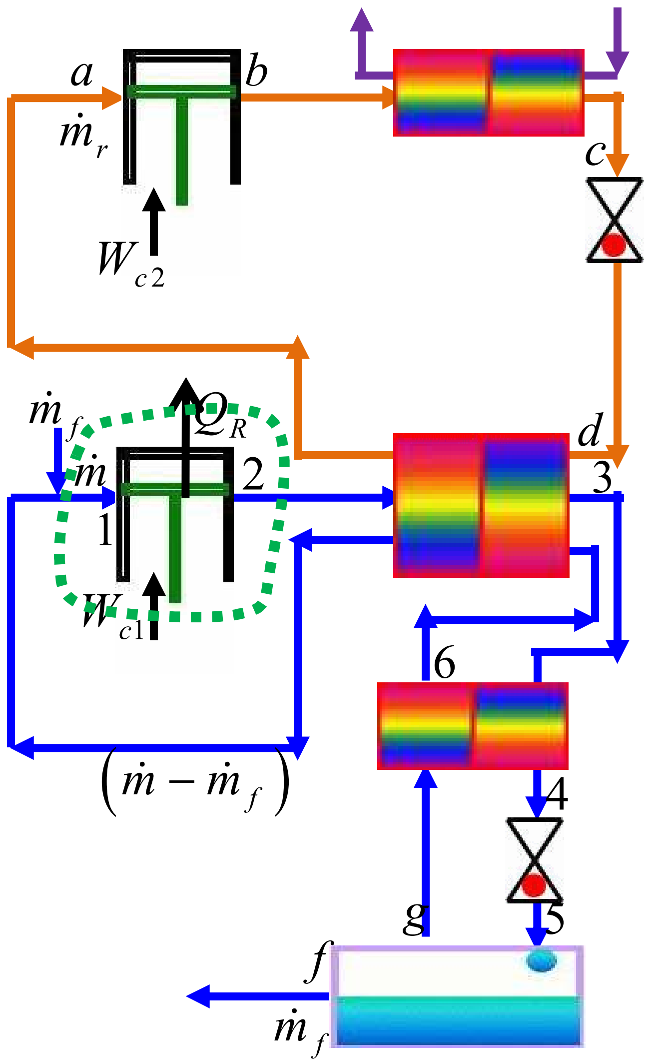
Precooled L – H Cycle



- Consider a control volume for the compressor in the liquefaction cycle as shown in the figure.
- The quantities entering and leaving this control volume are as given below.

IN	OUT
m @ 1	m @ 2
$-W_{c1}$	$-Q_R$

Precooled L – H Cycle



- Using 1st Law for the following table, we get

IN	OUT
m @ 1	m @ 2
$-W_{c1}$	$-Q_R$

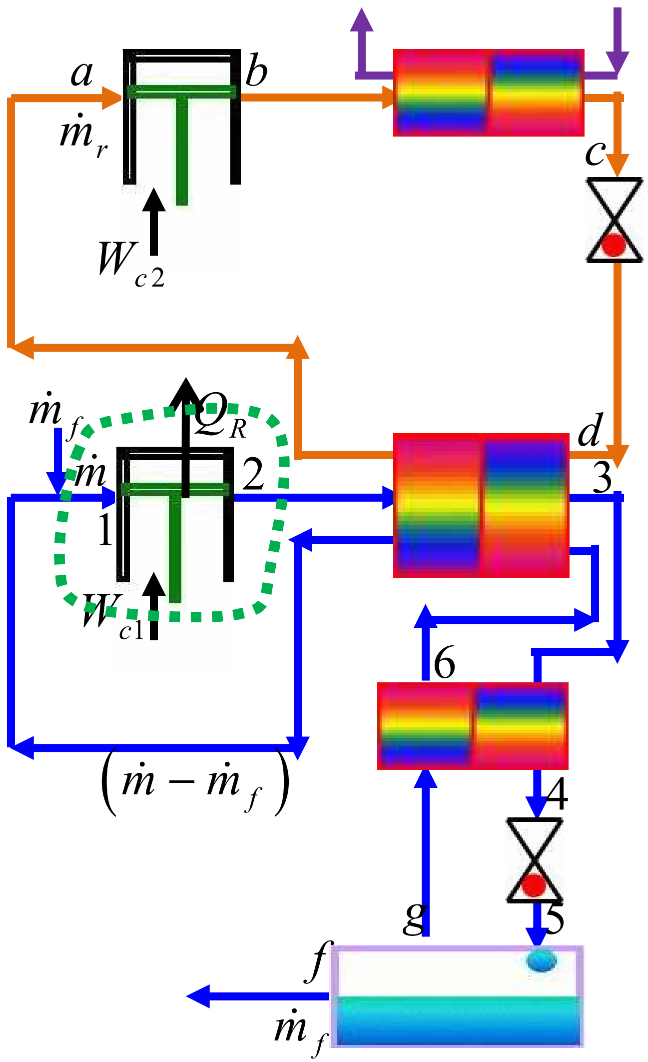
$$E_{in} = E_{out}$$

$$\dot{m}h_1 - W_{c1} = \dot{m}h_2 - Q_R$$

- Rearranging the terms, we have

$$Q_R - W_{c1} = \dot{m}\left(h_2 - h_1\right)$$

Precooled L – H Cycle



$$Q_R - W_{c1} = \dot{m}(h_2 - h_1)$$

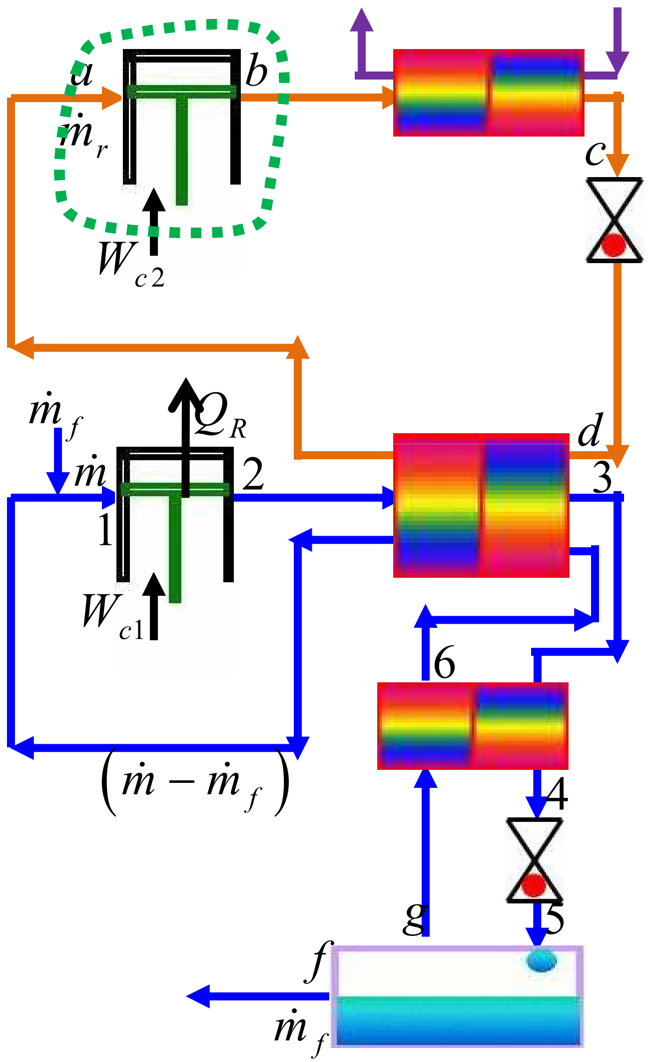
- The heat of compression Q_R can be obtained by using 2nd Law for an isothermal compression. It is given by,

$$Q_R = \dot{m}T_1(s_2 - s_1)$$

- Combining the above equations, we have

$$-W_{c1} = \dot{m}T_1(s_1 - s_2) - \dot{m}(h_1 - h_2)$$

Precooled L – H Cycle

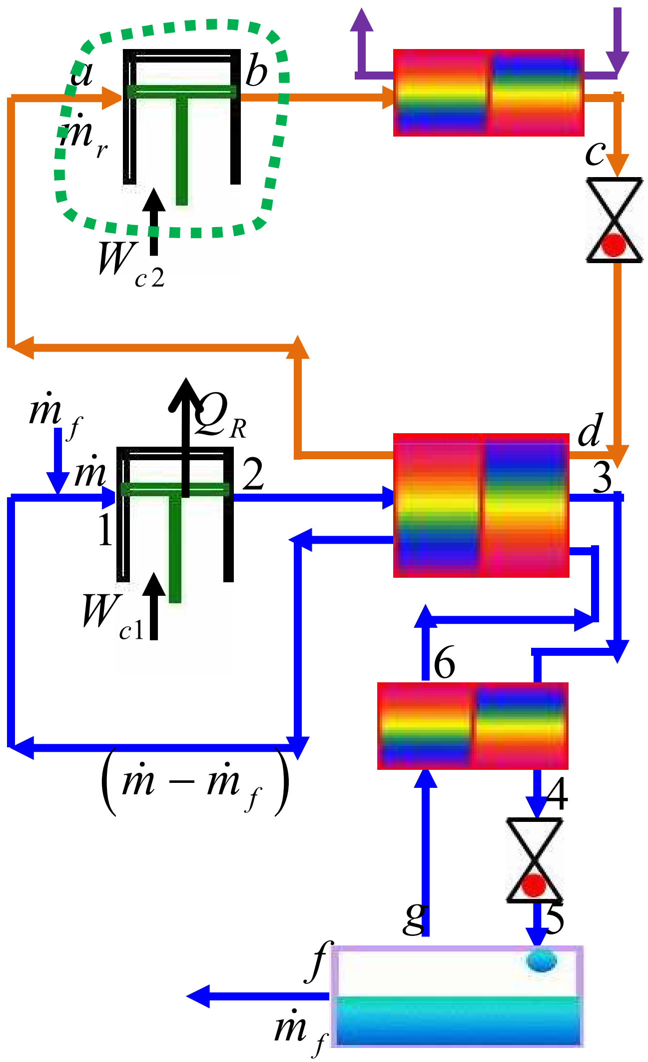


- Similarly, a control volume is taken enclosing the refrigerating compressor.
- The quantities entering and leaving this control volume are as given below.

IN	OUT
m_r @ a	m_r @ b
$-W_{c2}$	0

- The heat of compression is zero because the process is adiabatic.

Precooled L – H Cycle



- Using 1st Law for the following table, we get

IN	OUT
m_r @ a	m_r @ b
$-W_{c2}$	0

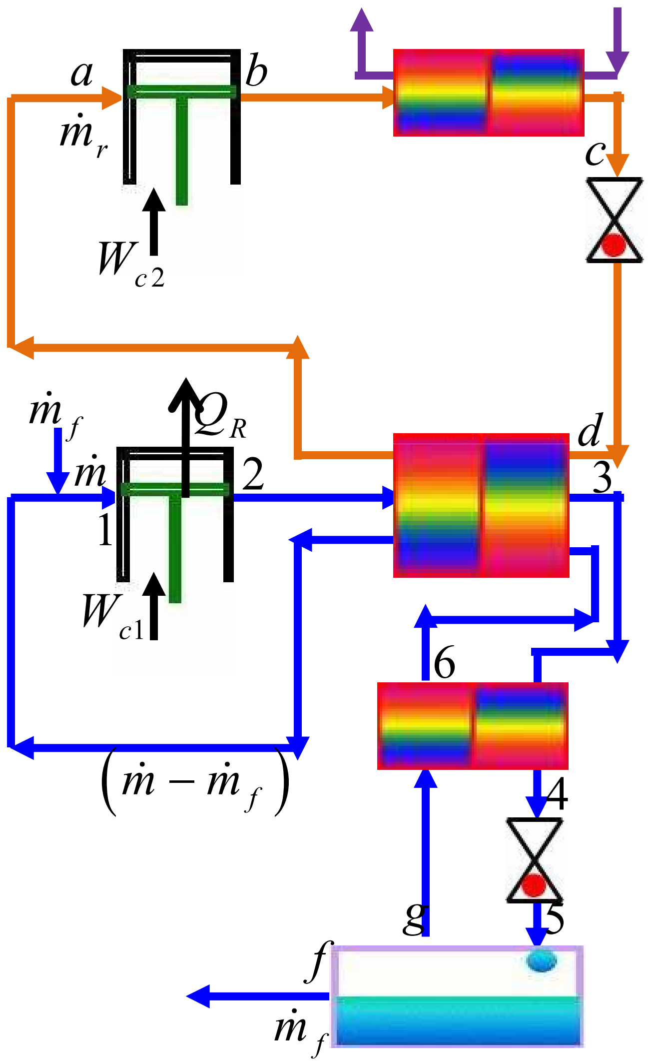
$$E_{in} = E_{out}$$

$$\dot{m}_r h_{a,r} - W_{c2} = \dot{m}_r h_{b,r}$$

- Rearranging the terms, we have

$$-W_{c2} = \dot{m}_r \left(h_{b,r} - h_{a,r} \right)$$

Precooled L – H Cycle



- The total work requirement for this system is

$$W_c = W_{c1} + W_{c2}$$

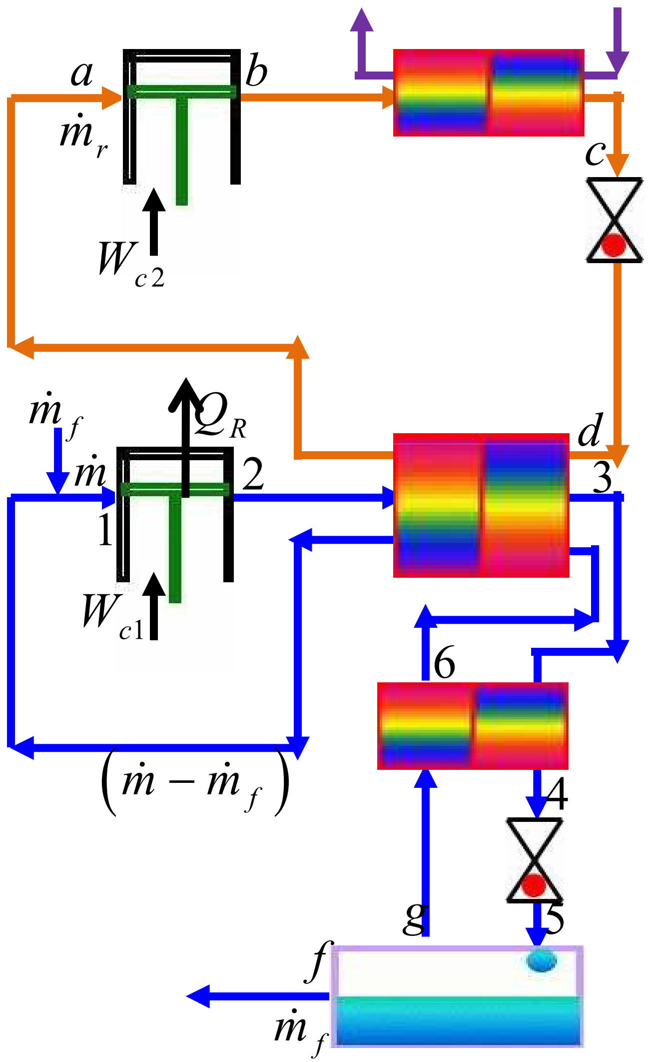
- Substituting the following values, we have

$$-W_{c1} = \dot{m}T_1\left(s_1 - s_2\right) - \dot{m}\left(h_1 - h_2\right)$$

$$-W_{c2} = \dot{m}_r\left(h_{b,r} - h_{a,r}\right)$$

$$-W_c = \dot{m}T_1\left(s_1 - s_2\right) - \dot{m}\left(h_1 - h_2\right) + \dot{m}_r\left(h_{b,r} - h_{a,r}\right)$$

Precooled L – H Cycle

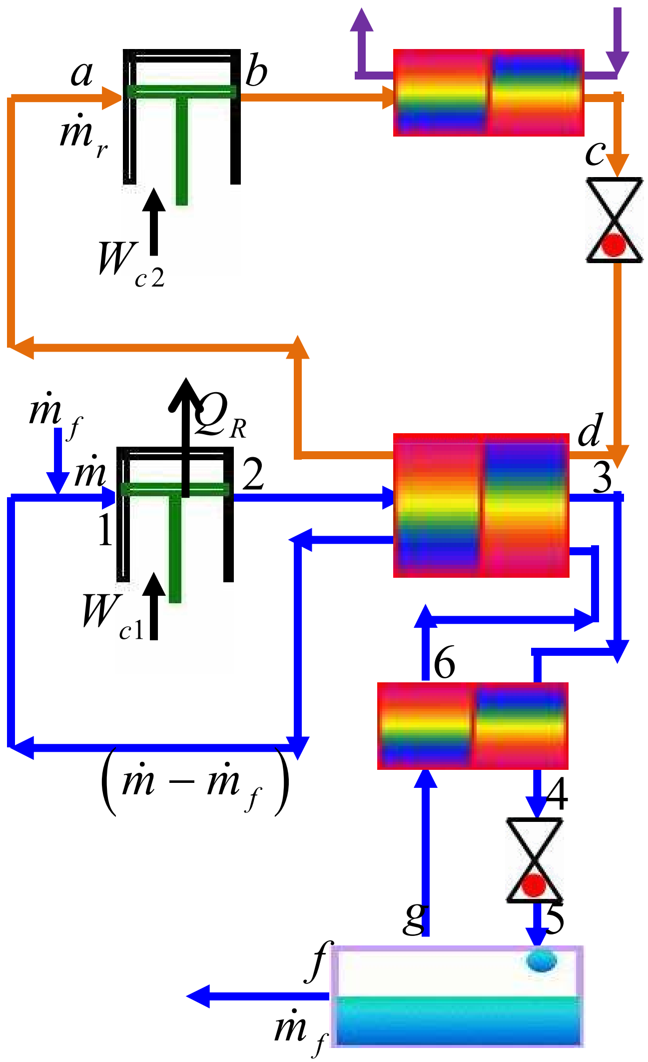


$$-W_c = \dot{m}T_1\left(s_1 - s_2\right) - \dot{m}\left(h_1 - h_2\right) + \dot{m}_r\left(h_{b,r} - h_{a,r}\right)$$

- The work required for a unit mass of primary gas compressed is given as

$$-\frac{W_c}{\dot{m}} = T_1\left(s_1 - s_2\right) - \left(h_1 - h_2\right) + \frac{\dot{m}_r}{\dot{m}}\left(h_{b,r} - h_{a,r}\right)$$

Precooled L – H Cycle

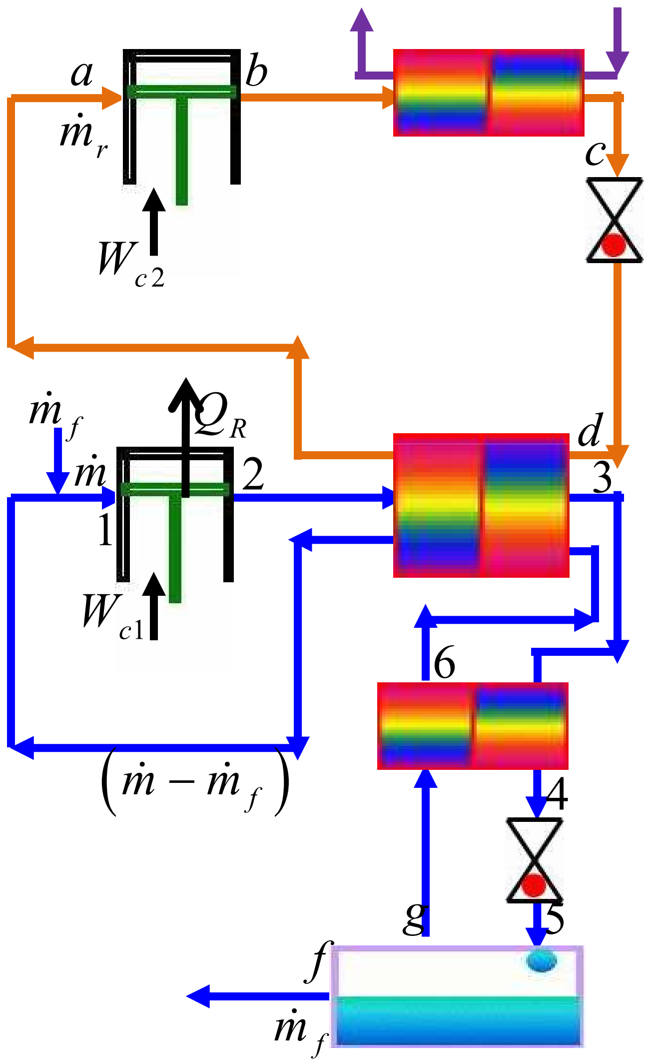


- Denoting the ratio $\frac{\dot{m}_r}{\dot{m}} = r$

$$-\frac{W_c}{\dot{m}} = T_1(s_1 - s_2) - (h_1 - h_2) + \frac{\dot{m}_r}{\dot{m}}(h_{b,r} - h_{a,r})$$

$$-\frac{W_c}{\dot{m}} = T_1(s_1 - s_2) - (h_1 - h_2) + r(h_{b,r} - h_{a,r})$$

Precooled L – H Cycle



$$-\frac{W_c}{\dot{m}} = T_1(s_1 - s_2) - (h_1 - h_2) + r(h_{b,r} - h_{a,r})$$

- The first and second terms are the work requirement in a Simple Linde – Hampson system.
- The third term is the additional work required to precool the system.

Tutorial – 1

- Determine the **y**, $\mathbf{y_{max}}$, the work/unit mass compressed, work/unit mass liquefied and FOM for the Simple and Precooled Linde – Hampson systems with Nitrogen as working fluid. The R134A is the refrigerant for the precooling system with ratio **r** as 0.08. The liquefaction system is operated between 1.013 bar (1 atm) and 101.3 bar (100 atm) at 300 K. The following is the data for R134a. Comment on the results.

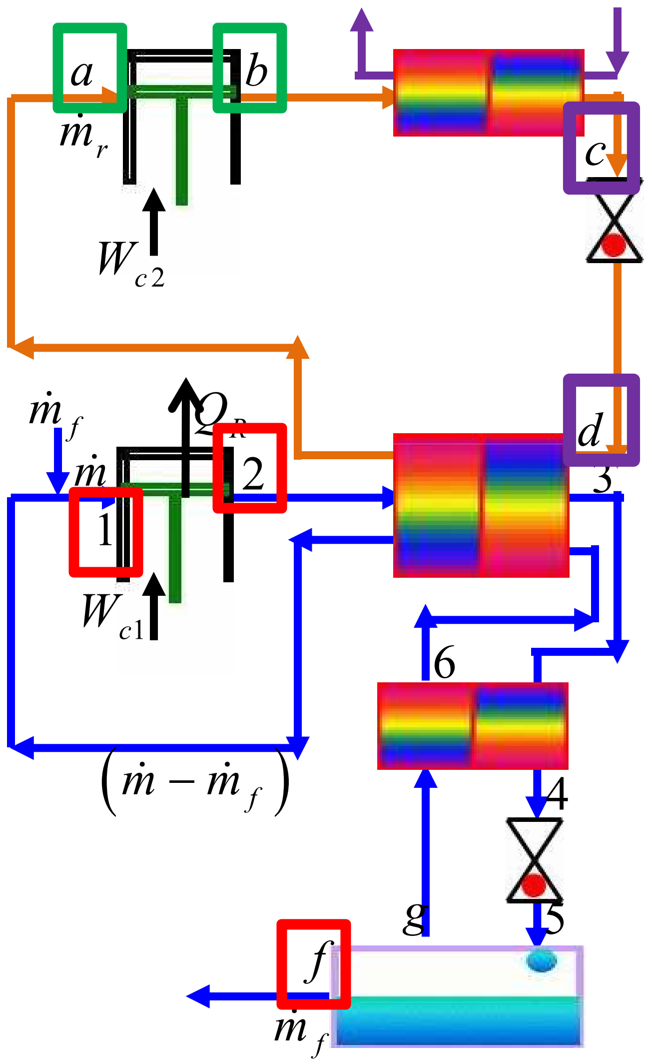
	a	b	c
p (bar)	1.013	10.13	10.13
T (K)	300	373	300
h (J/g)	390	482	260

Tutorial – 1

Given
Cycle : Simple and Precooled L – H System
Working Fluid : Nitrogen
Pressure : 1 atm → 100 atm
Temperature : 300 K
Refrigerant : R134a, 1 atm → 10 atm
Mass ratio(r) : 0.08

For above cycles, Calculate and comment	
1	Liquid Yield **y**, $\mathbf{y_{max}}$
2	Work/unit mass of gas compressed
3	Work/unit mass of gas liquefied
4	FOM

Tutorial – 1



	1	2	f
p (bar)	1.013	101.3	1.013
T (K)	300	300	77
h (J/g)	462	445	29
s (J/gK)	4.42	3.1	0.42

	a	b	c
p (bar)	1.013	10.13	10.13
T (K)	300	373	300
h (J/g)	390	482	260
s (J/gK)	R134a		

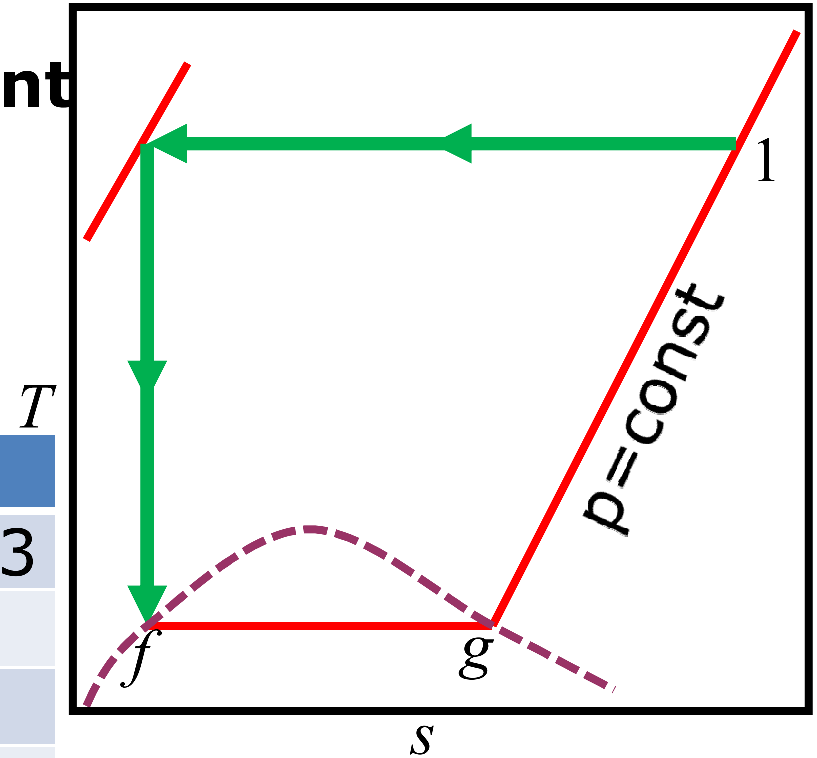
- $h_d = h_c$, since the expansion is isenthalpic.

Tutorial – 1

- **Ideal Work Requirement**

$$-\frac{\dot{W}_i}{\dot{m}} = T_1\left(s_1 - s_f\right) - \left(h_1 - h_f\right)$$

	1	2	f
p (bar)	1.013	101.3	1.013
T (K)	300	300	77
h (J/g)	462	445	29
s (J/gK)	4.42	3.1	0.42

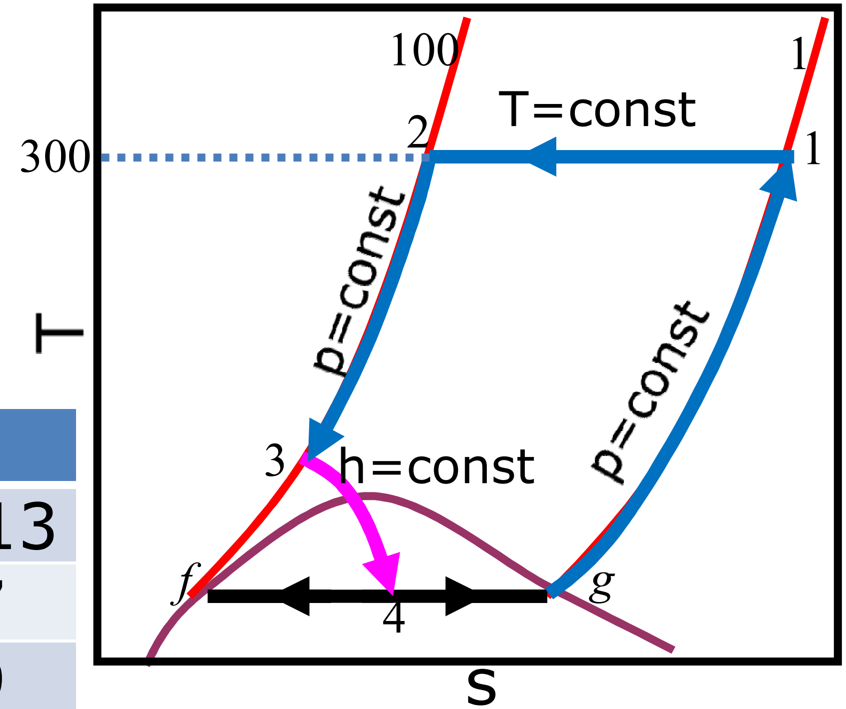


$$-\frac{W_c}{\dot{m}} = 300\left(4.42 - 0.42\right) - \left(462 - 29\right) = 767\ J/g$$

Tutorial – 1

- **Liquid yield**

$$y = \left(\frac{h_1 - h_2}{h_1 - h_f} \right)$$



	1	2	f
p (bar)	1.013	101.3	1.013
T (K)	300	300	77
h (J/g)	462	445	29
s (J/gK)	4.42	3.1	0.42

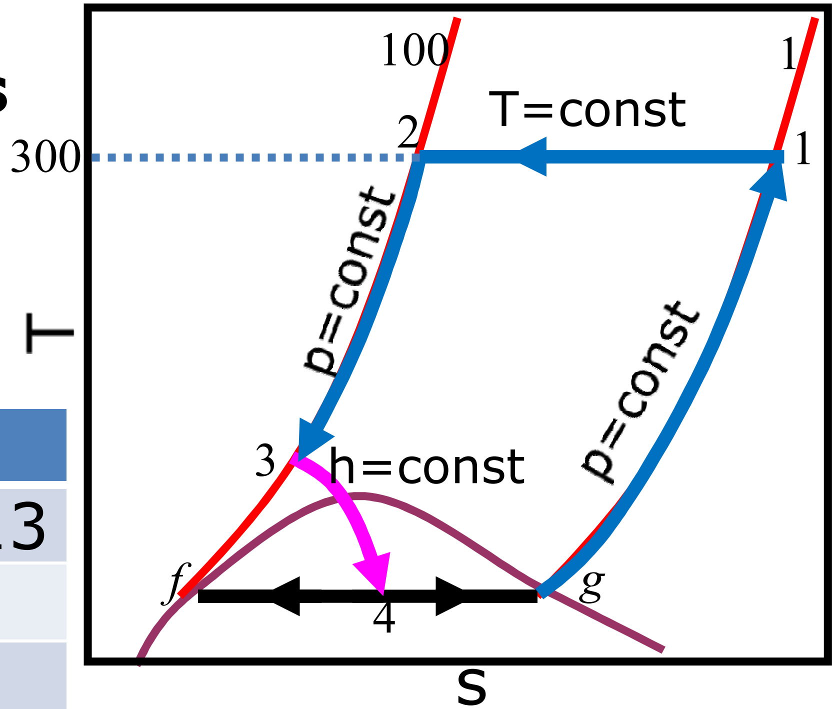
$$y = \left(\frac{h_1 - h_2}{h_1 - h_f} \right) = \left(\frac{462 - 445}{462 - 29} \right) = \left(\frac{17}{433} \right) = 0.04$$

Tutorial – 1

- **Work/unit mass of gas compressed**

$$-\frac{W_c}{\dot{m}} = T_1\left(s_1 - s_2\right) - \left(h_1 - h_2\right)$$

	1	2	f
p (bar)	1.013	101.3	1.013
T (K)	300	300	77
h (J/g)	462	445	29
s (J/gK)	4.42	3.1	0.42



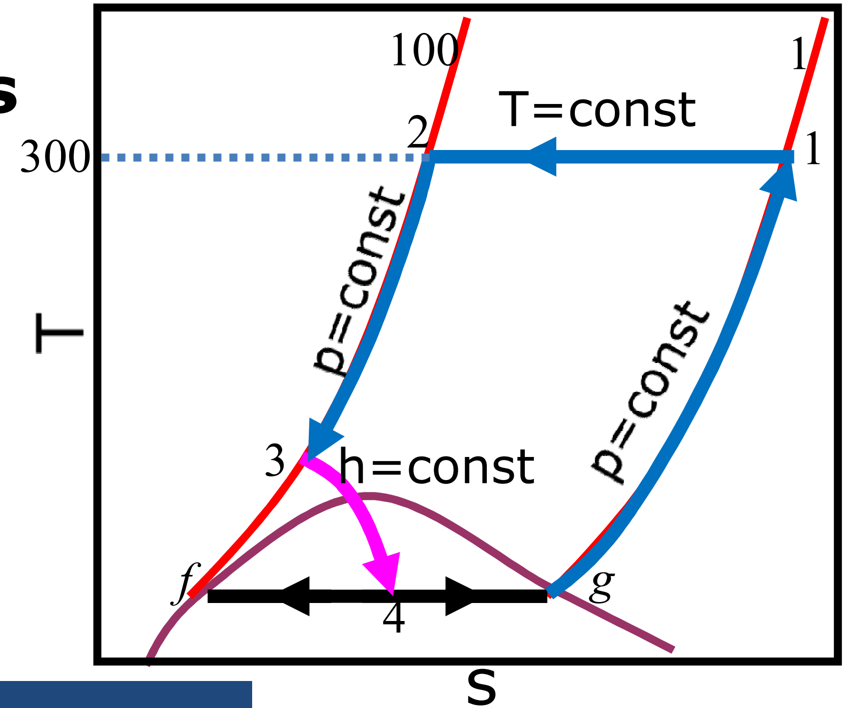
$$-\frac{W_c}{\dot{m}} = 300\left(4.42 - 3.1\right) - \left(462 - 445\right) \quad = 379\ J/g$$

Tutorial – 1

- **Work/unit mass of gas liquefied**

$$-\frac{W_c}{\dot{m}} = 379$$

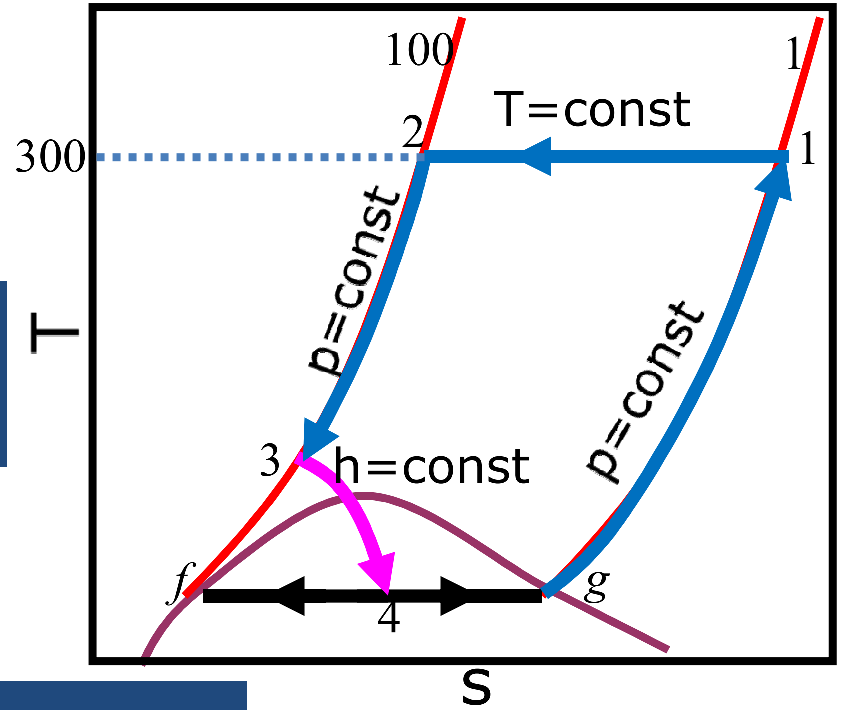
$$y = 0.04$$



$$-\frac{W_c}{\dot{m}_f} = -\frac{W_c}{y\dot{m}} \qquad = \frac{379}{0.04} \qquad = 9475\ J/g$$

Tutorial – 1

- **Figure of Merit (FOM)**



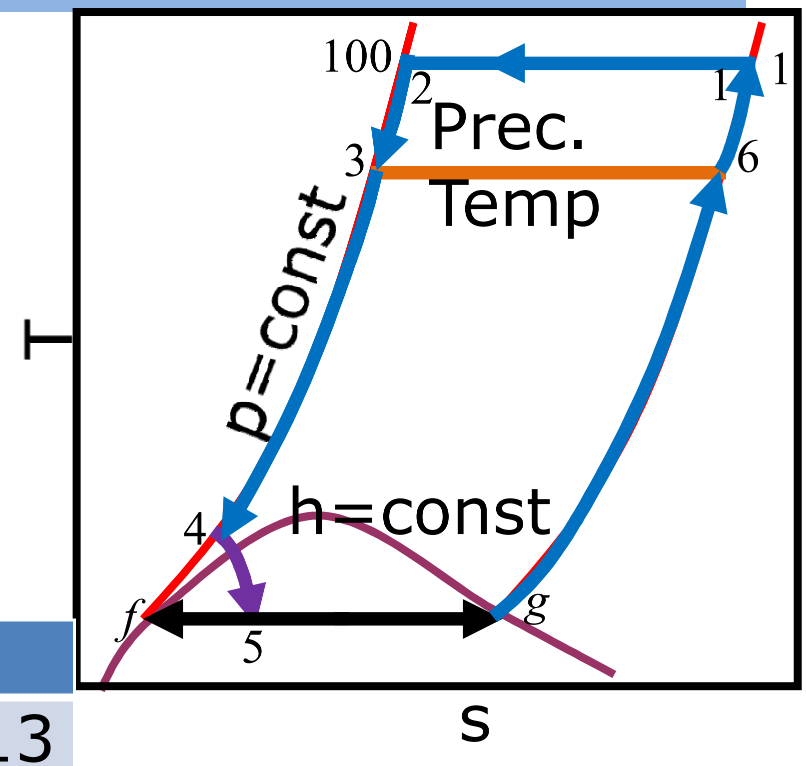
$$-\frac{W_c}{\dot{m}_f} = 9475$$

$$-\frac{W_i}{\dot{m}_f} = 767$$

$$FOM = \frac{W_i}{\dot{m}_f} \Big/ \frac{W_c}{\dot{m}_f} = \frac{767}{9475} = 0.081$$

Tutorial – 1

- The T – s diagram for a Precooled Linde – Hampson system is as shown.



- The state properties are as tabulated below.

	1	2	f
p (bar)	1.013	101.3	1.013
T (K)	300	300	77
h (J/g)	462	445	29
s (J/gK)	4.42	3.1	0.42

Tutorial – 1

- **Liquid yield**

$$y=\frac{\dot{m}_f}{\dot{m}}=\frac{h_1-h_2}{h_1-h_f}+r\left(\frac{h_{a,r}-h_{d,r}}{h_1-h_f}\right)$$

$$r = 0.08$$

	1	2	f	a	b	c
p (bar)	1.013	101.3	1.013	1.013	10.13	10.13
T (K)	300	300	77	300	373	300
h (J/g)	462	445	29	390	482	260
s (J/gK)	4.42	3.1	0.42	**R134a**		

$$y=\frac{(462-445)}{(462-29)}+0.08\frac{(390-260)}{(462-29)}=\frac{(17)}{(433)}+0.08\frac{(130)}{(433)}=0.063$$

Tutorial – 1

- **Work/unit mass of N_2 compressed**

$$-\frac{W_c}{\dot{m}} = T_1(s_1 - s_2) - (h_1 - h_2) + r(h_{b,r} - h_{a,r})$$

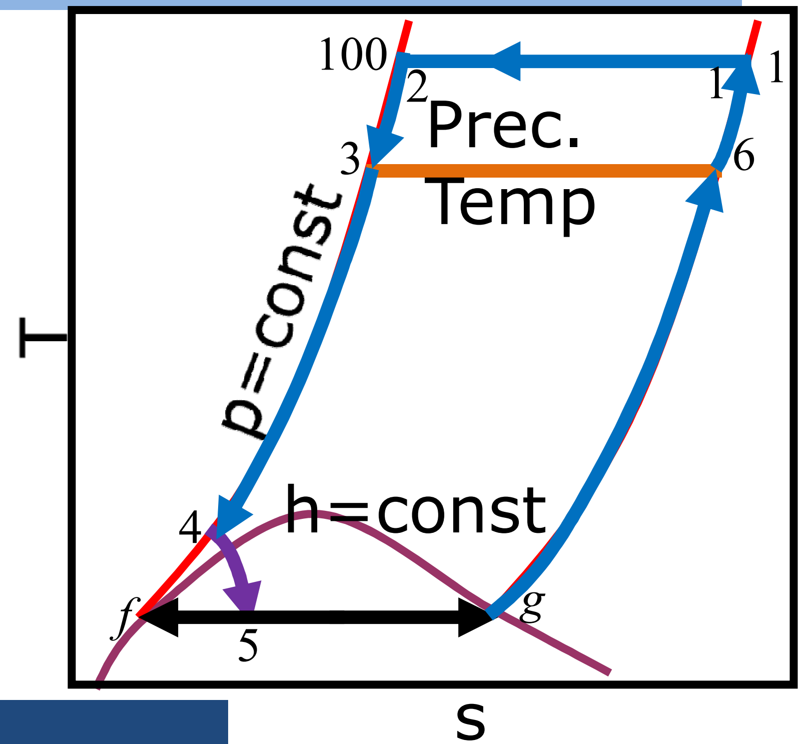
$$r = 0.08$$

	1	2	f	a	b	c
p (bar)	1.013	101.3	1.013	1.013	10.13	10.13
T (K)	300	300	77	300	373	300
h (J/g)	462	445	29	390	482	260
s (J/gK)	4.42	3.1	0.42	**R134a**		

$$-\frac{W_c}{\dot{m}} = 300(4.42 - 3.1) - (462 - 445) + 0.08(482 - 390) = 386.3\ J/g$$

Tutorial – 1

- **Work/unit mass of N_2 liquefied**



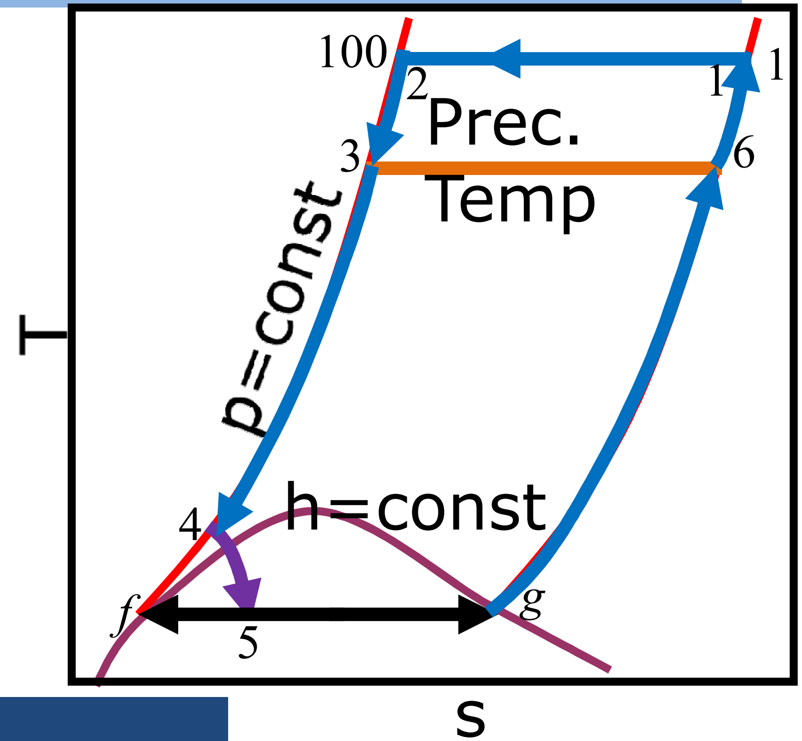
$$-\frac{W_c}{\dot{m}} = 386.3$$

$$y = 0.063$$

$$-\frac{W_c}{\dot{m}_f} = -\frac{W_c}{y\dot{m}} = \frac{386.3}{0.063} = 6131.7\ J/g$$

Tutorial – 1

- **Figure of Merit (FOM)**



$$-\frac{W_c}{\dot{m}_f} = 6131.7$$

$$-\frac{W_i}{\dot{m}_f} = 767$$

$$FOM = \frac{W_i}{\dot{m}_f} \bigg/ \frac{W_c}{\dot{m}_f} = \frac{767}{6131.7} = 0.1251$$

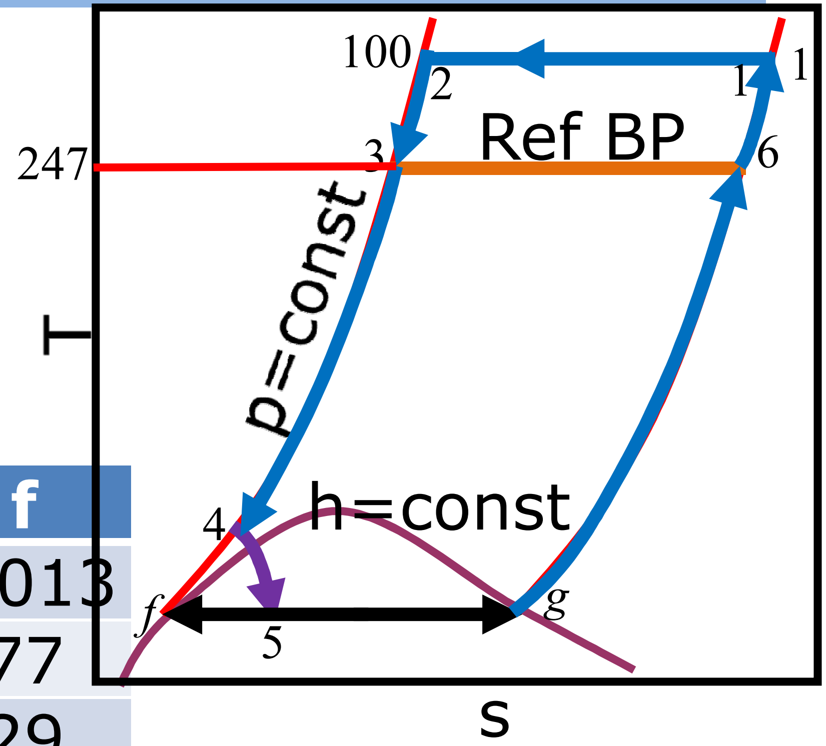
Tutorial – 1

- **Maximum Liquid yield**

$y = y_{max}$ $\quad T_3 = T_6 = T_d = 247\,K$

$$y_{max} = \frac{h_6 - h_3}{h_6 - h_f}$$

$r = 0.08$



	3	6	f
p (bar)	101.3	1.013	1.013
T (K)	247	247	77
h (J/g)	380	408	29

$$y_{max} = \frac{(408-380)}{(408-29)} = \frac{(28)}{(379)} = 0.074$$

Tutorial – 1

	Simple	Precooled	Max.
y	0.04	0.063	0.074
$-\frac{W_c}{\dot{m}}$	379	386.3	386.3
$-\frac{W_c}{\dot{m}_f}$	9475	6131.7	5220.2
FOM	0.081	0.1251	0.147

- Tabulating the results, we have the above comparison for Simple and Precooled Linde – Hampson System.

Assignment

1. Compare and comment on the following for both Simple and Precooled Linde – Hampson systems with Air as working fluid when the system is operated between 1.013 bar (1 atm) and 202.6 bar (200 atm) at 300 K. The effectiveness of HX is 100% and r=0.1.

- Ideal Work requirement
- Liquid yield
- Work/unit mass compressed
- Work/unit mass liquefied
- FOM

Summary

- The method of cooling the gas after the compression or before the entrance to the heat exchanger is called as precooling.
- The Linde – Hampson cycle with a precooling arrangement is called as Precooled Linde – Hampson cycle.
- In a Precooled Linde – Hampson system, a closed cycle refrigerator is thermally coupled to a simple Linde – Hampson system through a 3 – fluid heat exchanger.

Summary

- Compression process is isothermal in Liquefaction cycle but it is adiabatic in precooling system of a Precooled Linde – Hampson system.
- The precooling limit of the precooling cycle is governed by the boiling point of refrigerant at its suction pressure.
- The yield for a Precooled Linde – Hampson system is

$$y = \frac{\dot{m}_f}{\dot{m}} = \frac{h_1 - h_2}{h_1 - h_f} + r\left(\frac{h_{a,r} - h_{d,r}}{h_1 - h_f}\right)$$

$$\frac{\dot{m}_r}{\dot{m}} = r$$

Summary

$$y_{\max} = \frac{\dot{m}_f}{\dot{m}} = \frac{h_6 - h_3}{h_6 - h_f}$$

- The maximum liquid yield is given by the above expression. The enthalpy values are evaluated at the boiling point of the refrigerant.

- The work requirement for the unit mass of primary fluid compressed is

$$-\frac{W_c}{\dot{m}} = T_1\left(s_1 - s_2\right) - \left(h_1 - h_2\right) + r\left(h_{b,r} - h_{a,r}\right)$$

- From the tutorial, the yield of the precooled system is more than that of a simple system.

Thank You!