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Lecture No - 13



### **Earlier Lecture**

- In the earlier lecture, we have seen that the precooling of a Simple Linde – Hampson system improved the liquid yield.
- In a Precooled Linde Hampson system, a closed cycle refrigerator is thermally coupled to a simple Linde – Hampson system through a 3 – fluid heat exchanger.
- The precooling limit of the precooling cycle is governed by the boiling point of the refrigerant at its suction pressure.

### **Earlier Lecture**

- From the tutorial in the last lecture, we saw that the yield of a Precooled cycle was more than that of the Simple System.
- The maximum liquid yield in the Precooled system occurs, when the effectiveness of the 3 – fluid heat exchanger is 100%.



• In the above equation, the values of  $\mathbf{h_6}$  and  $\mathbf{h_3}$  are evaluated at boiling point of the refrigerant.

## **Outline of the Lecture**

#### **Topic : Gas Liquefaction and Refrigeration Systems (contd)**

- Precooled Linde Hampson system
  - Effect of Flow ratio r
  - Yield v/s mass ratio **r**
  - Work requirement v/s mass ratio r
  - FOM v/s mass ratio r

### Introduction

 The work requirement for a Precooled Linde – Hampson System is given by

$$-\frac{W_c}{\dot{m}} = \left(T_1(s_1 - s_2) - (h_1 - h_2)\right) + r(h_{b,r} - h_{a,r})$$

- The first term is the work requirement in a Simple Linde – Hampson system.
- The second term is the additional work required to precool the system.

### Introduction

 The yield for a Precooled Linde – Hampson system is as given below.

$$y = \frac{\dot{m}_{f}}{\dot{m}} = \frac{h_{1} - h_{2}}{h_{1} - h_{f}} + r\left(\frac{h_{a,r} - h_{d,r}}{h_{1} - h_{f}}\right)$$

Where, the mass ratio is given by



- The first term in the above expression is the yield for a simple Linde Hampson system.
- The second term is the additional yield occurring due to the precooling of the Simple system.

### Introduction

• The increment in the yield is related to the

$$y = \frac{\dot{m}_{f}}{\dot{m}} = \frac{h_{1} - h_{2}}{h_{1} - h_{f}} + r \begin{pmatrix} h_{a,r} - h_{d,r} \\ h_{1} - h_{f} \end{pmatrix} \qquad \frac{\dot{m}_{r}}{\dot{m}} = r$$

- The change in enthalpy values from  $(\mathbf{h}_d \rightarrow \mathbf{h}_a)$  of the refrigerant.
- The refrigerant flow (m<sub>r</sub>) rate across the 3 fluid heat exchanger.

- From the above equations, it is clear that the liquid yield and work requirement are dependent on the parameters like refrigerant flow rate (m<sub>r</sub>), compression pressure and precooling temperature.
- By varying these parameters, the performance of the system can be optimized.
- Hence, there is a need to study the effect of the various parameters on the performance of the system for the proper design.



- One such parameter which is of great importance is the refrigerant flow rate ratio **r**.
- The state of the working fluid entering the refrigeration compressor is very important.



- Let the heat change of the refrigerant be represented as  $Q_{ref} = r(h_{r, d} h_{r, a})$ .
- Similarly, the required heat change for Linde – Hampson cycle be denoted as Q<sub>LHS</sub>.
- The relative values of Q<sub>ref</sub> and Q<sub>LHS</sub> determine the state of the refrigerant at the point a.

## Introduction





- In the 1<sup>st</sup> case, the value of  $T_3$  would not be equal to  $T_d$ .
  - The 2<sup>nd</sup> case is the condition to achieve **y<sub>max</sub>.**
- Since Q<sub>ref</sub> > Q<sub>LHS</sub> in the 3<sup>rd</sup> case, the liquid would enter the refrigerating compressor.

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### Introduction



- That is any excess flow of refrigerant than required, results into an excess available heat content.
- As a result, the state of fluid at the point **a** would be a two – phase mixture which is unfavorable.
- Hence, for a given operating conditions, there is an optimum r.



- This is better explained through a tutorial solved in the subsequent slides.
- Various flow ratio r values are taken both below and above the limiting value to explain the principle.

### **Tutorial**

#### Part 1

 A Precooled Linde – Hampson System has Nitrogen and R134a as primary and secondary fluids respectively. Determine the Liquid yield and FOM. The operating conditions and other useful data are as given below.

$N_2$	r	Point 2		а	b	С	
Ι	0.05	101.3 bar	p (bar)	1.013	8.104	8.104	
II	0.07	101.3 bar	T (K)	247	314	305	
III	0.05	202.6 bar	h (J/g)	380	420	240	
IV	0.1	202.6 bar	R134a				

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### **Tutorial**

#### Part 2

 Also, calculate the y<sub>max</sub> for each of the pressures mentioned and their corresponding r values. Plot the data graphically and comment on the nature of y, work requirement, FOM versus r.

$N_2$	r	Point 2		а	b	С	
Ι	0.05	101.3 bar	p (bar)	1.013	8.104	8.104	
II	0.07	101.3 bar	T (K)	247	314	305	
III	0.05	202.6 bar	h (J/g)	380	420	240	
IV	0.1	202.6 bar	R134a				

### **Tutorial**

#### **Given : Part 1**

Cycle : Precooled L – H Cycle with N<sub>2</sub>. Temperature : 300 K Refrigerant : R134a, 1 atm  $\rightarrow$  8 atm

#### For this cycle, Calculate and comment

- **1** Liquid Yield **y**
- **2** Work/unit mass of gas compressed
- **3** Work/unit mass of gas liquefied
- **4** FOM



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### **Tutorial**

#### Given : Part 2

Cycle : Precooled L – H Cycle with N<sub>2</sub>. Temperature : 300 K Refrigerant : R134a, 1 atm  $\rightarrow$  8 atm

#### For this cycle, Calculate and comment

- 1 Liquid Yield **y**<sub>max</sub>
- **2** Work/unit mass of gas compressed
- **3** Work/unit mass of gas liquefied
- **4** FOM



### Methodology

- The two pressures conditions under study are 101.3 bar and 202.6 bar.
- The Liquid yield and FOM are calculated only for 101.3 bar pressure condition.
- Also, the calculations for y<sub>max</sub> and for an r value beyond y<sub>max</sub> condition are calculated only for 101.3 bar pressure condition.
- Calculations pertaining to 202.6 bar condition are left as an exercise to students.

### **Tutorial**



			_	
$N_2$	1	2	f	
p (bar)	1.013	101.3	1.013	
T (K)	300	300	77	
h (J/g)	462	445	29	
s (J/gK)	4.42	3.1	0.42	



•  $h_d = h_c$ , since the expansion is isenthalpic.

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$$-\frac{W_c}{\dot{m}} = 300(4.42 - 0.42) - (462 - 29) = 767 J/g$$

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### Tutorial : Part - 1

- The T s diagram for a Precooled Linde – Hampson system is as shown.
- The state properties are as tabulated below.



$N_2$	1	2	f	а	b	С
p (bar)	1.013	101.3	1.013	1.013	8.104	8.104
T (K)	300	300	77	247	314	305
h (J/g)	462	445	29	380	420	240
s (J/gK)	4.42	3.1	0.42		R134a	

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### Tutorial : Part – 1

Liquid yield



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#### **CRYOGENIC** ENGINEERING Tutorial : Part – 1 Work/unit mass of N<sub>2</sub> compressed Point 2 N- $-\frac{W_{c}}{\dot{m}} = T_{1}(s_{1} - s_{2}) - (h_{1} - h_{2}) + r(h_{b,r} - h_{a,r})$ 0.05 101.3 bar $N_{2}$ 2 b а C 1.013 1.013 8.104 8.104 p (bar) 1.013 101.3 T (K) 300 300 314 305 247 77 h (J/g) 462 445 29 380 420 240 3.1 0.42 s(J/qK)4.42 **R134a** $-\frac{W_c}{\dot{m}}\Big|_1 = 300(4.42 - 3.1) - (462 - 445) + 0.05(420 - 380) = 381J / g$

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### Tutorial : Part – 1

Liquid yield



## Tutorial : Part – 1

 Work/unit mass of N<sub>2</sub> compressed

$$-\frac{W_{c}}{\dot{m}} = T_{1}(s_{1}-s_{2}) - (h_{1}-h_{2})$$



$$\frac{W_c}{\dot{m}}\Big|_2 = 300(4.42 - 3.1) - (462 - 445) + 0.07(420 - 380) = 381.8$$





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### Tutorial : Part – 2

r corresponding to y<sub>max</sub>



### CRYOGENIC ENGINEERING Tutorial : Part – 2

 Work/unit mass of N<sub>2</sub> compressed

$$-\frac{W_c}{\dot{m}} = T_1(s_1 - s_2) - (h_1 - h_2)$$

 $+r(h_{b,r}-h_{a,r})$ 

32

$$\frac{W_c}{\dot{m}}\Big|_3 = 300(4.42 - 3.1) - (462 - 445) + 0.11(420 - 380) = 384J / g$$

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### Tutorial : Part – 2



- From the above calculations, the value of **r** corresponding to  $y_{max}$  is 0.11 at the compression pressure of 101.3 bar.
- For r= 0.12, the enthalpy of the refrigerant at the state
   a is calculated by applying the energy balance across the 3 fluid heat exchanger.

### Tutorial : Part – 2



Consider a control volume enclosing the 3 fluid heat exchanger.

IN	OUT
m <sub>r</sub> @ d	m <sub>r</sub> @ a
m @ 2	m @ 3
m – m <sub>f</sub> @ 6	m – m <sub>f</sub> @ 1

 Applying the heat balance, we have

$$\dot{m}_{r}h_{d,r} + \dot{m}h_{2} + (\dot{m} - \dot{m}_{f})h_{6}$$
$$= \dot{m}_{r}h_{a,r} + \dot{m}_{3}h_{3} + (\dot{m} - \dot{m}_{f})h_{1}$$

### Tutorial : Part – 2



Rearranging the terms,

$$\dot{h}_{r} \left( h_{a,r} - h_{d,r} \right) + \dot{m} \left( h_{3} - h_{2} + h_{1} - h_{6} \right)$$
$$= \dot{m}_{f} \left( h_{1} - h_{6} \right)$$

#### Denoting the ratios

$$\frac{\dot{m}_r}{\dot{m}} = r \qquad y = \frac{\dot{m}_f}{\dot{m}}$$

$$r(h_{a,r} - h_{d,r}) + (h_3 - h_2 + h_1 - h_6)$$
  
=  $y(h_1 - h_6)$ 

37

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**CRYOGENIC ENGINEERING**  
**Tutorial : Part – 2**  

$$r(h_{a,r} - h_{d,r}) + (h_3 - h_2 + h_1 - h_6) = y(h_1 - h_6)$$

The equation of **y** at this refrigerant flow rate **r** is given by

$$y = \frac{h_1 - h_2}{h_1 - h_f} + r \left( \frac{h_{a,r} - h_{d,r}}{h_1 - h_f} \right)$$

- The only unknowns in these two equations are h<sub>a,r</sub> and y.
- The values of h<sub>a,r</sub> and y are obtained by solving these two simulations equations.

### Tutorial : Part – 2

• Substituting the values, we have

N <sub>2</sub>	1	2	f	6	а	С
p (bar)	1.013	101.3	1.013	1.013	1.013	8.104
T (K)	300	300	77	247	247	305
h (J/g)	462	445	29	408	h <sub>a.r</sub>	240
					<b>Ŕ1</b>	34a

$$y = r \left( \frac{h_{a,r} - h_{d,r}}{h_1 - h_6} \right) + \left( \frac{h_3 - h_2 + h_1 - h_6}{h_1 - h_6} \right) 54y - 0.12h_{a,r} = -39.8$$

$$y = \frac{h_1 - h_2 + r(h_{a,r} - h_{d,r})}{h_1 - h_f}$$

$$h_{a,r} - 3610.1y = 98.55$$

## Tutorial : Part – 2

- Solving the simultaneous equations we have values as  $y_4 = 0.074$   $h_{a,r} = 364.9$
- It is important to note that the value of y is same as y<sub>max</sub> = 0.074.
- Also, the value of enthalpy at point a after the heat exchanger for r=0.12 is 364.9 J/g.
- This value is less than the value at the saturated vapor (380 J/g) indicating that the fluid is now a two – phase mixture.

#### **CRYOGENIC ENGINEERING** Tutorial : Part – 2 Work/unit mass of N<sub>2</sub> compressed Point 2 above y<sub>max</sub> $-\frac{W_{c}}{\dot{m}} = T_{1}(s_{1} - s_{2}) - (h_{1} - h_{2}) + r(h_{b,r} - h_{a,r})$ r=0.12 101.3 bar $N_{2}$ 2 b а C p (bar) 1.013 1.013 8.104 8.104 1.013 101.3 T (K) 300 300 314 305 247 77 h (J/g) 420 462 445 29 364.9 240 3.1 0.42 s(J/gK)4.42 **R134a** $\frac{W_c}{\dot{m}}\Big|_{4} = 300(4.42 - 3.1) - (462 - 445) + 0.12(420 - 364.8) = 385.6J / g$

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### Tutorial : Part – 1

- Tabulating the results for 101.3 bar pressure condition, we have the following comparison for the various values of
  - Refrigerant flow rate (m<sub>r</sub>).

	r	У	$-\frac{W}{\cdot}$	$-\frac{W}{\dot{w}}$	FOM
			m	$m_{f}$	
I	0.05	0.055	381.0	6927.2	0.111
II	0.07	0.062	381.8	6158.1	0.125
III (y <sub>max</sub> )	0.11	0.074	384.0	5189.2	0.148
IV	0.12	0.074	385.6	5239.1	0.146

### Tutorial : Part – 1

- Similarly, calculating the results for 202.6 bar pressure condition, we have the following comparison for the various values of
  - Refrigerant flow rate (m<sub>r</sub>).

	r	У	$-\frac{W}{m}$	$-\frac{W}{\dot{w}}$	FOM
			m	$m_{f}$	
I	0.05	0.085	476.0	5600.0	0.137
II	0.1	0.102	478.0	4704.7	0.163
III (y <sub>max</sub> )	0.17	0.127	479.0	3783.5	0.203
IV	0.18	0.127	483.5	3819.6	0.201

### Tutorial : Part – 2





### Tutorial : Part – 2



- It is clear that the yield of the system increases with the increase in the refrigerant flow rate for a given compression pressure.
- As the compression pressure increases, the yield increases for a given amount of the refrigerant flow rate.

### Tutorial : Part – 2



- For each compression pressure, the yield reaches to a maximum values and thereafter, it remains constant.
- This value of **r** is the limiting value.

### Tutorial : Part – 2



- Any additional increase in r leads to the liquid flow into the refrigerant compressor, which is not a desirable condition.
- Also, the limiting value increases with the increase in the compression pressure.

### Tutorial : Part – 2

Work/unit mass compressed v/s.r



### Tutorial : Part – 2

Work/unit mass compressed v/s. r



- We see that the work/unit mass of gas compressed increases with the increase in the refrigerant flow rate for a given compression pressure.
- As the compression pressure increases, work requirement also increases.

### Tutorial : Part – 2

Work/unit mass compressed v/s. r





 It is clear that the work requirement is increased when the r value is increased beyond the limiting value.

### Tutorial : Part – 2

Work/unit mass compressed v/s. r



- For each compression pressure, the increase in the work requirement is very small.
  - Hence, the work requirement for the precooling compressor is negligible as compared to that of liquefaction compressor.

### Tutorial : Part – 2

Work/unit mass Liquified v/s. r



### Tutorial : Part – 2

Work/unit mass Liquified v/s. r



- For each compression pressure, the work requirement decreases with the increase in the refrigerant flow rate.
- As the compression pressure increases, the work requirement decreases for a given amount of the refrigerant flow rate r.

### Tutorial : Part – 2

Work/unit mass Liquified v/s. r



- The limiting values of **r** are as shown.
- Plotting the values of r above the limiting values we have as shown



### Tutorial : Part – 2

Work/unit mass Liquified v/s. r



- Any further increase in the r, increases the work input. But under such conditions, the liquid refrigerant would enter the precooling compressor.
- This is undesirable for compressor operation.

### Tutorial : Part – 2





0.18

IV

0.201

### Tutorial : Part – 2



• FOM v/s.r

- For each compression pressure, the FOM increases with the increase in the refrigerant flow rate.
- As the compression pressure increases, the FOM increase for a given amount of the refrigerant flow rate r.

### Tutorial : Part – 2



• FOM v/s.r

 The limiting values of r are as shown.



• Any further increase in the **r**, decreases the FOM. But, the liquid refrigerant would enter the precooling compressor.

### **Summary**

- For a Precooled Linde Hampson system, the liquid yield and work requirement are dependent on the parameters like refrigerant flow rate (m<sub>r</sub>), compression pressure and precooling temperature.
- It is important to note that the working fluid entering the refrigeration compressor should be in the gaseous state.
- If Q<sub>ref</sub> > Q<sub>LHS</sub>, the liquid enters the refrigerating compressor.

### Summary

- The yield of the system increases with the increase in the refrigerant flow rate and the compression pressure.
- The value of **r** corresponding to maximum yield is called as the limiting value.
- This limiting value of **r** increases with the increase in the compression pressure.
- Work/unit mass of gas compressed increases with the increase in the refrigerant flow rate and compression pressure.

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### Summary

- The work requirement for the precooling compressor is negligible as compared to that of liquefaction compressor.
- Work/unit mass of the gas liquefied decreases with the increase in the refrigerant flow rate and compression pressure.
- For a given compression pressure, this work falls to the minimum at the limiting value of r.

### Summary

- Figure of Merit (FOM) increases with the increase in the refrigerant flow rate and the compression pressure.
- For a given compression pressure, FOM reaches to a maxima at the limiting value of r.



### **Thank You!**

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