
CRYOGENIC ENGINEERING
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Lecture No - 14

Earlier Lecture

- In the earlier lectures, we have seen an Ideal Thermodynamic cycle, in which all the gas that is compressed is liquefied.
- In a Linde – Hampson system, a heat exchanger is used to conserve cold. In this system, only a part of the gas that is compressed is liquefied.
- In a Precooled Linde – Hampson system, the liquid yield and FOM are improved by precooling the working fluid using an independent refrigerating system.

Earlier Lecture

- In a Precooled Linde – Hampson system, the liquid yield and FOM are dependent on refrigerant flow rate (m_r), compression pressure and precooling temperature.

- The yield of the system increases with the increase in the refrigerant flow rate and the compression pressure.

- In this system, the mass ratio **(r)** corresponding to maximum yield is called as the limiting value. It increases with the increase in the compression pressure.

Earlier Lecture

- Work/unit mass of gas compressed increases with the increase in the refrigerant flow rate and compression pressure.

- Work/unit mass of the gas liquefied decreases with the increase in the refrigerant flow rate and compression pressure.

- Figure of Merit (FOM) increases with the increase in the refrigerant flow rate and the compression pressure.

Outline of the Lecture

Topic : Gas Liquefaction and Refrigeration Systems (contd)

- Linde Dual – Pressure System
  - Liquid yield
  - Work requirement
- Parametric study

Introduction

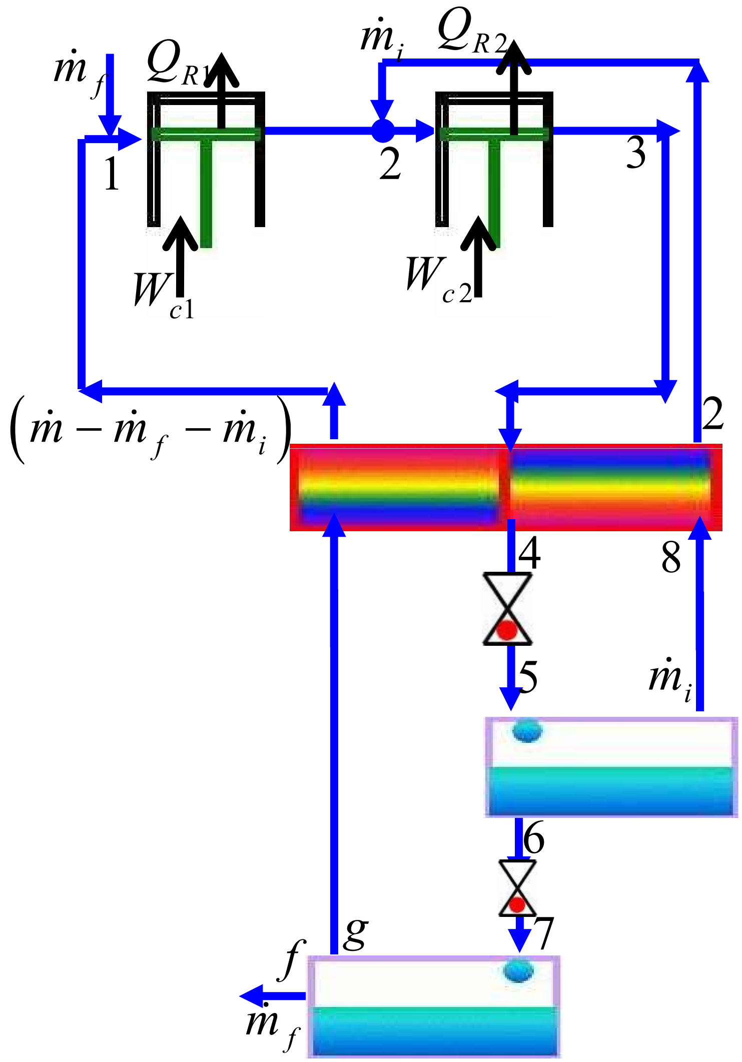
- Mathematically, the work requirement for an ideal isothermal compression process is given by,

$$\dot{W} = \dot{m} R T_1 \ln\left(\frac{P_2}{P_1}\right)$$

- The work requirement decreases either with the decrease in the mass flow rate or with the decrease in the compression ratio.

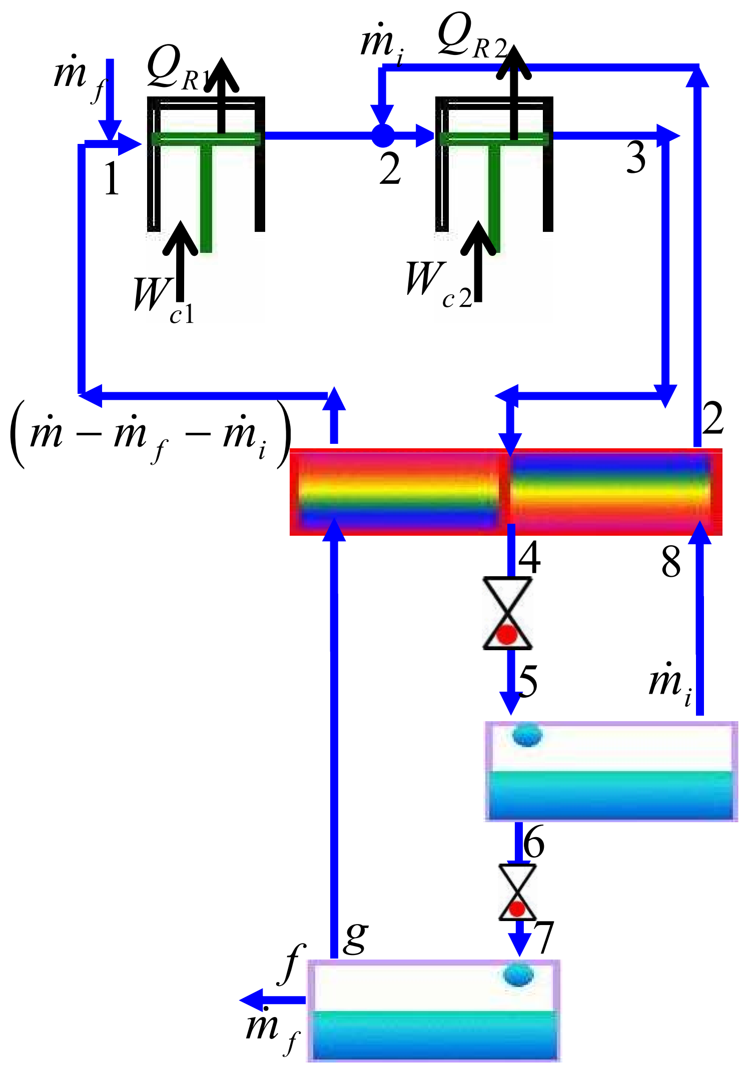
- In a Linde Dual – Pressure system, the work requirement decreases, when the compression of fluid is done in two stages and for different mass flow rates (m).

Linde Dual – Pressure System



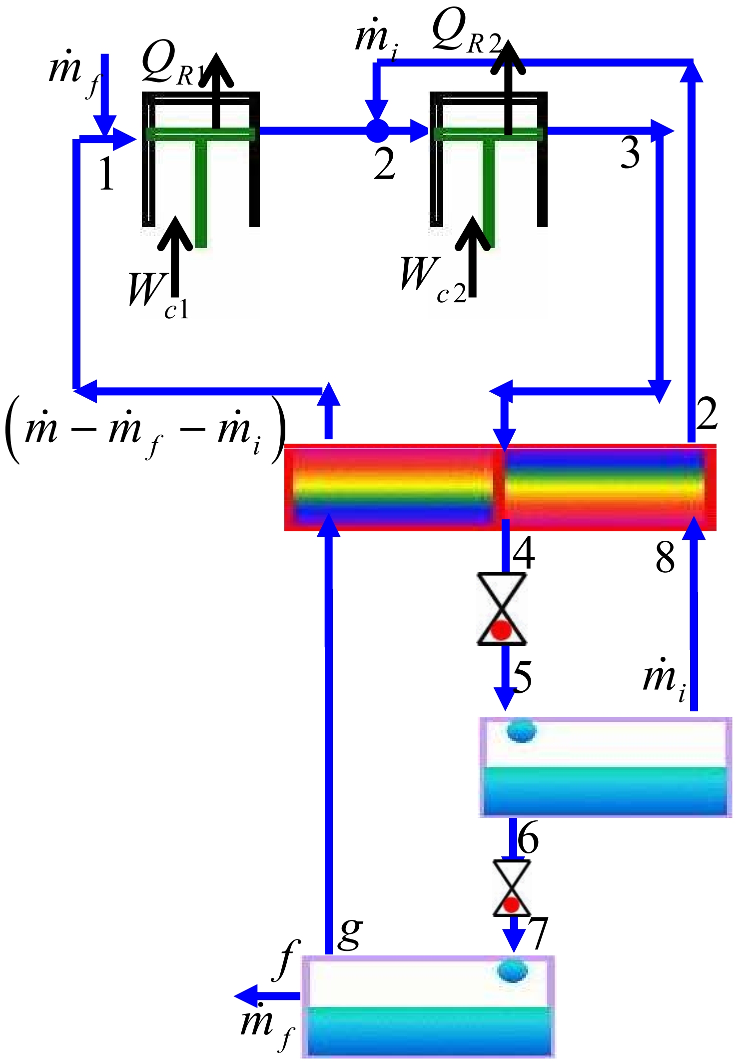
- The system consists of two compressors, a 3 – fluid heat exchanger, two J – T expansion devices, two liquid containers and a makeup gas connection.
- In this system, the entire mass flow rate of the gas is not compressed to the required high pressure, as was the case in the previous systems.

Linde Dual – Pressure System



- Only a part of the mass flow rate **(m – m$_i$)**, is compressed to an intermediate pressure from **1** → **2**.
- Thereafter, mass flow rate **m$_i$** is added to the above stream and it is then compressed from **2** → **3**.

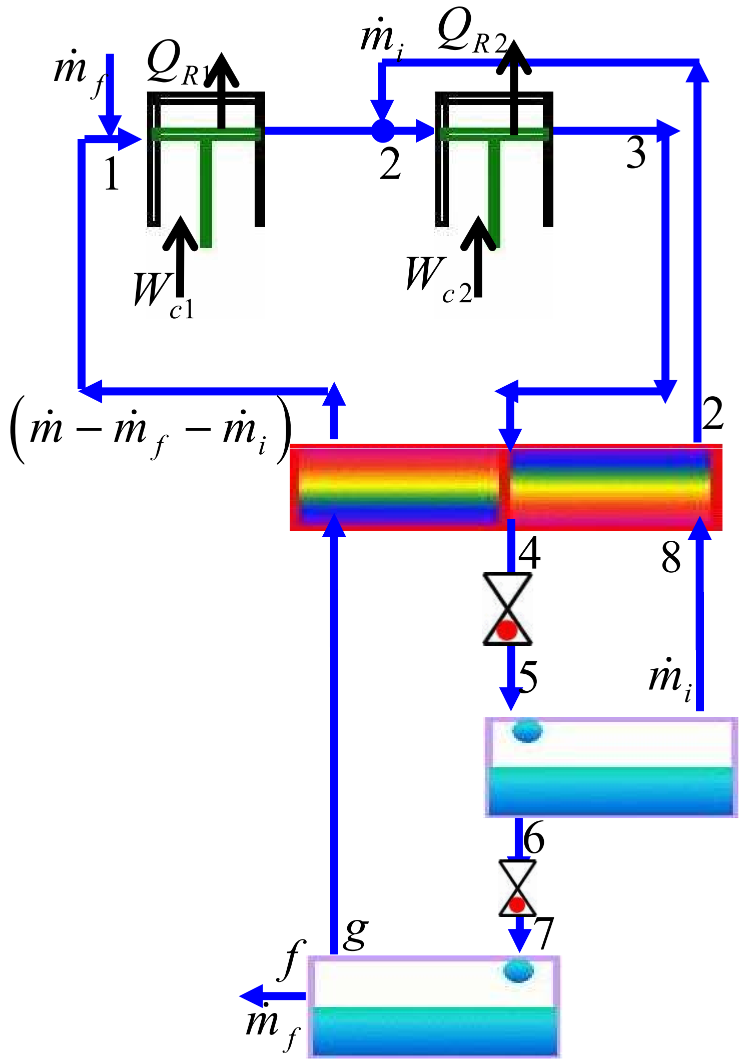
Linde Dual – Pressure System



- This arrangement not only compresses the gas in two stages but also reduces the work requirement.

- The stream **m$_i$** along with the return stream from the container – **2** is used to precool the gas at point **3** in the 3 – fluid heat exchanger.

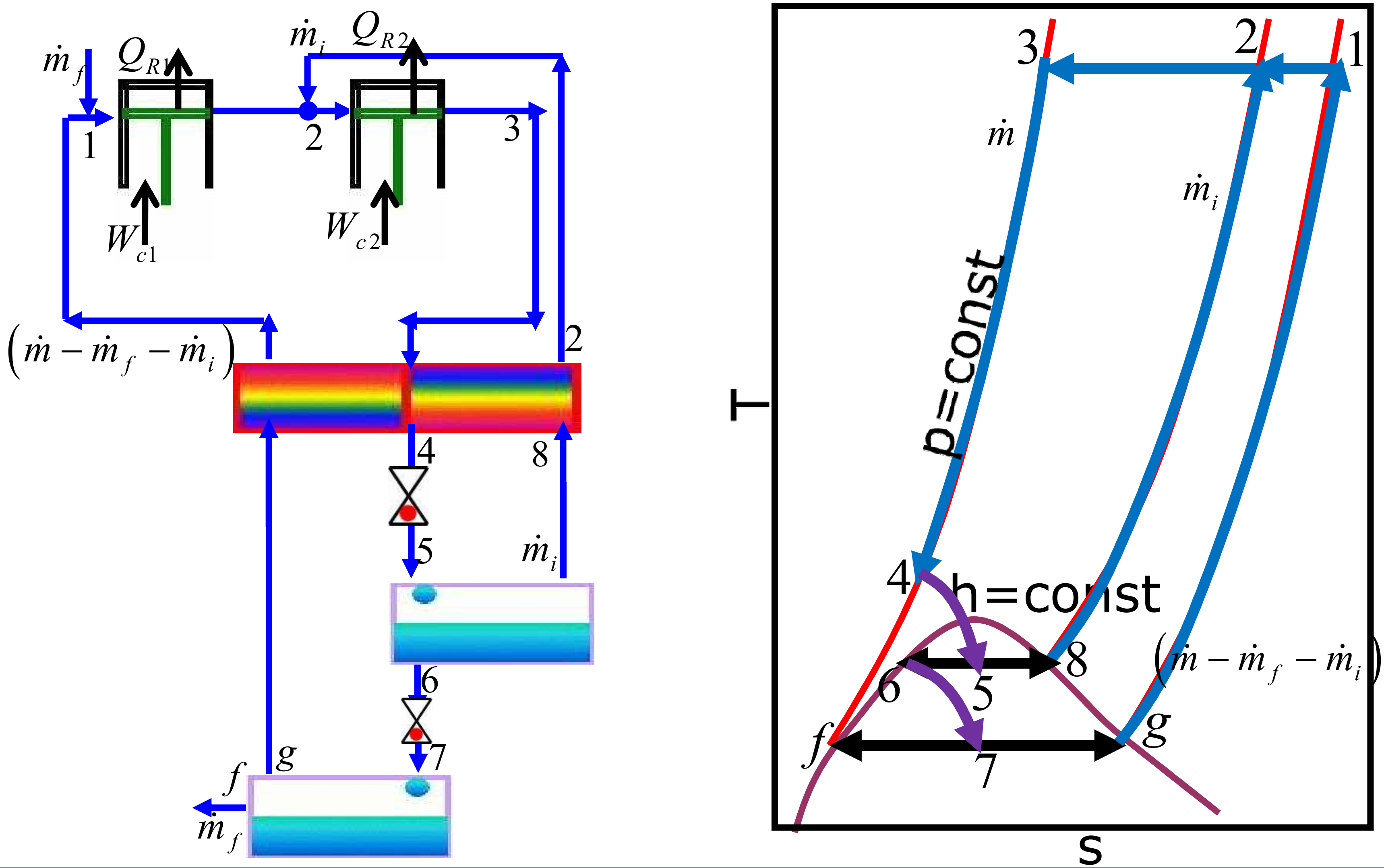
Linde Dual – Pressure System



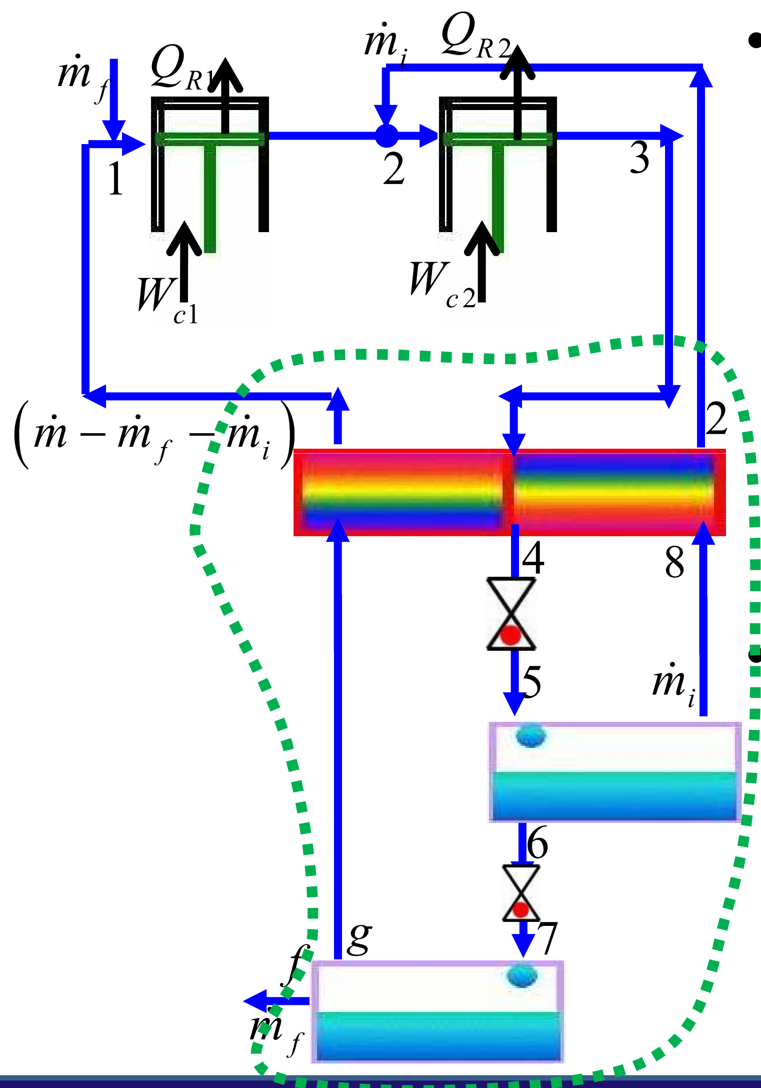
- It is important to note the 3 – fluid heat exchanger has three streams with the three different flow rates. They are

  - **(m)**
  - **(m_i)**
  - **($m - m_f - m_i$)**

Linde Dual – Pressure System



Linde Dual – Pressure System



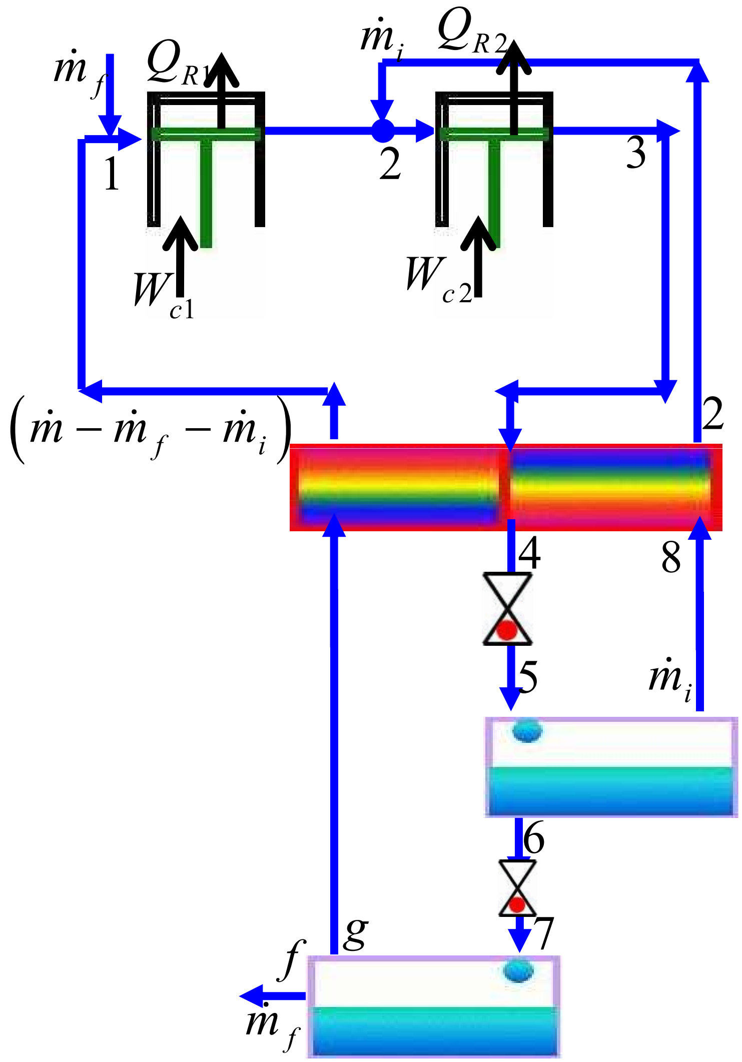
- Consider a control volume for this system as shown in the figure.

IN	OUT
m @ 3	m_i @ 2
	$m - m_f - m_i$ @ 1
	m_f @ f

- Applying the 1st Law, we have

$$\dot{m}h_3 = \dot{m}_i h_2 + \left(\dot{m}-\dot{m}_f-\dot{m}_i\right)h_1 + \dot{m}_f h_f$$

Linde Dual – Pressure System



- Rearranging the terms, we have

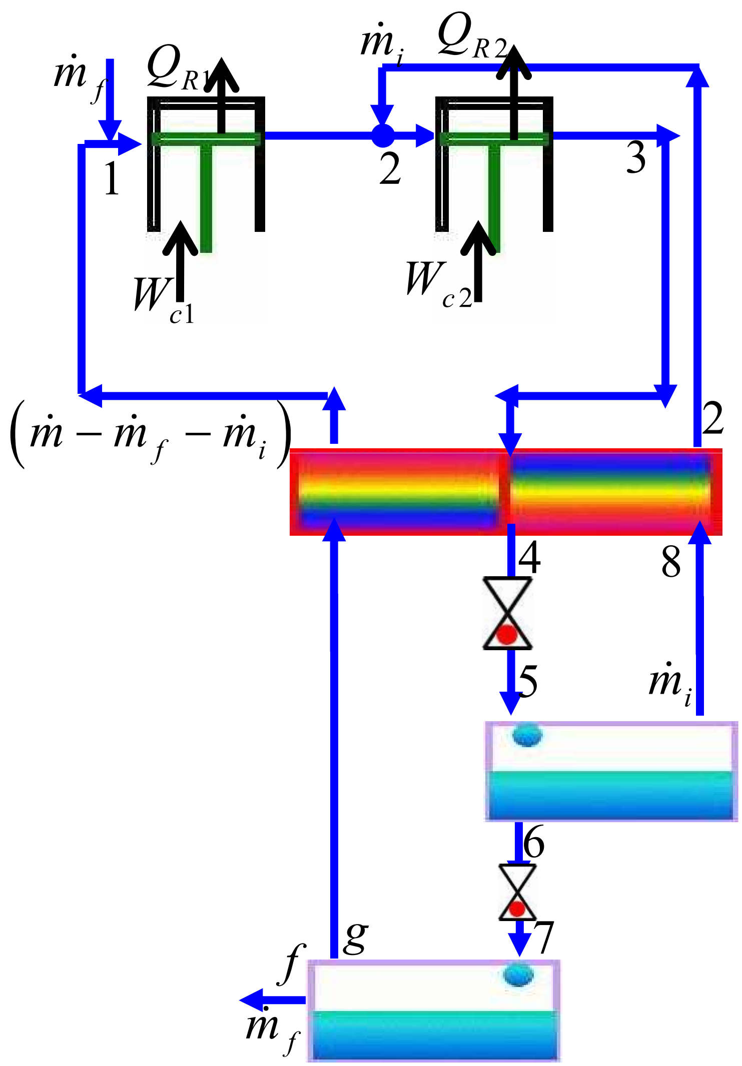
$$\frac{\dot{m}_f}{\dot{m}}=\left(\frac{h_1-h_3}{h_1-h_f}\right)-\frac{\dot{m}_i}{\dot{m}}\left(\frac{h_1-h_2}{h_1-h_f}\right)$$

- Denoting the intermediate mass ratio $\frac{\dot{m}_i}{\dot{m}}=i$

- We have the liquid yield as,

$$y=\frac{h_1-h_3}{h_1-h_f}-i\left(\frac{h_1-h_2}{h_1-h_f}\right)$$

Linde Dual – Pressure System

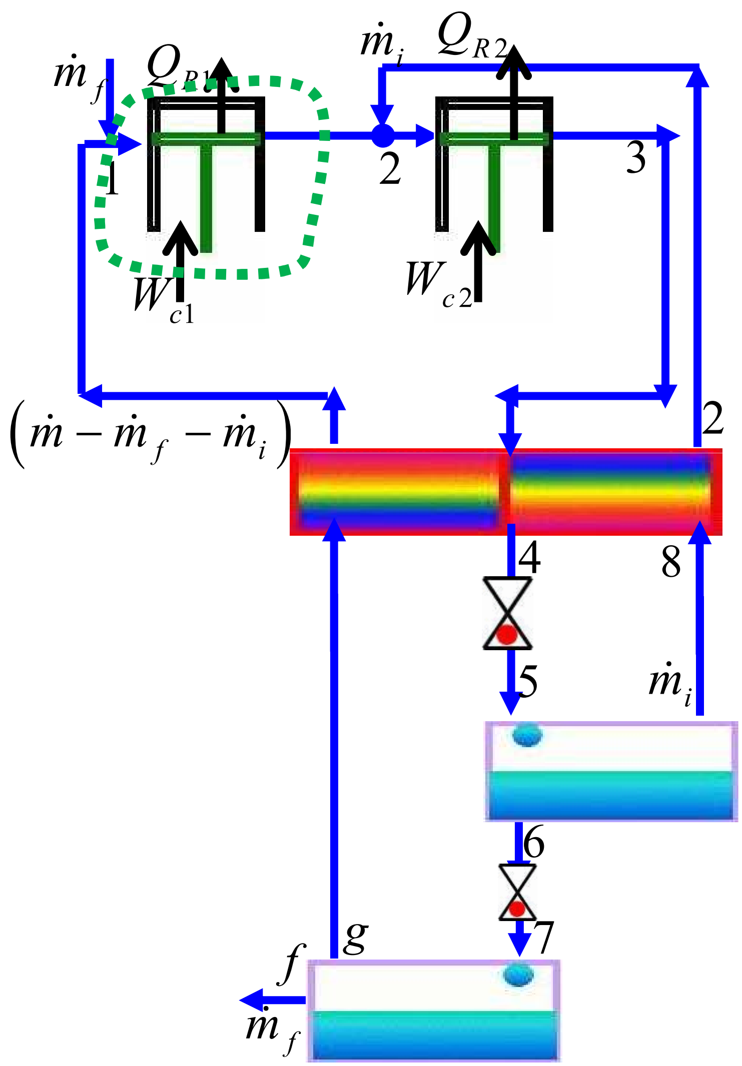


$$y = \frac{h_1 - h_3}{h_1 - h_f} - i\left(\frac{h_1 - h_2}{h_1 - h_f}\right)$$

- The first term is the yield for a simple L – H system considering that the entire mass of the gas is compressed from **1 → 3**.

- The second term is the reduction in the liquid yield occurring due to the modification.

Linde Dual – Pressure System



- For the work requirement, consider a control volume for the compressor – **1** as shown in the figure.

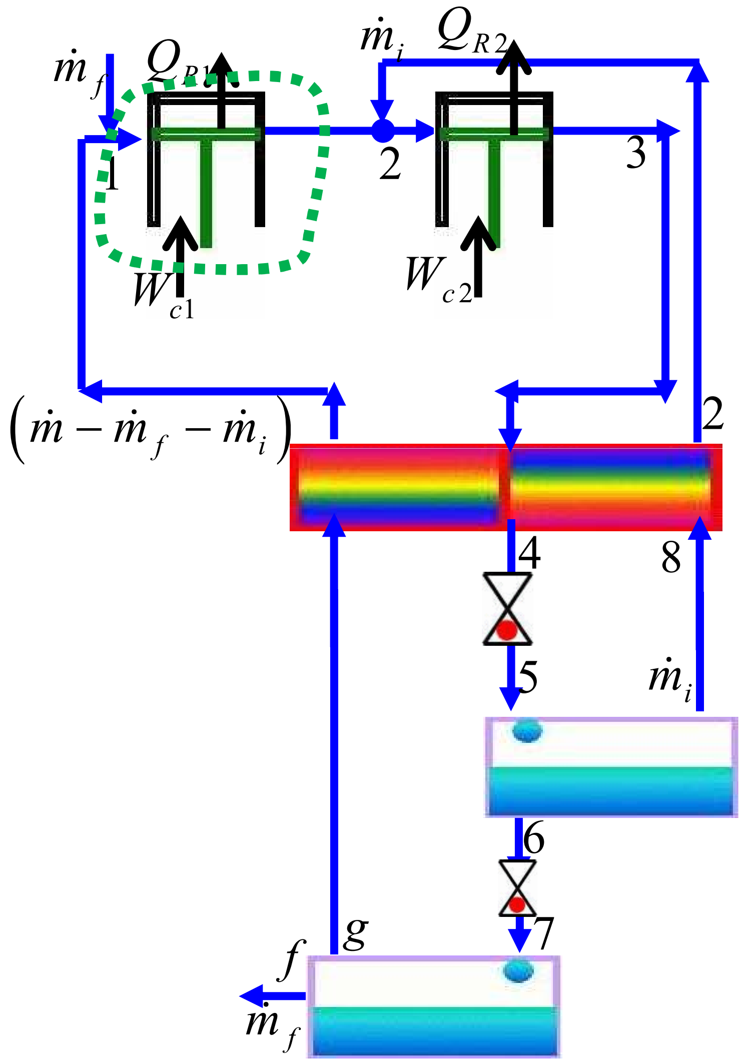
IN	OUT
m – m_i @ 1	m – m_i @ 2
$-W_{c1}$	$-Q_{R1}$

- Using 1st Law for the above table, we get

$$E_{in} = E_{out}$$

$$\left(\dot{m}-\dot{m}_i\right)h_1 - W_{c1} = \left(\dot{m}-\dot{m}_i\right)h_2 - Q_{R1}$$

Linde Dual – Pressure System



- Rearranging the terms, we have

$$Q_{R1}-W_{c1}=\left(\dot{m}-\dot{m}_i\right)\left(h_2-h_1\right)$$

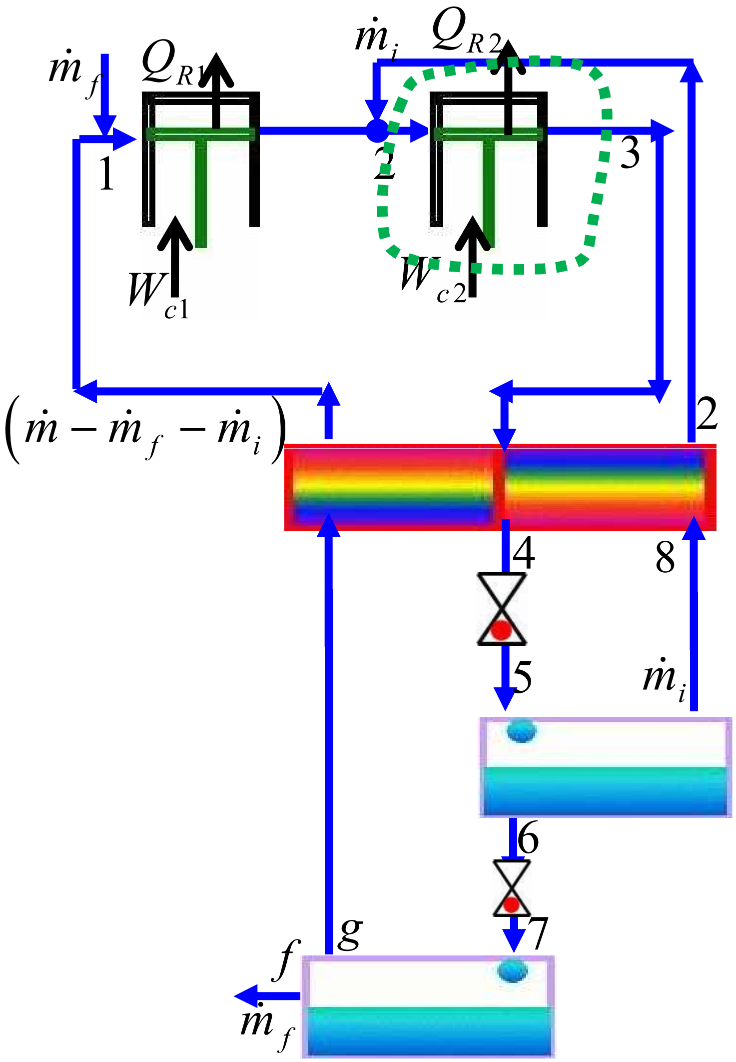
- By 2nd Law, the Q_{R1} is given by,

$$Q_{R1}=\left(\dot{m}-\dot{m}_i\right)T_1\left(s_2-s_1\right)$$

- Combining the above equations, we have

$$-W_{c1}=\left(\dot{m}-\dot{m}_i\right)\left(T_1\left(s_1-s_2\right)-\left(h_1-h_2\right)\right)$$

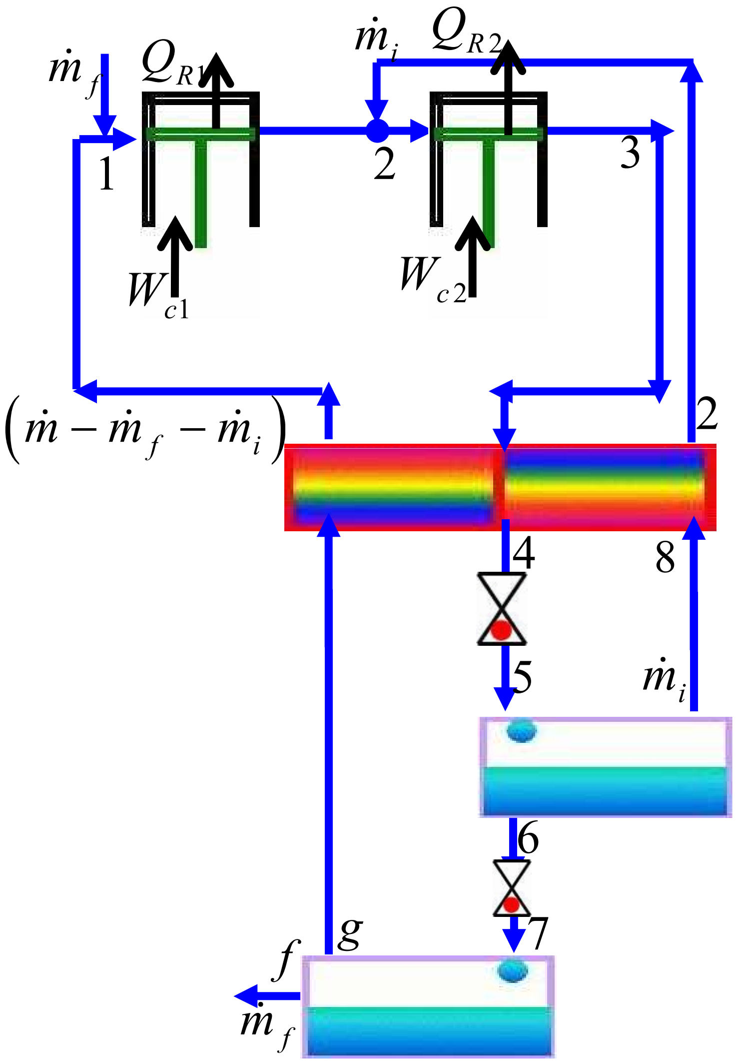
Linde Dual – Pressure System



- The mass flow rate across the compressor – **2** is **(m)**.
- Following the similar procedure for the work requirement for the compressor – **2**, we have,

$$-W_{c2} = \dot{m}\left(T_1\left(s_2 - s_3\right) - \left(h_2 - h_3\right)\right)$$

Linde Dual – Pressure System



- The total work requirement is given by

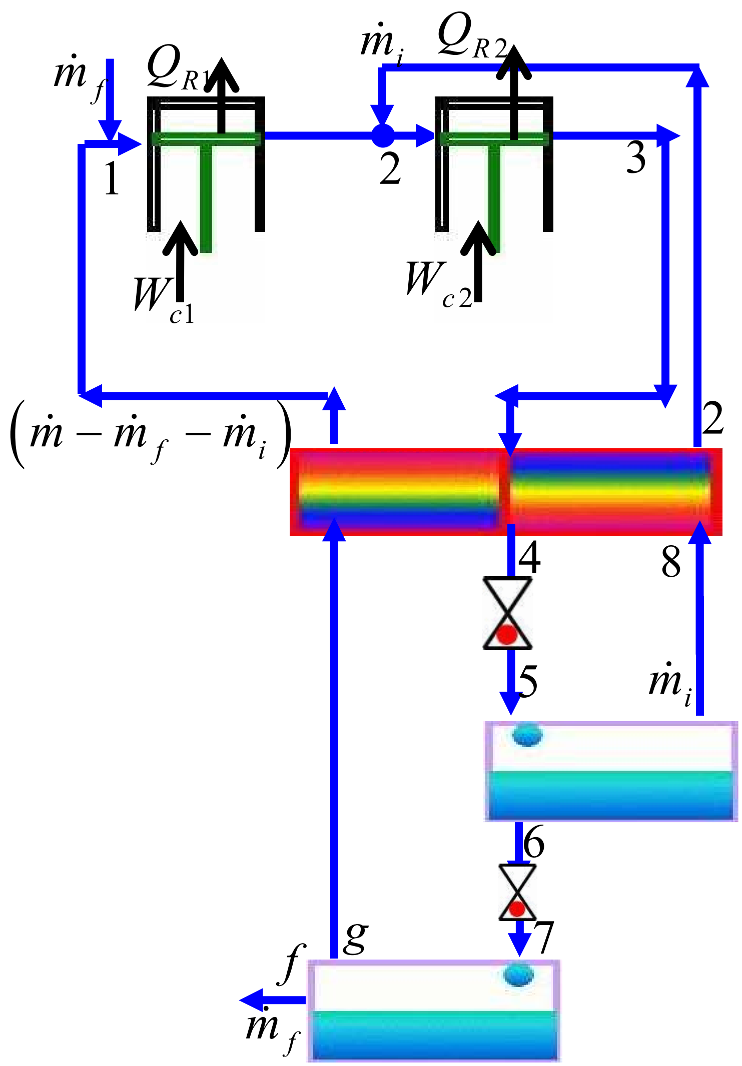
$$W_c = W_{c1} + W_{c2}$$

- Denoting the ratio $\dfrac{\dot{m}_i}{\dot{m}} = i$

- We have, the work/unit mass of gas compressed as given by

$$-\frac{W_c}{\dot{m}} = T_1(s_1 - s_3) - (h_1 - h_3) - i\left(T_1(s_1 - s_2) - (h_1 - h_2)\right)$$

Linde Dual – Pressure System



$$-\frac{W_c}{\dot{m}} = T_1(s_1 - s_3) - (h_1 - h_3)$$
$$-i\left(T_1(s_1 - s_2) - (h_1 - h_2)\right)$$

- The first term is the work requirement for simple system considering that the entire mass of the gas is compressed from **1 → 3**.
- The second term is the reduction in the work requirement occurring due to the modification.

Tutorial

- Determine **W/m_f** & **FOM** for a Linde Dual – Pressure System with Argon as working fluid for the following intermediate pressures. The system operates between 1.013 bar (1 atm) and 121.5 bar (120 atm). The intermediate mass ratio **i** is **0.6**.

Ar	Int. Pr. 2
I	4.05 bar
II	20.3 bar
III	75.9 bar
IV	101.3 bar

- Repeat the above problem for **i** = 0.7. Plot the data graphically and comment on the nature of **y**, **W/m_f**, **FOM** versus **i**.

Tutorial

Given
Cycle : Linde Dual – Pressure System
Working Pressure : 1 atm → P_i → 120 atm
Working Fluid : Argon
Temperature : 300 K
Intermediate mass ratio : i = 0.6 & 0.7

For above System, Calculate	
1	Work/unit mass of gas liquefied and FOM

Ar	Int. Pr. 2
I	4.05 bar
II	20.3 bar
III	75.9 bar
IV	101.3 bar

Methodology

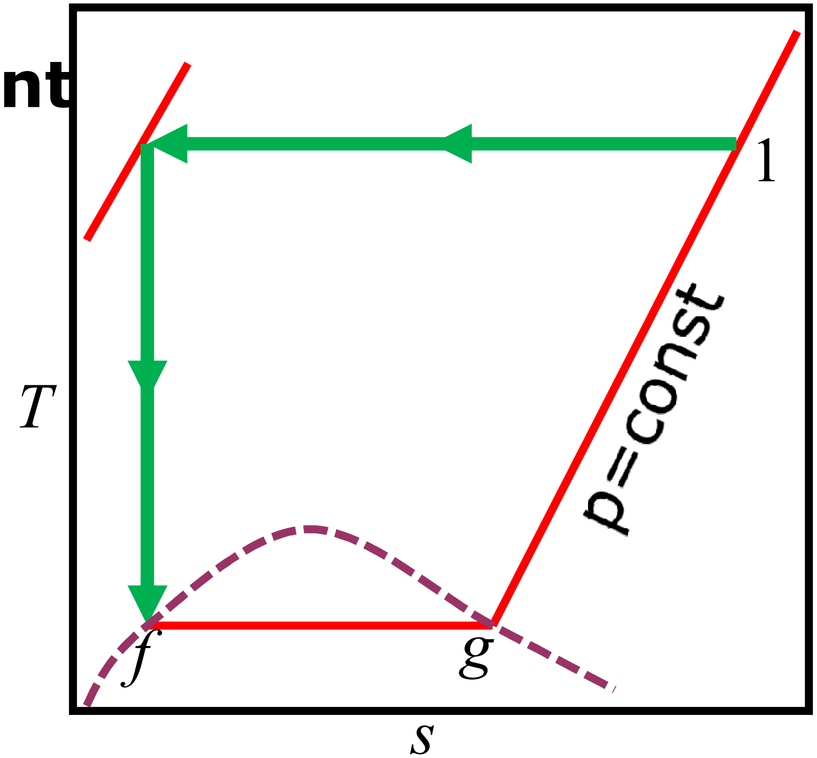
- The two mass ratio **(i)** conditions under study are 0.6 and 0.7.
- In this tutorial, the liquid yield and work/unit mass of gas liquefied are calculated only for **i = 0.6** and **4.05 bar** as intermediate pressure condition.
- All other calculations pertaining to **i = 0.6 & 0.7** and for all other intermediate pressure conditions are left as an exercise to students.

Tutorial

- **Ideal Work Requirement**

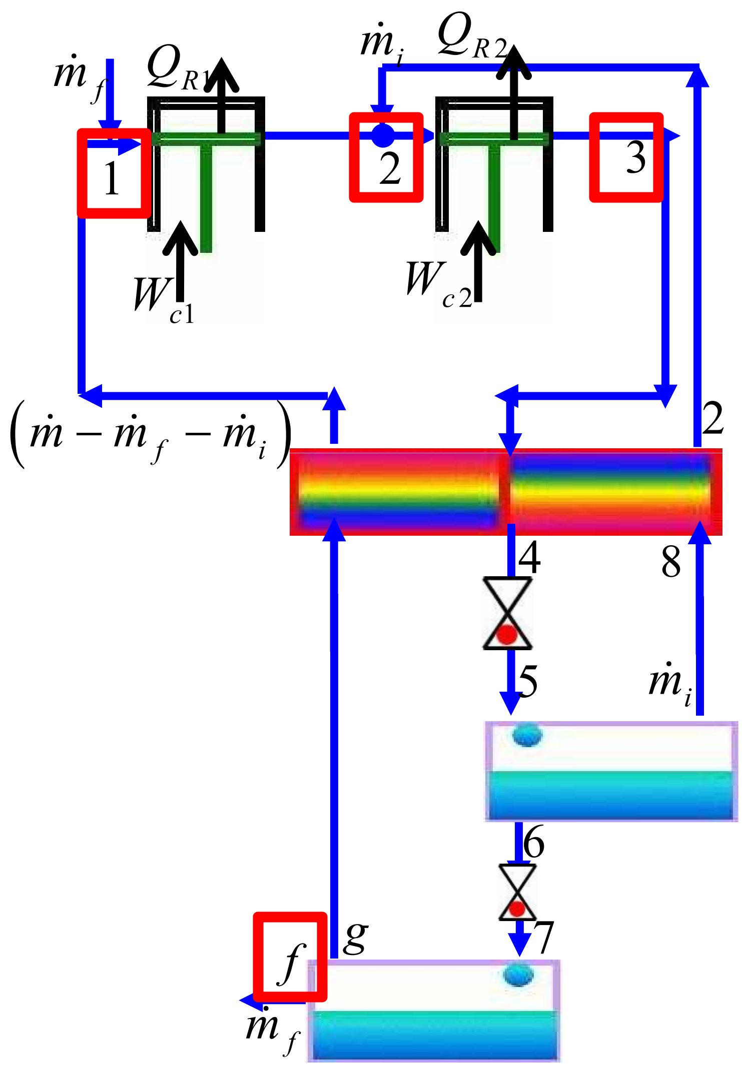
$$-\frac{\dot{W}_i}{\dot{m}} = T_1\left(s_1 - s_f\right) - \left(h_1 - h_f\right)$$

	1	f
p (bar)	1.013	1.013
T (K)	300	87.3
h (J/g)	349	75
s (J/gK)	3.85	1.4



$$-\frac{W_c}{\dot{m}} = 300\left(3.85 - 1.4\right) - \left(349 - 75\right) \quad = 461\ J/g$$

Tutorial



- The enthalpies and entropies are as given below.

	1	2
p (bar)	1.013	4.05
T (K)	300	300
h (J/g)	349	348
s (J/gK)	3.85	3.6

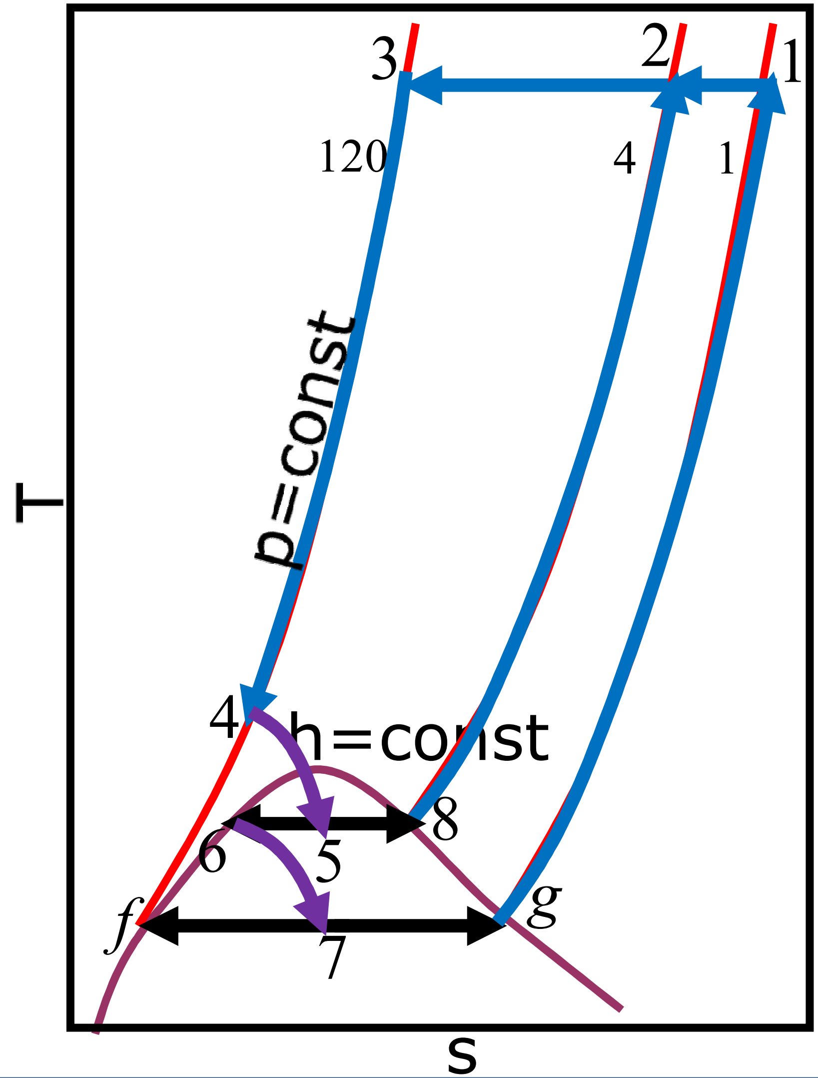
	3	f
p (bar)	121.5	1.013
T (K)	300	87.3
h (J/g)	326	75
s (J/gK)	2.84	1.4

Tutorial

Ar	i	Int. Pr. 2
I	0.6	4.05 bar

- The T – s diagram for a Linde Dual – Pressure system is as shown.

- The compression process is from **1 atm → 4 atm → 120 atm**, As shown in the figure.



Tutorial

- **Liquid yield**

$$y = \frac{\dot{m}_f}{\dot{m}} = \frac{h_1 - h_3}{h_1 - h_f} - i\left(\frac{h_1 - h_2}{h_1 - h_f}\right)$$

Ar	i	Int. Pr. 2
I	0.6	4.05 bar

	1	2	3	f
p (bar)	1.013	4.05	121.5	1.013
T (K)	300	300	300	87.3
h (J/g)	349	348	326	75
s (J/gK)	3.85	3.6	2.84	1.4

$$y = \frac{(349 - 326)}{(349 - 75)} - 0.6\frac{(349 - 348)}{(349 - 75)} = \frac{(23)}{(274)} - 0.6\frac{(1)}{(274)} = 0.0817$$

Tutorial

- **Work/unit mass of Ar compressed**

$$i = 0.6$$

$$-\frac{W_c}{\dot{m}} = T_1(s_1 - s_3) - (h_1 - h_3) - i\left(T_1(s_1 - s_2) - (h_1 - h_2)\right)$$

	1	2	3	f
p (bar)	1.013	4.05	121.5	1.013
T (K)	300	300	300	87.3
h (J/g)	349	348	326	75
s (J/gK)	3.85	3.6	2.84	1.4

$$-\frac{W_c}{\dot{m}} = \begin{matrix} 300(3.85 - 2.84) - (349 - 326) \\ -0.6\left(300(3.85 - 3.6) - (349 - 348)\right) \end{matrix} = 235.6\, J/g$$

Tutorial

- **Work/unit mass of Ar liquefied**

$$-\frac{W_c}{\dot{m}} = 235.6$$

$$y = 0.0817$$

$$-\frac{W_c}{\dot{m}_f} = -\frac{W_c}{y\dot{m}} = \frac{235.6}{0.0817} = 2883.7\ J/g$$

- **FOM**

$$-\frac{W_i}{\dot{m}_f} = 461$$

$$FOM = \frac{W_i / \dot{m}_f}{W_c / \dot{m}_f} = \frac{461}{2883.7} = 0.1598$$

Tutorial

- Tabulating the results for i = 0.6, we have the following comparison for the various values of
  - Intermediate pressure.

	Int. Pressure	y	$-\frac{W}{\dot{m}}$	$-\frac{W}{\dot{m}_f}$	FOM
I	4.05 bar	0.0817	235.6	2883.7	0.1598
II	20.3 bar	0.0752	172.6	2295.2	0.2008
III	75.9 bar	0.0512	118.0	2304.6	0.2000
IV	101.3 bar	0.0424	111.4	2627.4	0.1754

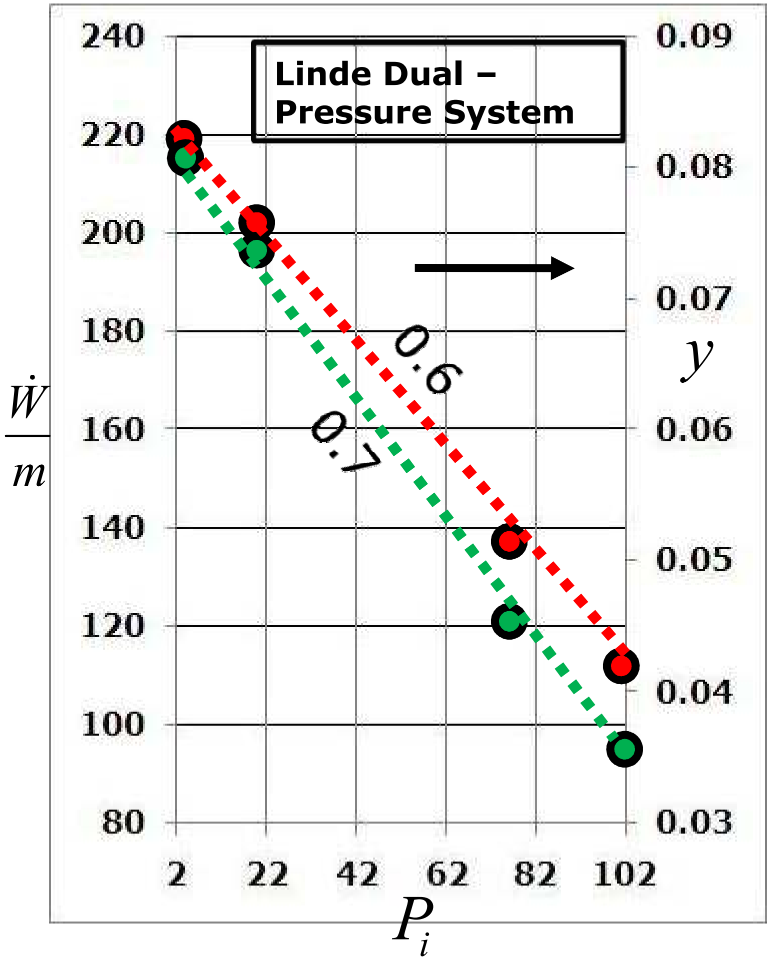
Tutorial

- Similarly, calculating the results for i = 0.7, we have the following comparison for the various values of
  - Intermediate pressure.

	Int. Pressure	y	$-\frac{W}{\dot{m}}$	$-\frac{W}{\dot{m}_f}$	FOM
I	4.05 bar	0.0814	228.2	2803.4	0.1644
II	20.3 bar	0.0738	154.7	2096.2	0.2199
III	75.9 bar	0.0457	91.0	1991.2	0.2315
IV	101.3 bar	0.0355	83.3	2346.5	0.1964

Tutorial

- **Liquid yield v/s. i**



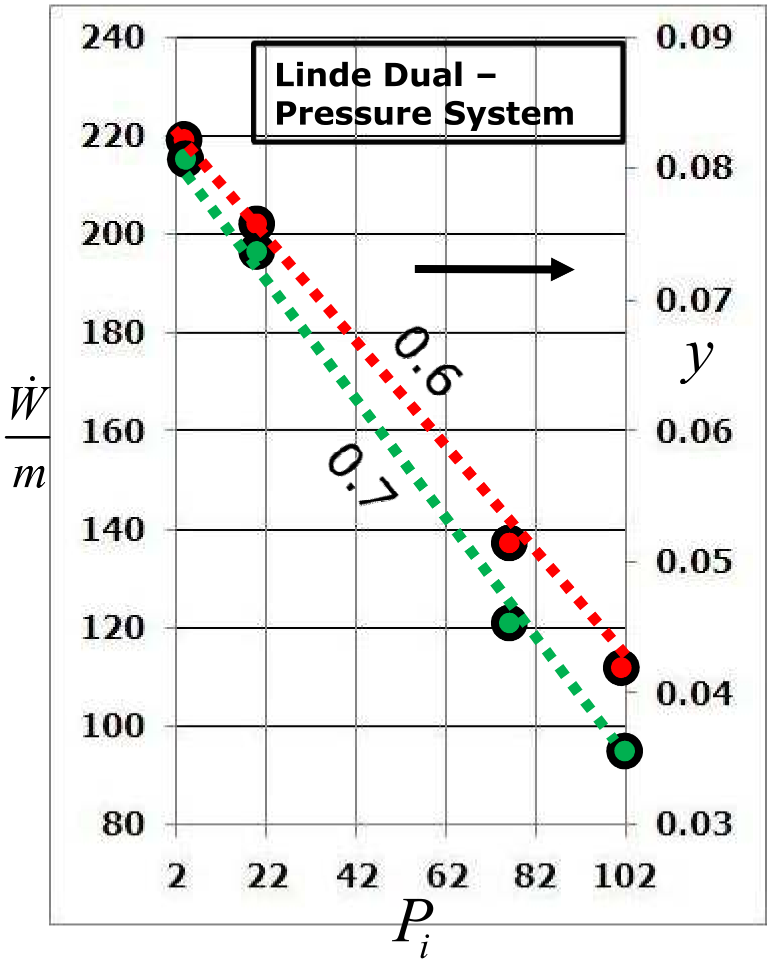
- The Plot for **y** versus **i** for different pressures is as shown.

i=0.6	P_i	y
I	4.05 bar	0.0817
II	20.3 bar	0.0752
III	75.9 bar	0.0512
IV	101.3 bar	0.0424

i=0.7	P_i	y
I	4.05 bar	0.0814
II	20.3 bar	0.0738
III	75.9 bar	0.0457
IV	101.3 bar	0.0355

Tutorial

- **Liquid yield v/s. i**

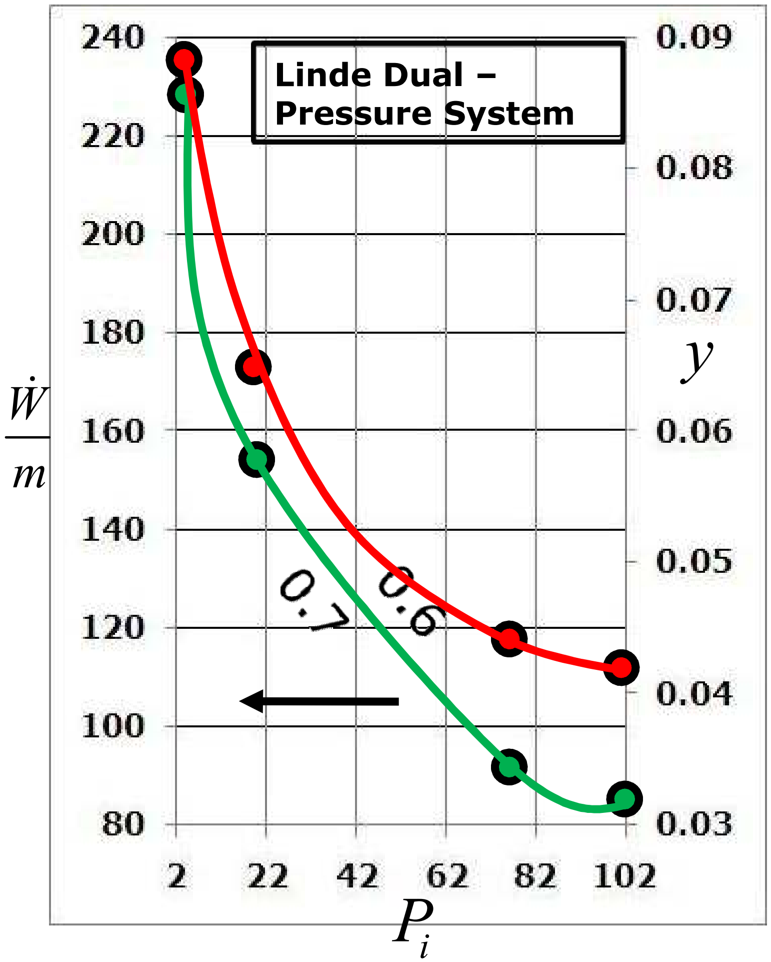


- For a given value of mass ratio **i**, the yield (dotted line) of the system decreases with the increase in the intermediate pressure.

- As the mass ratio **i** increases, the yield of the system decreases because, the mass of gas actually expanded in J – T device decreases.

Tutorial

- **W/m v/s. i**



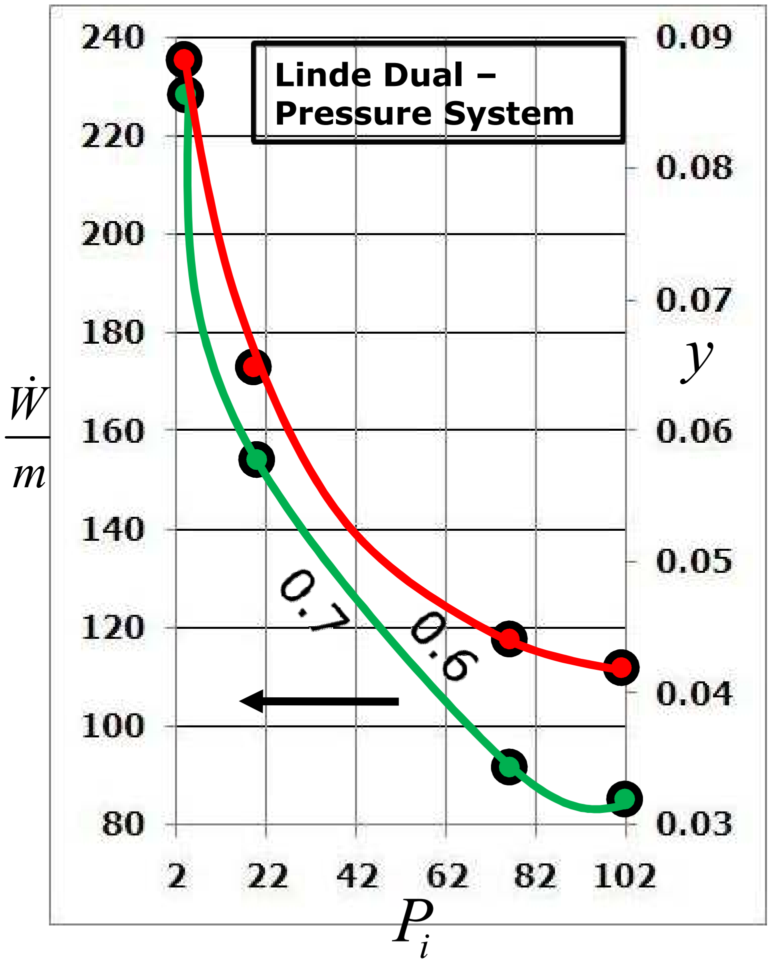
- The Plot for **W/m** versus **i** for different pressures is as shown.

i=0.6	P_i	$-\dot{W}/\dot{m}$
I	4.05 bar	235.6
II	20.3 bar	172.6
III	75.9 bar	118.0
IV	101.3 bar	111.4

i=0.7	P_i	$-\dot{W}/\dot{m}$
I	4.05 bar	228.2
II	20.3 bar	154.7
III	75.9 bar	91.0
IV	101.3 bar	83.3

Tutorial

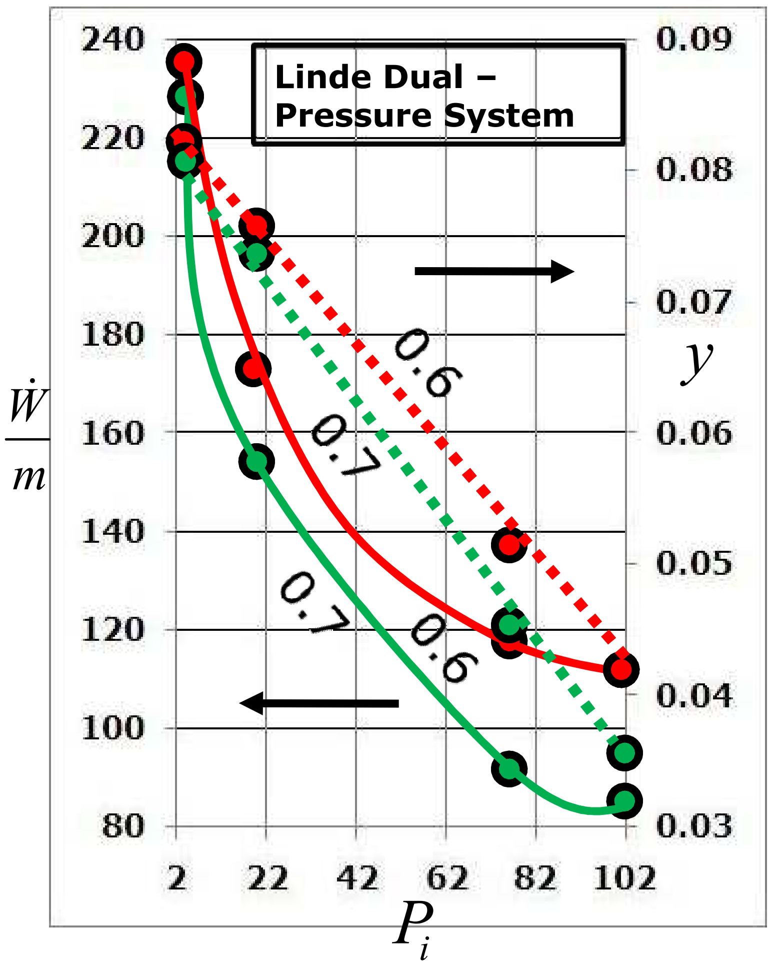
- **W/m v/s. i**



- For a given value of mass ratio **i**, the **W/m** (solid line) of the system decreases with the increase in the intermediate pressure.

- As the mass ratio **i** increases, the **W/m** decreases because, the more of the mass flow rate is bypassed from compressor – **1**.

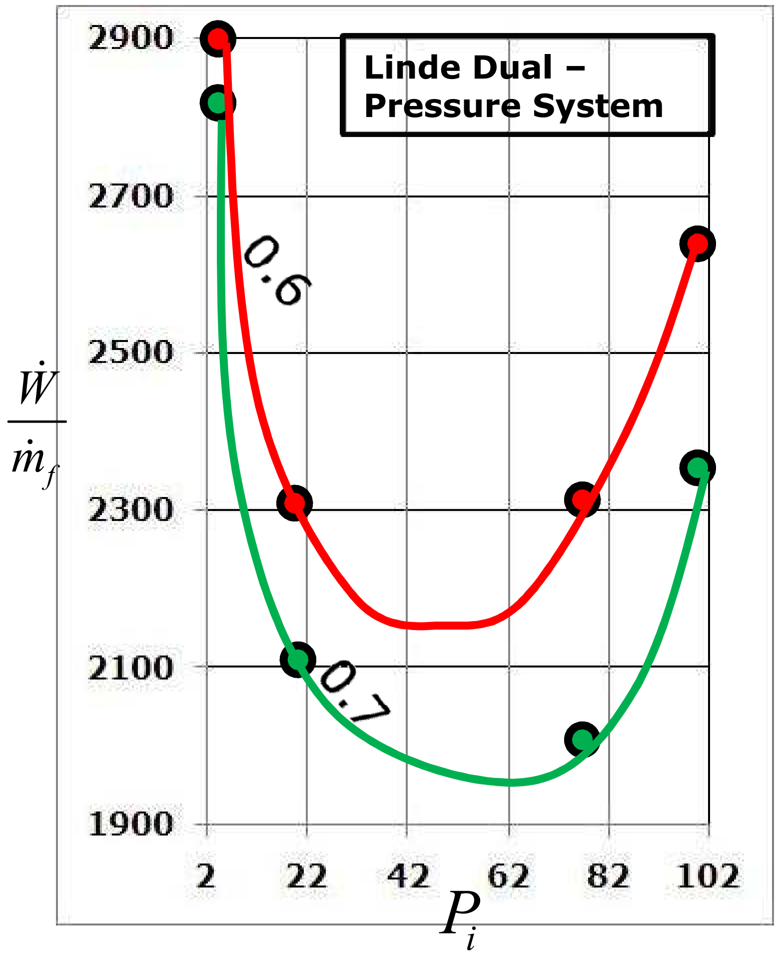
Tutorial



- It is important to note that, initially the slope of **W/m** (solid lines) is much steeper than that of **y** (dotted lines).
- Later on, as the intermediate pressure increases, the slope of **y** (dotted lines) is steeper while the slope **W/m** (solid lines) decreases.

Tutorial

- **W/m_f v/s. i**



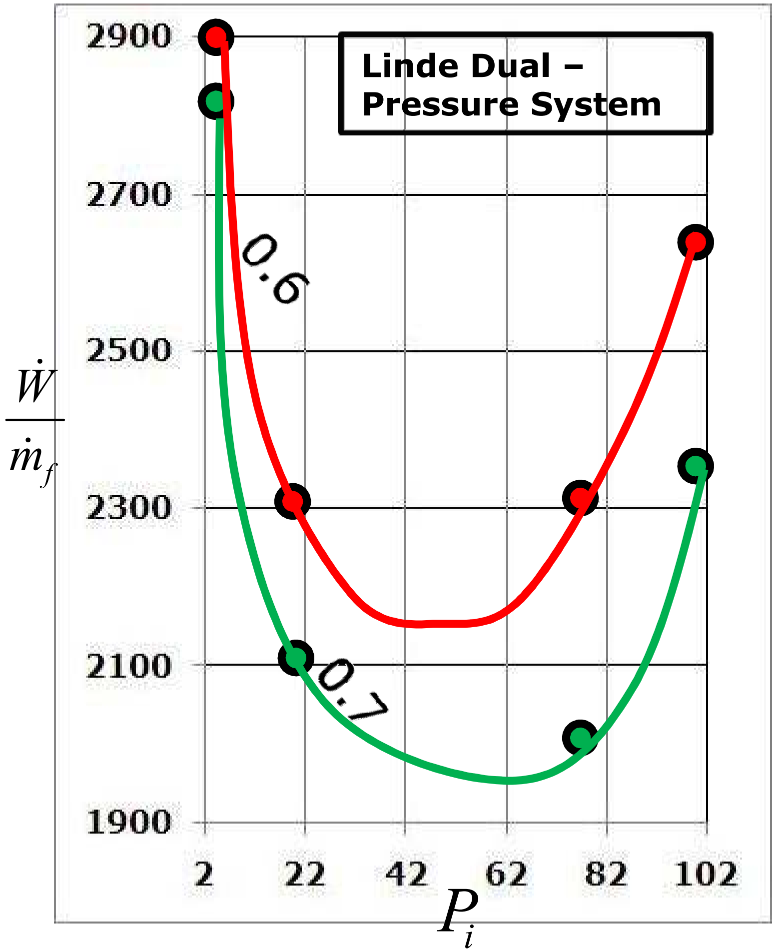
- The Plot for **W/m_f** versus **i** for different pressures is as shown.

i=0.6	P_i	$-W/\dot{m}_f$
I	4.05 bar	2883.7
II	20.3 bar	2295.2
III	75.9 bar	2304.6
IV	101.3 bar	2627.4

i=0.7	P_i	$-W/\dot{m}_f$
I	4.05 bar	2803.4
II	20.3 bar	2096.2
III	75.9 bar	1991.2
IV	101.3 bar	2346.5

Tutorial

- **W/m_f v/s. i**



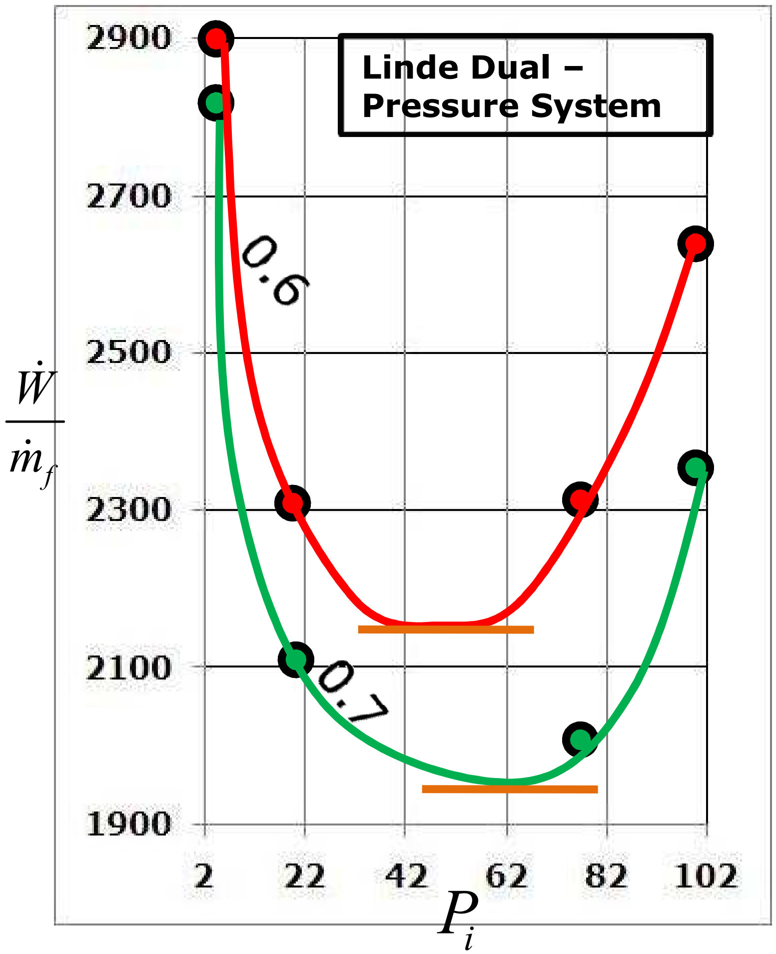
- Mathematically,

$$-\frac{W}{\dot{m}_f} = \frac{-\frac{W}{\dot{m}}}{y}$$

- **W/m_f** being a ratio of **W/m** and liquid **y**, the relative decrease in the numerator and denominator determines the slope of the curve of **W/m_f**.

Tutorial

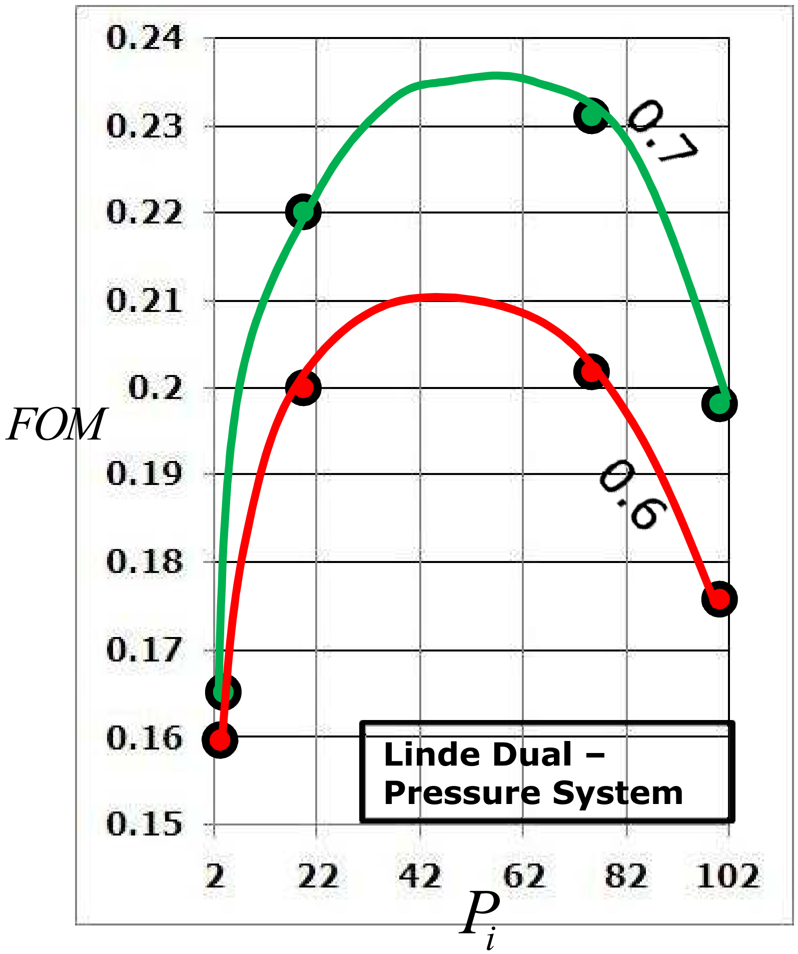
- **W/m_f v/s. i**



- For a mass ratio **i**, the **W/m_f** decreases with the increase in the intermediate pressure.
- This work falls to a minima and then increases with the increase in the intermediate pressure.
- The working point is a compromised value between **y** and **$(W/m_f)_{min.}$**

Tutorial

- **FOM v/s. i**



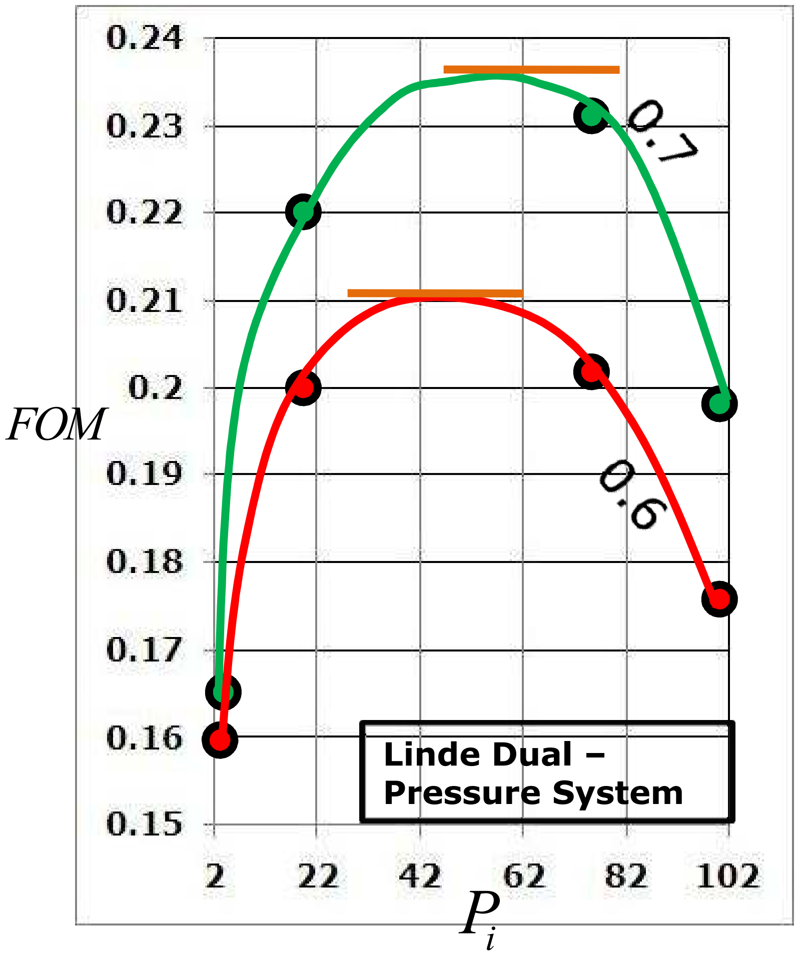
- The Plot for **FOM** versus **i** for different pressures is as shown.

i=0.6	P_i	FOM
I	4.05 bar	0.1598
II	20.3 bar	0.2008
III	75.9 bar	0.2000
IV	101.3 bar	0.1754

i=0.7	P_i	FOM
I	4.05 bar	0.1644
II	20.3 bar	0.2199
III	75.9 bar	0.2315
IV	101.3 bar	0.1964

Tutorial

- **FOM v/s. i**

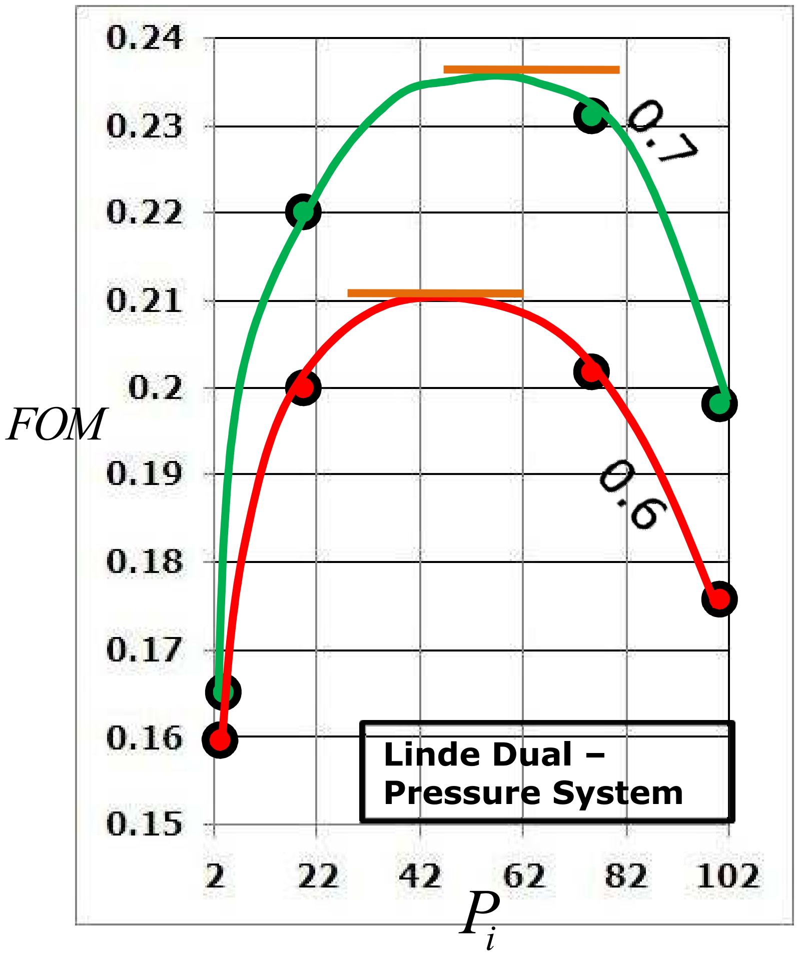


- For a mass ratio **i**, the **FOM** increases with the increase in the intermediate pressure.

- With the further increase in the intermediate pressure, the FOM reaches a maxima value and thereby it decreases.

Tutorial

- **FOM v/s. i**



- It is important to note that the **FOM** reaches a maxima value at the same intermediate pressure at which the $\mathbf{W/m_f}$ reaches a minima, for a given value of mass ratio **i**.

Assignment

- Determine **y**, **W/m**, **W/m_f** and **FOM** for a Linde Dual – Pressure System with Argon as working fluid. The system operates between 1.013 bar (1 atm) and 202.6 bar (200 atm). The intermediate mass ratio **i=0.6**.

- Ans : 0.09115, 155 J/g, 1700.49 J/g, 0.2711.

Summary

- Linde Dual – pressure system is a modification of a Simple Linde – Hampson system in order to reduce the work requirement.

- In this system, the entire mass flow rate of the gas is not compressed to the required high pressure, as was the case in the previous systems.

- In this system, the work requirement decreases when the compression of fluid is done in two stages and for different mass flow rates.

Summary

- The yield of the system is given by the following equation.

$$y = \frac{h_1 - h_3}{h_1 - h_f} - i\left(\frac{h_1 - h_2}{h_1 - h_f}\right)$$

- The work requirement is given by

$$-\frac{W_c}{\dot{m}} = T_1\left(s_1 - s_3\right) - \left(h_1 - h_3\right) - i\left(T_1\left(s_1 - s_2\right) - \left(h_1 - h_2\right)\right)$$

- In the above equations, the first term corresponds to the L – H system and the second term is the reduction occurring due to the modification.

Summary

- For a given value of mass ratio **i**, the **y** and **W/m** of the system decreases with the increase in the intermediate pressure.

- For a mass ratio **i**, the $\mathbf{W/m_f}$ passes through a minima as the intermediate pressure increases.

- On the other hand, for a mass ratio **i**, the FOM passes through a maxima with the increase in the intermediate pressure.

Summary

- The operating point of the system is a compromised value between the **y** and the $(\mathbf{W/m_f})_{\mathbf{min}}$.

- It is important to note that the **FOM** reaches a maxima value at the same intermediate pressure at which the $\mathbf{W/m_f}$ reaches a minima, for a given value of mass ratio **i**.

Thank You!