CRYOGENIC ENGINEERING

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Lecture No - 15

Earlier Lecture

- In the earlier lectures, we have seen an Ideal Thermodynamic cycle, in which all the gas that is compressed is liquefied.
- In a Linde – Hampson system, a heat exchanger is used to conserve cold and only a part of the gas that is compressed is liquefied.
- In a Precooled Linde – Hampson system, an independent refrigerating system is used. The mass ratio **(r)** corresponding to the maximum yield is called as the limiting value.

Earlier Lecture

- The Linde Dual – Pressure system is a modification of the Simple Linde – Hampson system in order to reduce the work requirement.

- In this system, the work requirement/mass of gas liquefied decreases when the compression of fluid is done in two stages and for different mass flow rates.

Outline of the Lecture

Topic : Gas Liquefaction and Refrigeration Systems (contd)

- Claude System – In the year 1920, Claude developed an air liquefaction system and established l'Air Liquide.
    - Liquid yield
    - Work requirement
    - Parametric study

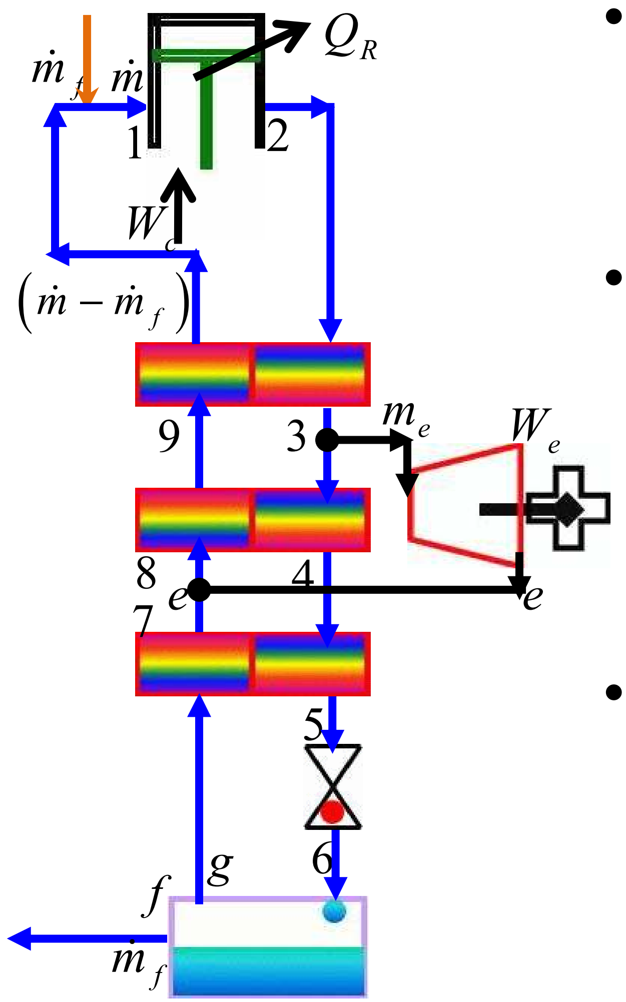
Introduction

- In order to achieve a better performance and to approach ideality, the expansion process should be a reversible process.

- In the earlier lecture, we have seen that a J – T expansion is an irreversible isenthalpic expansion and expansion using an expansion engine is an reversible isentropic process.

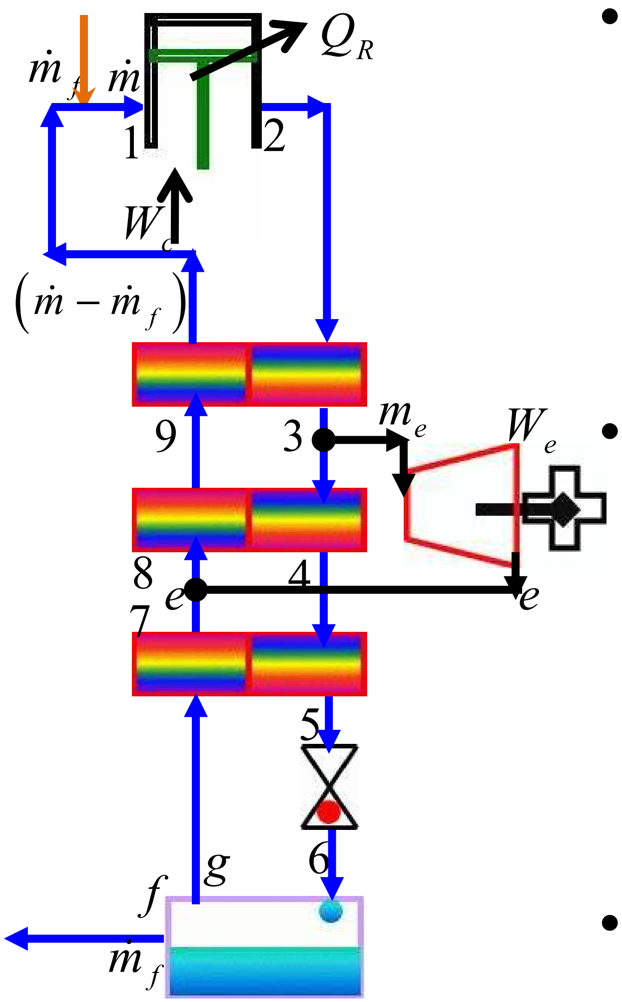
- For any gas, an isentropic expansion results in lower temperature irrespective of its inversion temperature ($\mathbf{T_{INV}}$).

Claude System



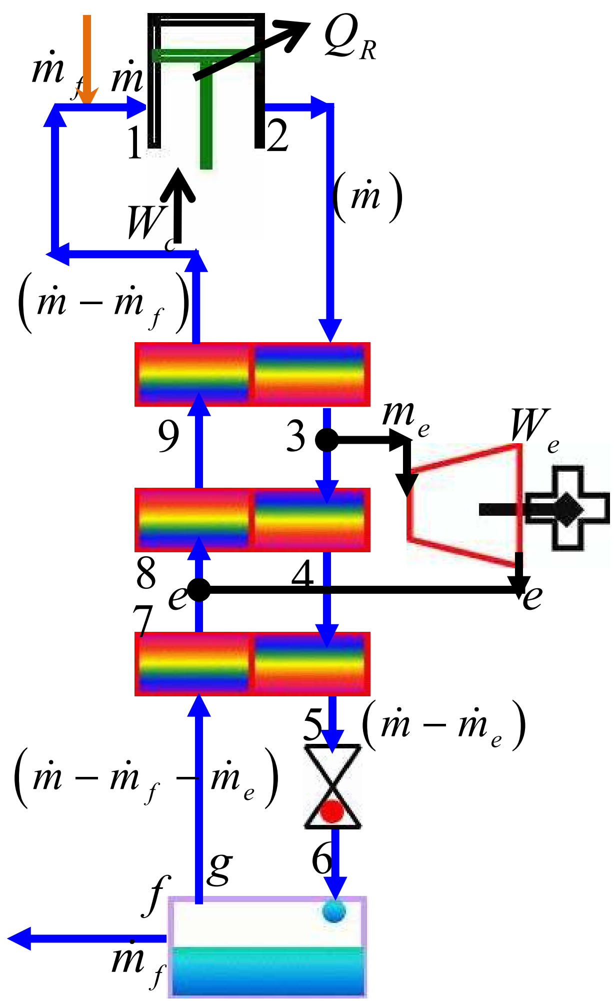
- The schematic of the Claude System is as shown.
- It consists of a compressor, three 2 – fluid heat exchangers, a J – T expansion device and a make up gas connection.
- The system also has an expansion engine operating across the second heat exchanger as shown in the figure.

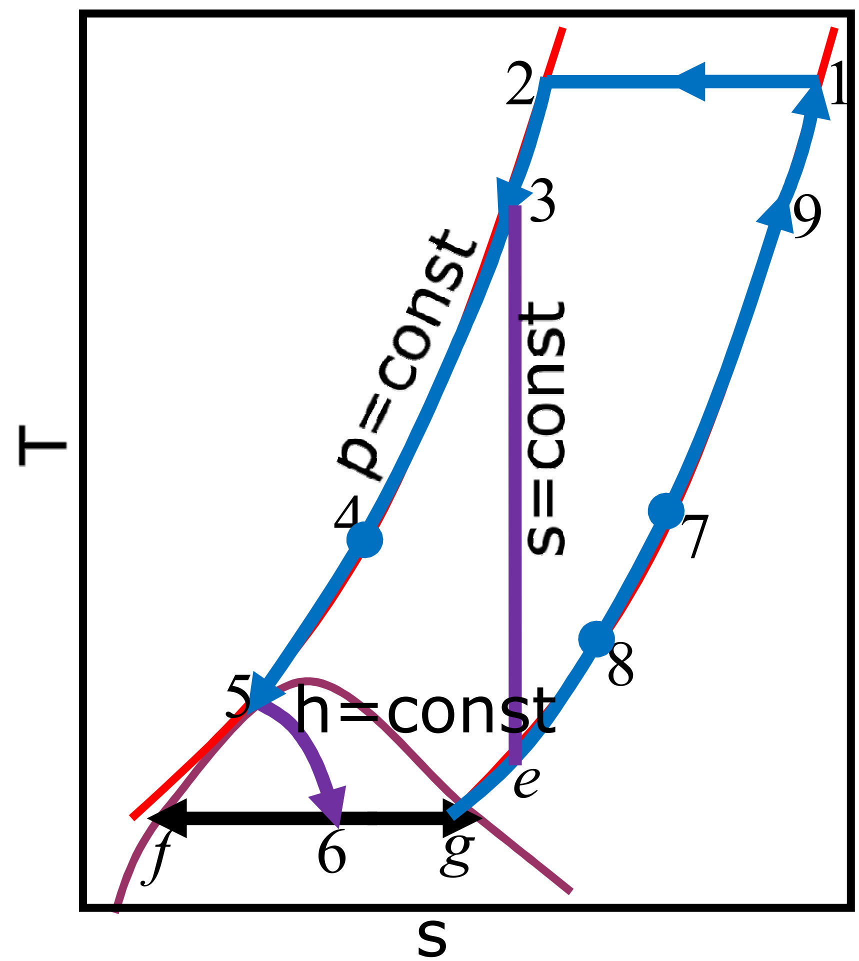
Claude System



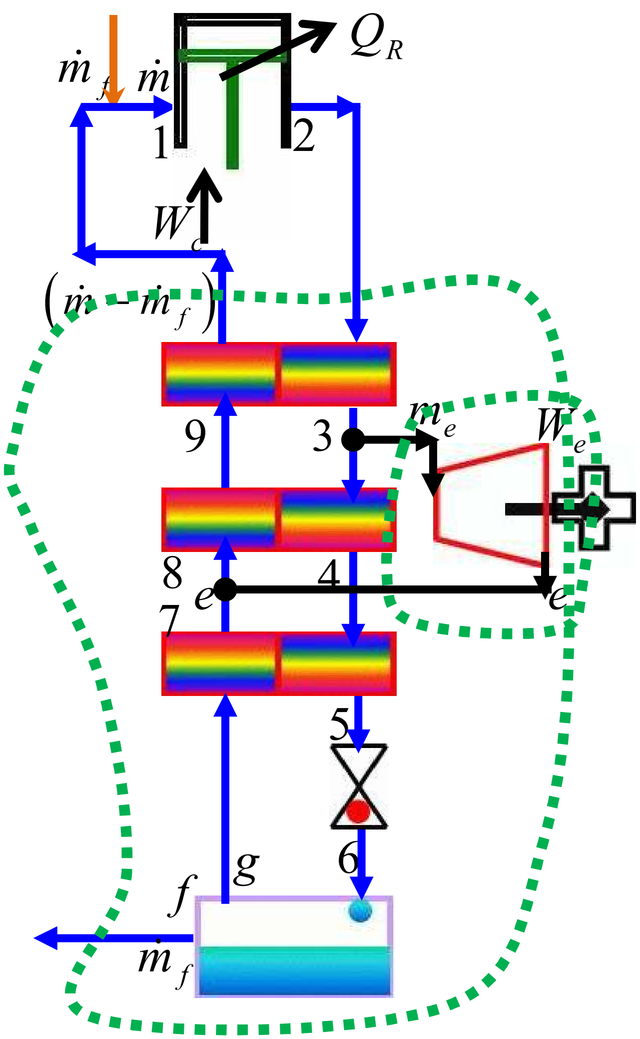
- In this system, the energy content in the gas is removed by allowing it to do some work in an expansion device.
- As shown in the figure, a part of the main stream of gas is expanded from **3 → e** and is reunited with the return stream.
- This process of expansion is an reversible isentropic expansion.

Claude System





Claude System



- Consider a control volume as shown in the figure. Applying 1st Law, we have

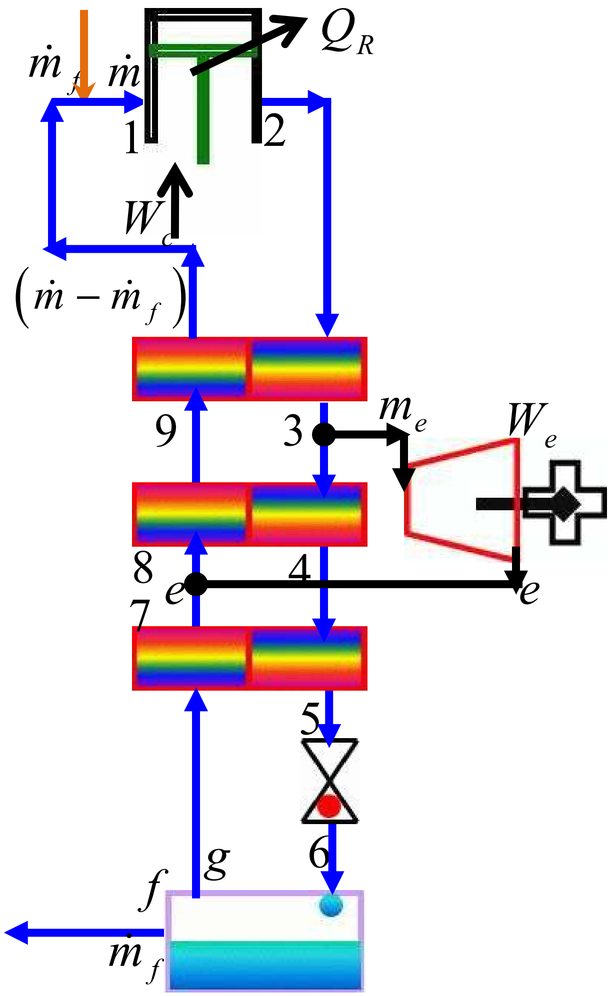
$$\dot{m}h_2 = W_e + \left(\dot{m} - \dot{m}_f\right)h_1 + \dot{m}_f h_f$$

- The Expander work output is given by $W_e = \dot{m}_e h_3 - \dot{m}_e h_e$

- Substituting the expression for $\mathbf{W_e}$, we have

$$\dot{m}h_2 = \left(\dot{m} - \dot{m}_f\right)h_1 + \dot{m}_f h_f + \dot{m}_e h_3 - \dot{m}_e h_e$$

Claude System



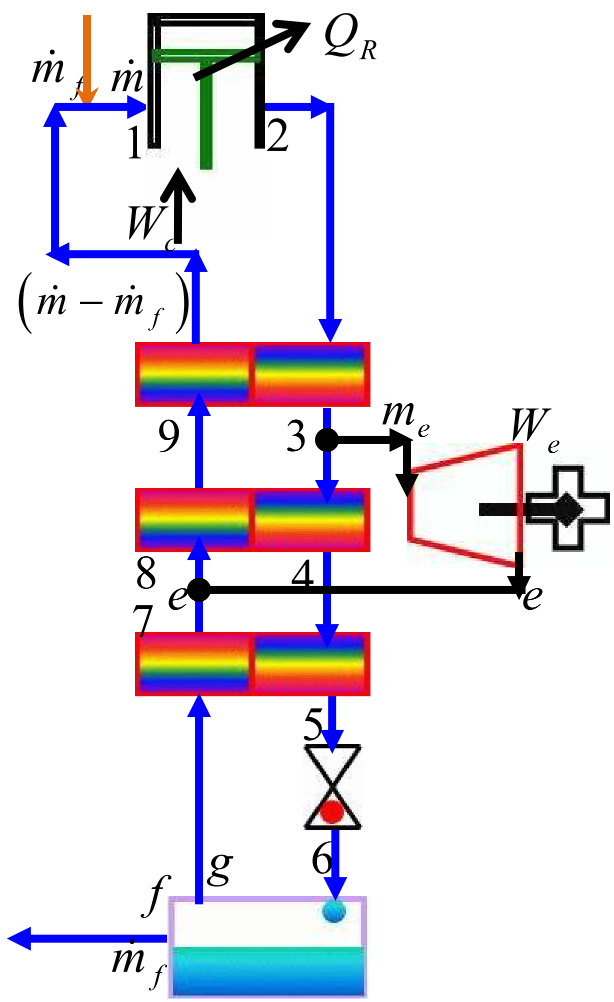
- Rearranging the terms, we have

$$y = \left(\frac{h_1 - h_2}{h_1 - h_f} \right) + x \left(\frac{h_3 - h_e}{h_1 - h_f} \right)$$

$$x = \frac{\dot{m}_e}{\dot{m}}$$

- Where, the expander mass flow ratio be denoted by **x**.

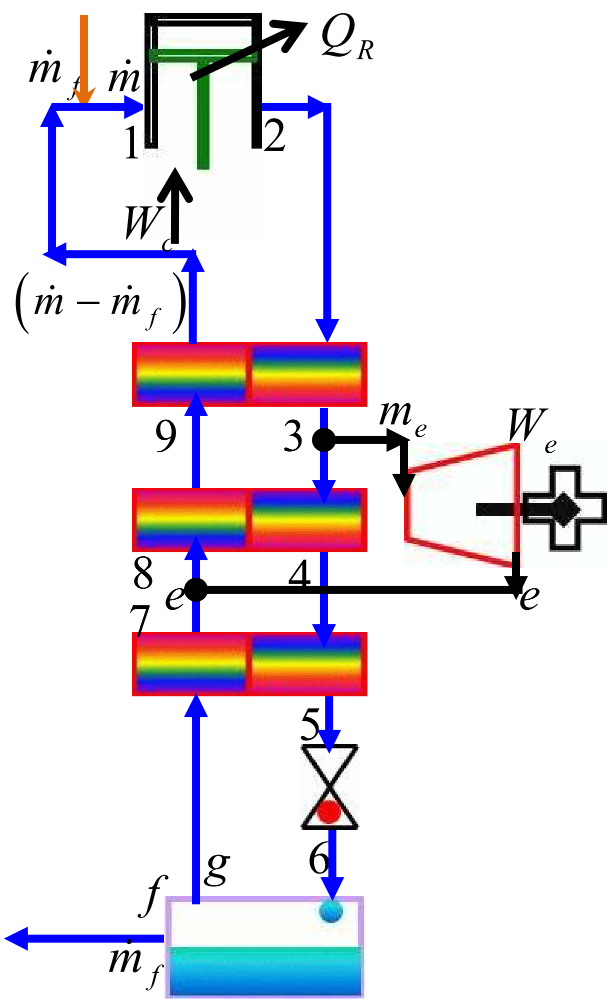
Claude System



$$y = \left(\frac{h_1 - h_2}{h_1 - h_f}\right) + x\left(\frac{h_3 - h_e}{h_1 - h_f}\right)$$

- The 1st term is the yield for a simple L – H system.
- The 2nd term is the change in the yield occurring due to the expansion engine in the cycle.
- For a given initial and final conditions of **p**, the yield **y** depends on $\mathbf{h_3(T_3)}$ and **x**.

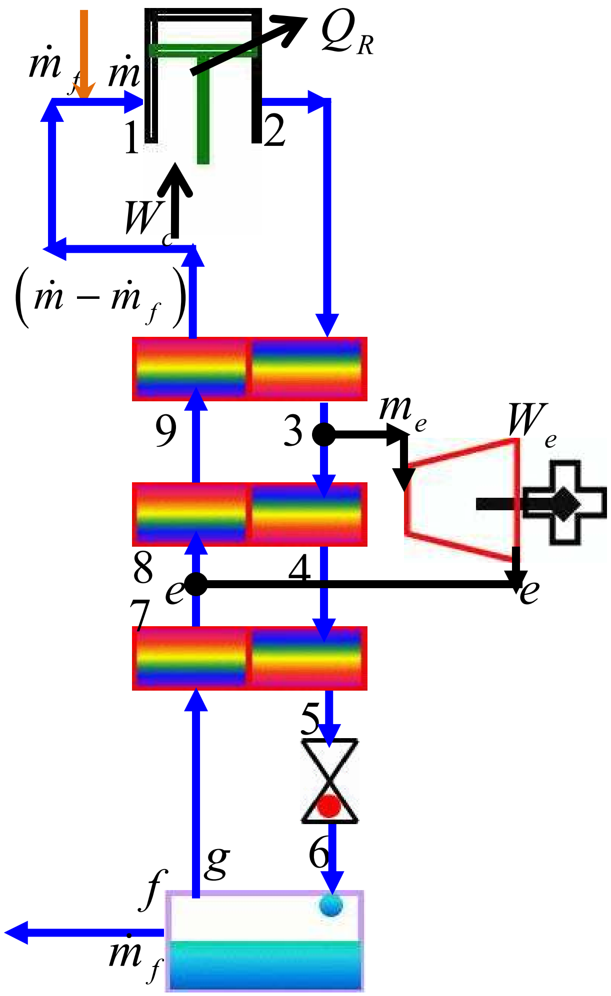
Claude System



$$y = \left(\frac{h_1 - h_2}{h_1 - h_f}\right) + x\left(\frac{h_3 - h_e}{h_1 - h_f}\right)$$

- However, if $\mathbf{T_3}$ is held constant, the yield **y** is a linear function of **x**.
- But for a case of **x=1**, the yield **y=0**, which is not governed by this equation.
- For **x=1**, the gas in the return stream (**m – $\mathbf{m_f}$ - $\mathbf{m_e}$**) is **0**.

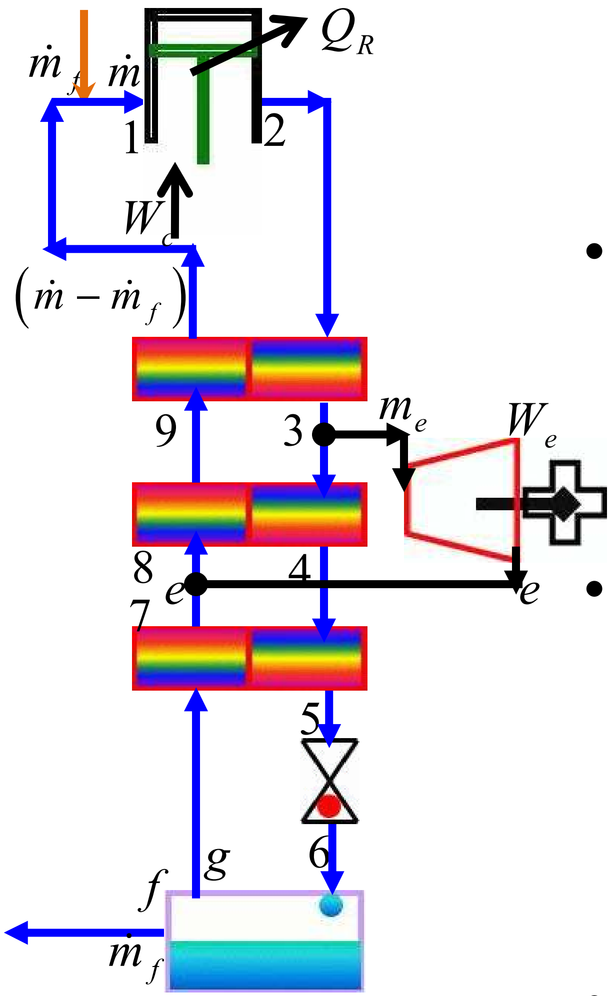
Claude System



$$y = \left(\frac{h_1 - h_2}{h_1 - h_f}\right) + x\left(\frac{h_3 - h_e}{h_1 - h_f}\right)$$

- It means that in order to have a finite yield, (**m – m$_f$ - m$_e$**) should always be **>0**.
- Dividing (**m – m$_f$ - m$_e$**)**>0** by **m**, we get **x+y<1**.
- Therefore, the above equation is valid only when **x+y<1**.

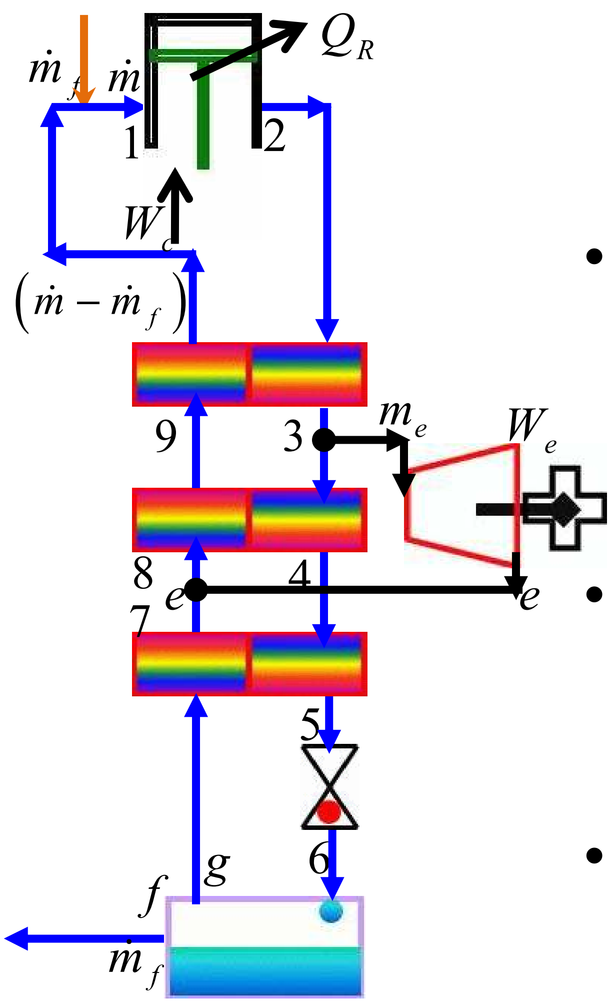
Claude System



$$y = \left(\frac{h_1 - h_2}{h_1 - h_f}\right) + x\left(\frac{h_3 - h_e}{h_1 - h_f}\right)$$

- The yield **y** of the system increases with the increase in the **x** for a constant value of $\mathbf{T_3}$.
- Based on **y** calculated from the above equation, when the sum is **x+y>1**, a limiting value of **y** may be calculated using **x+y=0.99**.
- Rearranging, we have **y=0.99-x**.

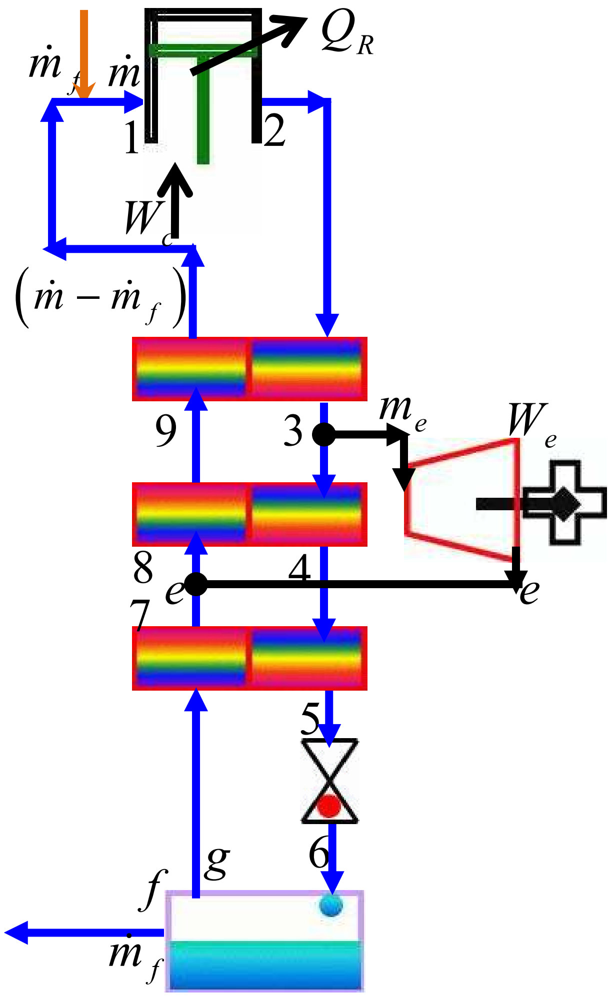
Claude System



$$y = \left(\frac{h_1 - h_2}{h_1 - h_f}\right) + x\left(\frac{h_3 - h_e}{h_1 - h_f}\right)$$

- In summary, **y** is calculated using the above equation until **x+y<1** or **=0.99** is valid.
- After which, a limiting value of **y** is given by **y=0.99-x**.
- This value is the maximum **y** that is possible, but the actual value may be less than this value.

Claude System



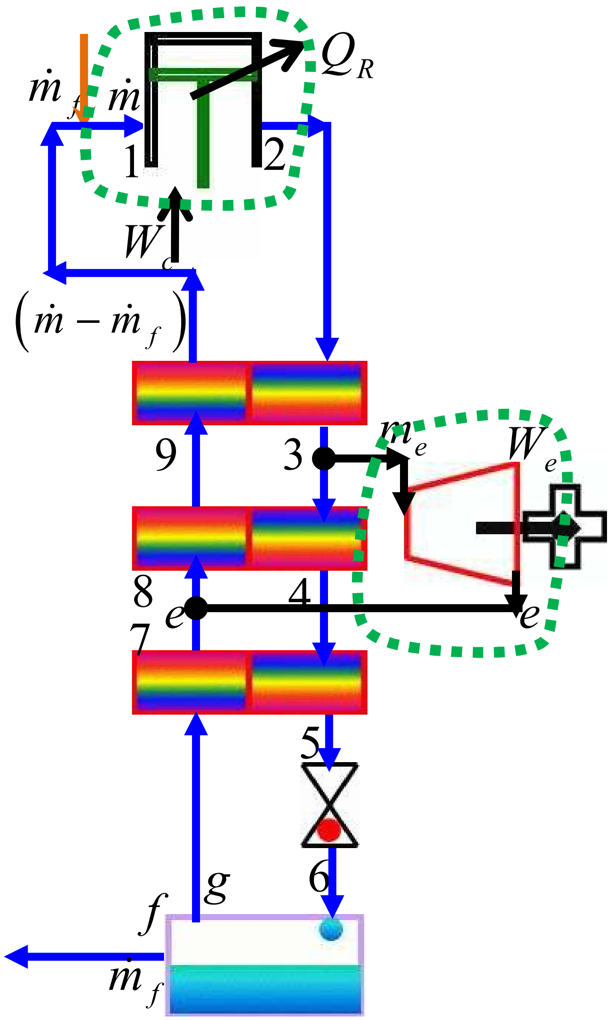
- It is clear that the work interaction of the system with the surroundings is due to
  - Compressor (inwards)
  - Expander (outwards)

- The net work requirement, if the expander work is used in compression process, is given by

$$-W_{net} = -W_c - W_e$$

- where, $-\mathbf{W_c}$ is the work done on the system (negative).

Claude System



- As stated earlier, using a control volume, 1st and 2nd Laws for a compressor, we get

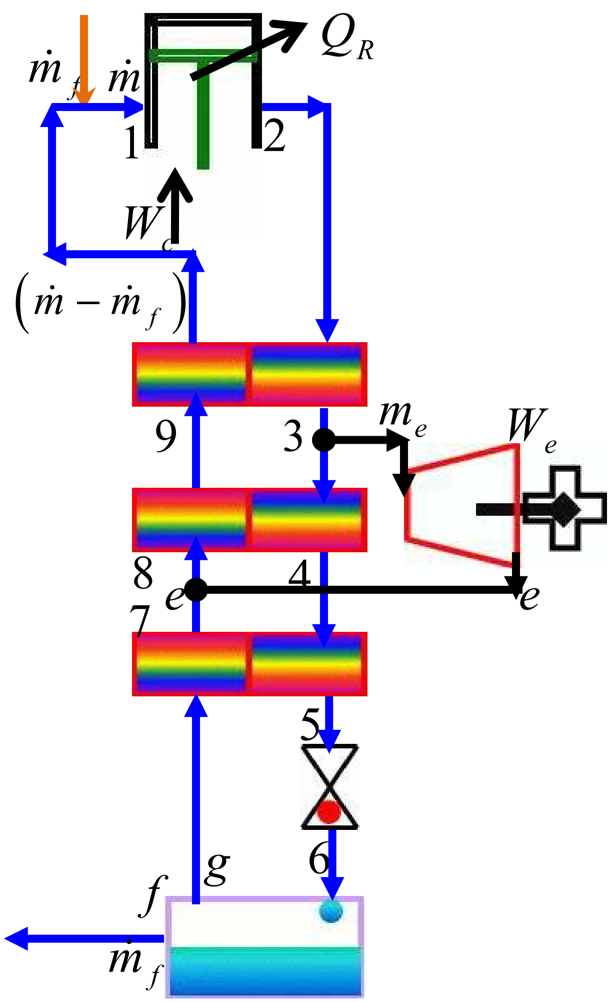
$$-W_c = \dot{m}\left(T_1\left(s_1 - s_2\right) - \left(h_1 - h_2\right)\right)$$

- Similarly, the control volume for an expansion engine, we get

IN	OUT
m_e @ 3	m_e @ e
	W_e

$$W_e = \dot{m}_e\left(h_3 - h_e\right)$$

Claude System



$$\therefore \frac{-W_{net}}{\dot{m}} = -\frac{W_c}{\dot{m}} - \frac{W_e}{\dot{m}}$$

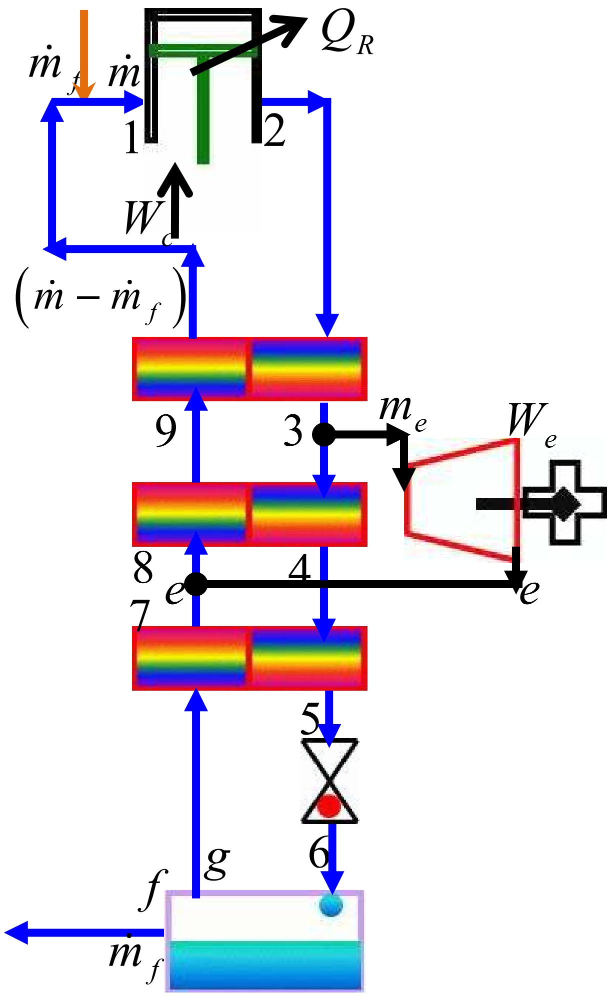
- Substituting the expressions, we have

$$-\frac{W_{net}}{\dot{m}} = \begin{Bmatrix} \left(T_1\left(s_1 - s_2\right) - \left(h_1 - h_2\right)\right) \\ -x\left(h_3 - h_e\right) \end{Bmatrix}$$

$$x = \frac{\dot{m}_e}{\dot{m}}$$

- Where, **x** is the expansion engine flow rate ratio.

Claude System



$$-\frac{W_{net}}{\dot{m}} = \left\{ \begin{array}{l} \left(T_1\left(s_1 - s_2\right) - \left(h_1 - h_2\right)\right) \\ -x\left(h_3 - h_e\right) \end{array} \right.$$

- The first term is the work requirement for simple Linde – Hampson system.
- The second term is the reduction in the work requirement occurring due to the modification.

Tutorial

A. Determine **W/m_f** for a Claude Cycle with N_2 as working fluid. The system operates between 1.013 bar (1 atm) and 40.52 bar (40 atm). The expander inlet **T_3** is at 225 K. The expander flow ratio is varied between 0.1 and 0.9.

B. Repeat the above problem for **T_3** = 300 K, 275 K, 250 K and 225 K. Plot the data **y**, **W/m_f** versus **x** graphically and comment on the results.

Tutorial

Given
Cycle : Claude System
Working Pressure : 1 atm → 40 atm
Working Fluid : Nitrogen
T_3 : 300 K, 275 K, 250 K, 225 K
Mass flow ratio : x = 0.1 → 0.9

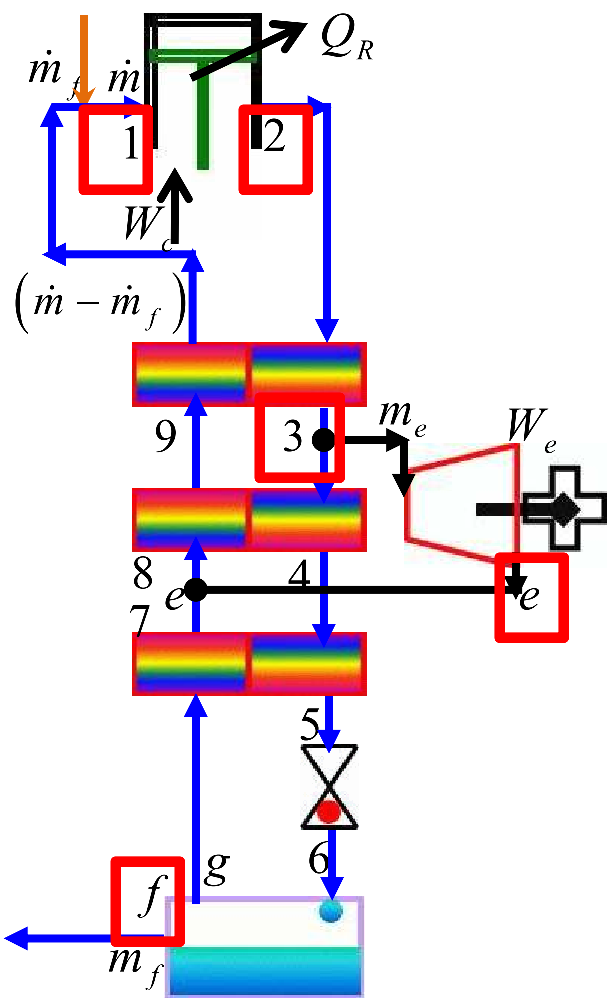
For above System, Calculate	
1	Work/unit mass of gas liquefied

N_2	Point 3
I	300 K
II	275 K
III	250 K
IV	225 K

Methodology

- In the part **A**, the expander inlet condition under study is 225 K at 40.52 bar.
- The expander mass flow ratio varies between 0.1 and 0.9.
- In this tutorial, the **y**, $\mathbf{W/m_f}$ is calculated only for **x = 0.2** and **225 K** as inlet condition.

Tutorial



	1	2	3
p (bar)	1.013	40.52	40.52
T (K)	300	300	225
h (J/g)	462	453	369
s (J/gK)	4.42	3.3	3.0

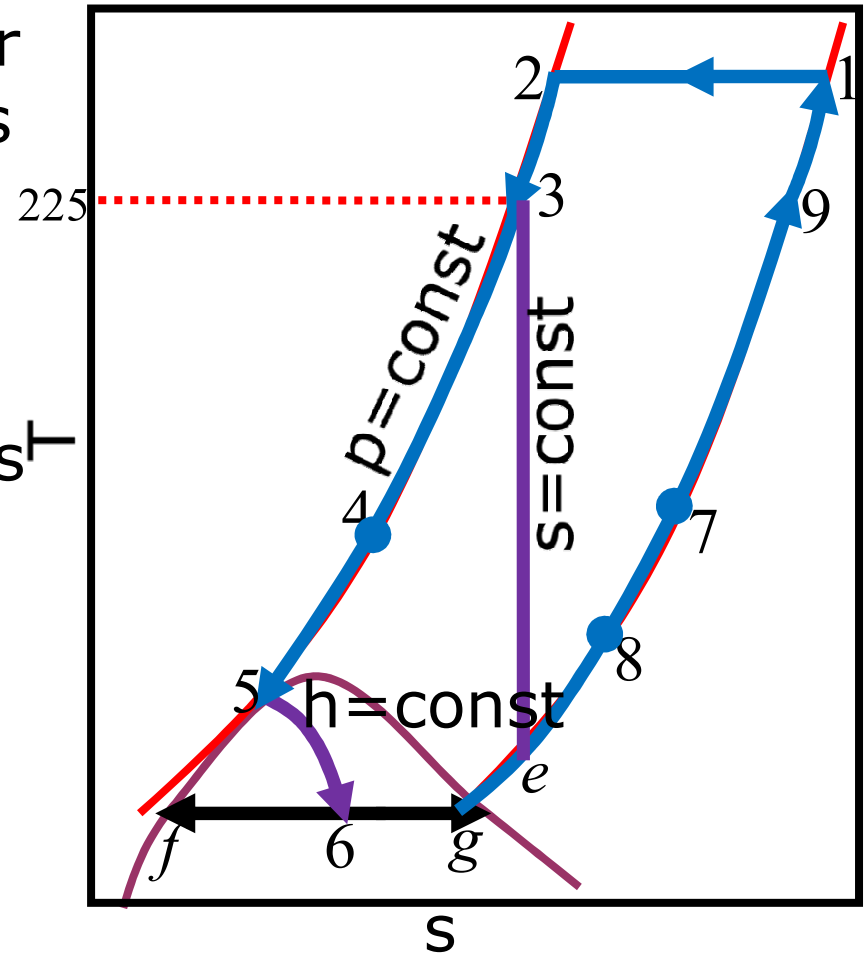
	e	f
p (bar)	1.013	1.013
T (K)	80*	77
h (J/g)	228	29
s (J/gK)	3.0	.25

* The point **e** is located on p=1bar isobar by drawing a vertical line from point **3**.

Tutorial

- The T – s diagram for a Claude System is as shown.

- The expander inlet condition and its mass flow ratio are **225 K** and **0.2** respectively.



Tutorial

- **Liquid yield**

$$y = \frac{h_1 - h_2}{h_1 - h_f} + x\left(\frac{h_3 - h_e}{h_1 - h_f}\right)$$

x	Point 3
0.2	225 K, 40 atm

	1	2	3	e	f
p (bar)	1.013	40.52	40.52	1.013	1.013
T (K)	300	300	225	80	77
h (J/g)	462	453	369	228	29
s (J/gK)	4.42	3.3	3.1	3.1	3.0

$$y = \frac{(462 - 453)}{(462 - 29)} + 0.2\frac{(369 - 228)}{(462 - 29)} = 0.021 + 0.065 = 0.086$$

Tutorial

- **Work/unit mass of N_2 compressed**

$$-\frac{W_c}{\dot{m}} = T_1\left(s_1 - s_2\right) - \left(h_1 - h_2\right) - x\left(h_3 - h_e\right)$$

	1	2	3	e	f
p (bar)	1.013	40.52	40.52	1.013	1.013
T (K)	300	300	225	80	77
h (J/g)	462	453	369	228	29
s (J/gK)	4.42	3.3	3.1	3.1	3.0

$$-\frac{W_c}{\dot{m}} = 300\left(4.42 - 3.3\right) - \left(462 - 453\right) - 0.2\left(369 - 228\right)$$

$$= 299\, J / g$$

Tutorial

- **Work/unit mass of N_2 liquefied**

$$-\frac{W_c}{\dot{m}} = 299$$

$$y = 0.086$$

$$-\frac{W_c}{\dot{m}_f} = -\frac{W_c}{y\dot{m}} = \frac{299}{0.086} = 3476.7\, J/g$$

Tutorial

- Extending the calculations for all other values of **x** and tabulating the results, we have

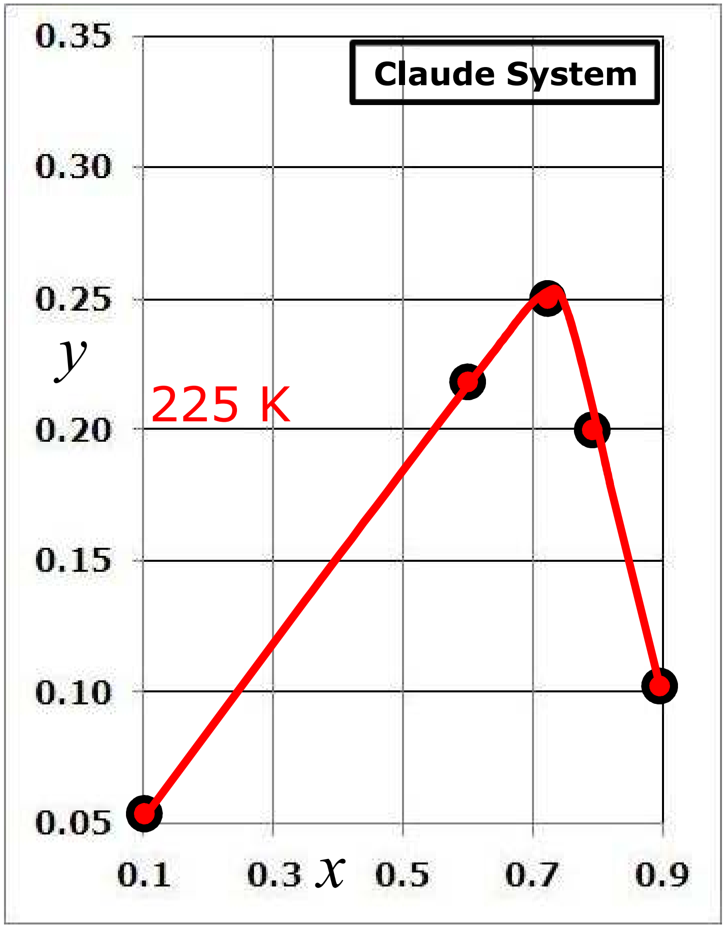
- In the adjacent table, the equation for **y** is used from **x=0.1** to **0.73**. Thereafter, **y=0.99-x** is used.

- Actual **y** may be less than this value.

225		
x	**y**	**W/m_f**
0.10	0.05	5865.2
0.20	0.09	3478.0
0.30	0.12	2403.0
0.40	0.15	1791.6
0.50	0.18	1397.0
0.60	0.22	1121.4
0.70	0.25	917.9
0.73	**0.26**	**866.8**
0.80	0.19	1127.4
0.90	0.09	2223.3

Tutorial

- **Liquid yield v/s. x**



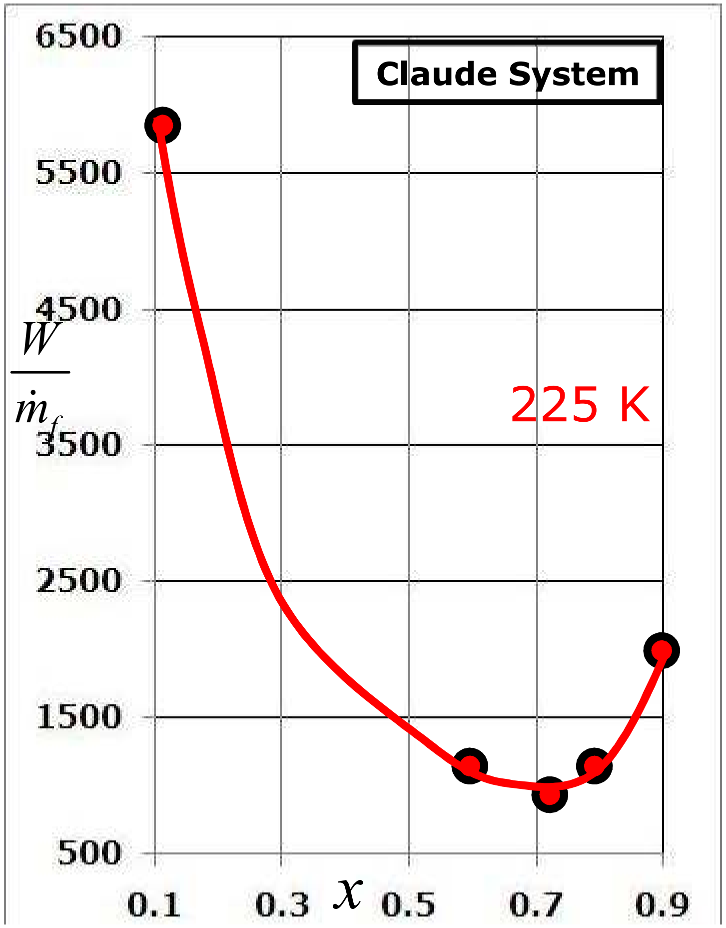
225 K	x	y
I	0.1	0.05
II	0.6	0.22
III	**0.73**	**0.26**
IV	0.8	0.19
V	0.9	0.09

- From the plot, it is clear that **y** crosses a maxima with the increasing **x**.

- Beyond this maxima, the **y** is estimated as limiting value of **y=0.99-x**.

Tutorial

- W/m_f v/s. x



225 K	x	$-W/\dot{m}_f$
I	0.1	5865.2
II	0.6	1121.4
III	**0.73**	**866.8**
IV	0.8	1127.4
V	0.9	2223.3

- The trend shows that the W/m_f crosses a minima with the increasing **x**.

- Beyond this minima, the W/m_f is estimated based on limiting value of **y**.

Tutorial

- All the calculations pertaining to part **B** are left as an exercise.
- The results for these calculations are as shown.

300		275		250	
x	**y**	**x**	**y**	**x**	**y**
0.10	0.07	0.10	0.06	0.10	0.06
0.20	0.11	0.20	0.10	0.20	0.09
0.30	0.16	0.30	0.15	0.30	0.13
0.40	0.20	0.40	0.19	0.40	0.16
0.50	0.25	0.50	0.23	0.50	0.20
0.60	0.29	0.60	0.27	0.60	0.24
0.67	**0.32**	**0.69**	**0.30**	0.70	0.27
0.70	0.29	0.70	0.29	**0.72**	**0.27**
0.80	0.19	0.80	0.19	0.80	0.19
0.90	0.09	0.90	0.09	0.90	0.09

Tutorial

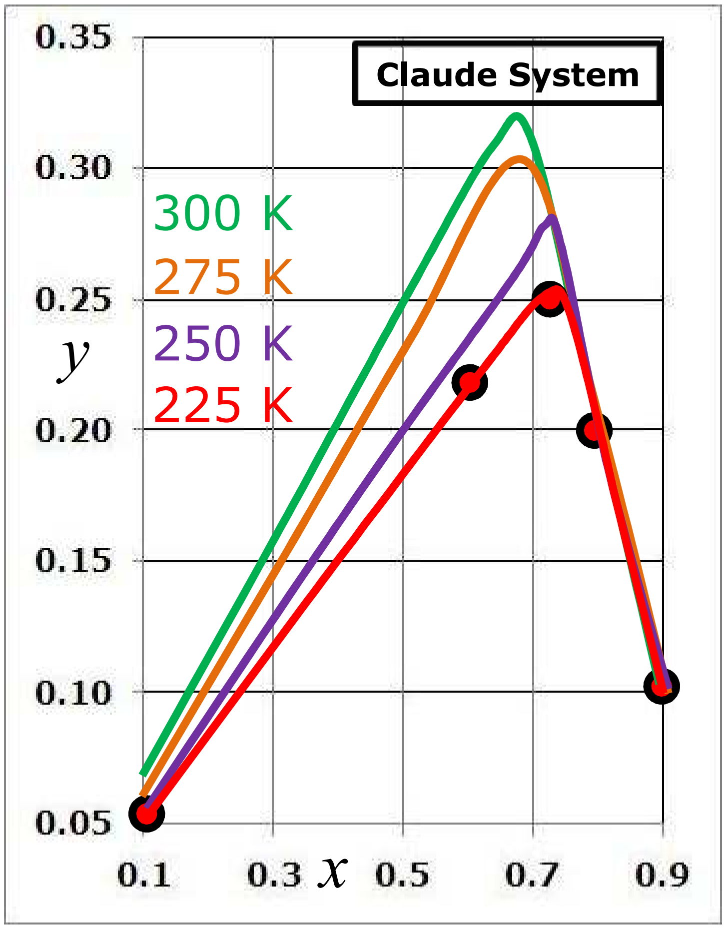
- All the calculations pertaining to part **B** are left as an exercise.

- The results for these calculations are as shown.

300		275		250	
x	W/m_f	x	W/m_f	x	W/m_f
0.10	4671.8	0.10	4955.4	0.10	5505.3
0.20	2598.0	0.20	2800.1	0.20	3204.2
0.30	1722.4	0.30	1876.3	0.30	2188.4
0.40	1239.3	0.40	1363.1	0.40	1616.1
0.50	933.1	0.50	1036.6	0.50	1248.9
0.60	721.7	0.60	810.5	0.60	993.4
0.67	**608.8**	**0.69**	**659.3**	0.70	805.2
0.70	656.9	0.70	693.1	**0.72**	**773.4**
0.80	900.0	0.80	963.2	0.80	1068.4
0.90	1683.3	0.90	1833.3	0.90	2083.3

Tutorial

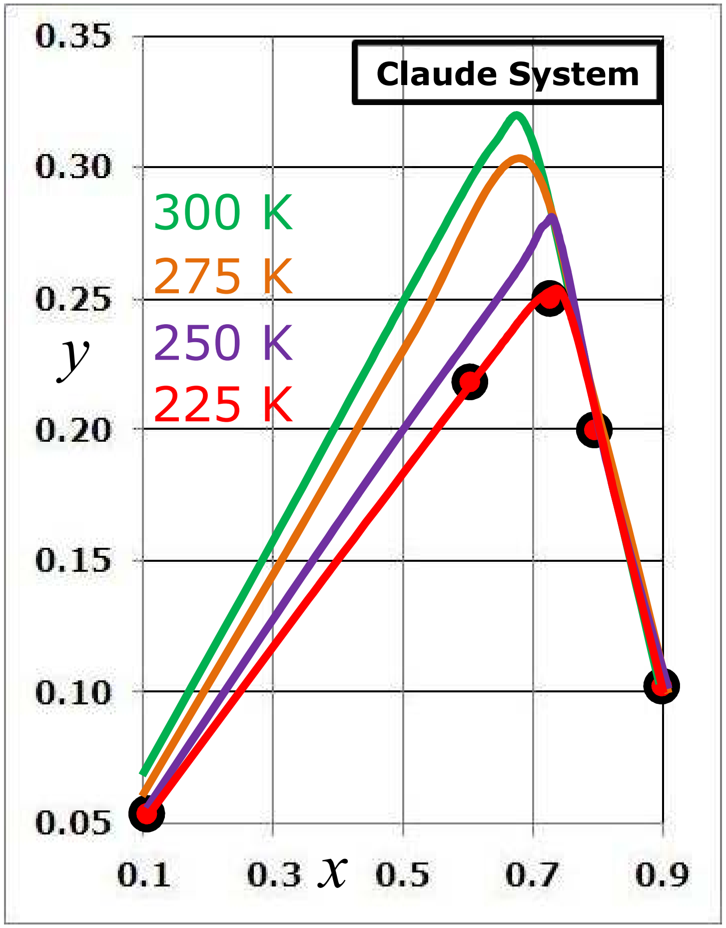
- **Liquid yield v/s. x**



- The plot for **y** versus **x** for all other values of $\mathbf{T_3}$ is as shown.
- As mentioned earlier, **y** crosses a maxima with the increase in the value of **x** for each of the $\mathbf{T_3}$.
- Also, the position of this maxima shifts to the right with the decrease in the value of $\mathbf{T_3}$.

Tutorial

- **Liquid yield v/s. x**



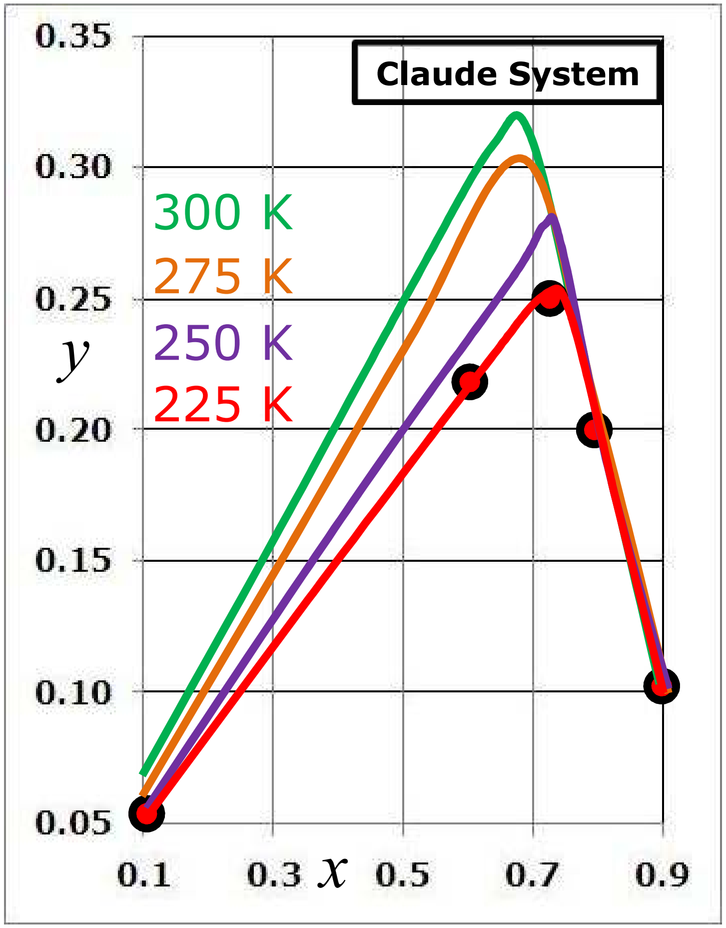
- This occurs because the expander work (**h_3-h_e**) decreases with the decrease in **T_3**.

- Also at the lower values of **T_3**, more amount of the gas can be diverted to the expander engine.

- This is because the product **x(h_3-h_e)** is maximized.

Tutorial

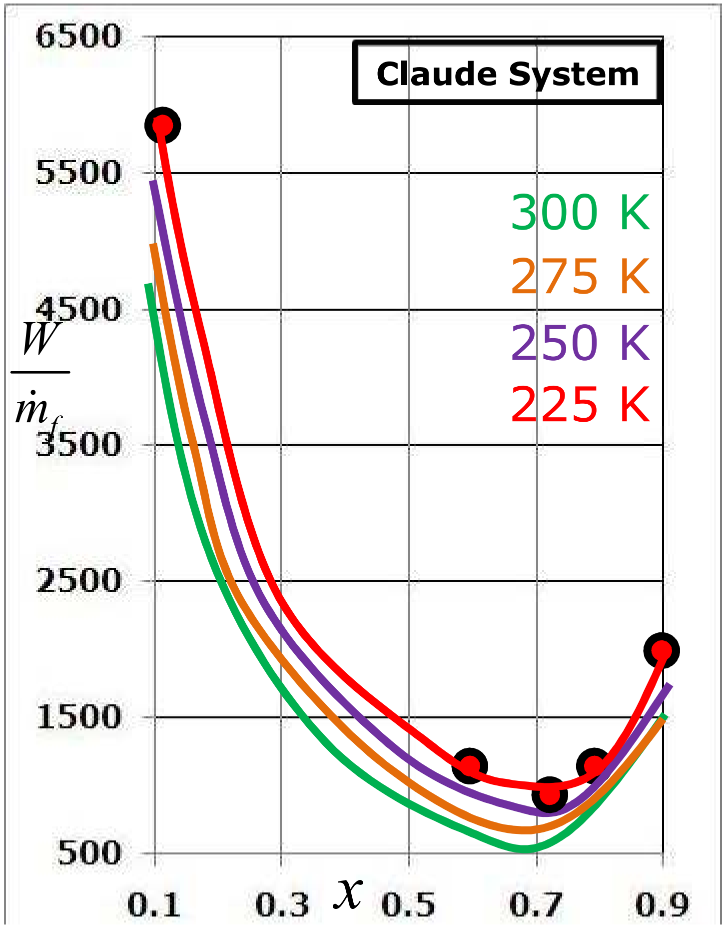
- **Liquid yield v/s. x**



- However, $\mathbf{T_3}$ is limited by the position of the point **e** on the T – s diagram.

Tutorial

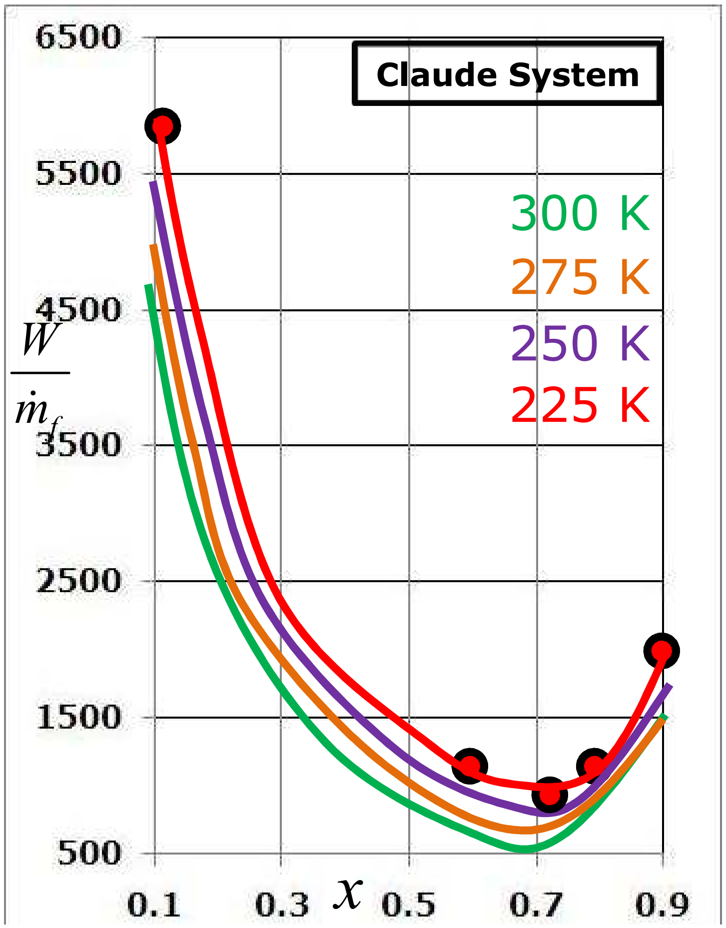
- **W/m_f v/s. x**



- The plot for **W/m_f** versus **x** for all other values of **T_3** is as shown.
- As stated earlier, the **W/m_f** crosses a minima with the increase in the **x**, for each of the **T_3**.
- Also, the position of this minima shifts to the right with the decrease in the value of **T_3**.

Tutorial

- **W/m$_f$ v/s. x**



- The minima shifts to the right because the expander work (**h_3-h_e**) decreases with the decrease in **T_3**.

- Also, at the lower values of **T_3**, more amount of the gas can be diverted so that the product **x(h_3-h_e)** is maximized.

Summary

- The J – T expansion is an irreversible isenthalpic expansion and an expansion by an expansion engine is an reversible isentropic process.
- In a Claude System, the energy content of the gas is removed by allowing it to undergo an isentropic expansion.
- The yield & work requirement of the system are

$$y = \left(\frac{h_1 - h_2}{h_1 - h_f}\right) + x\left(\frac{h_3 - h_e}{h_1 - h_f}\right)$$

$$-\frac{W_{net}}{\dot{m}} = \begin{cases} \left(T_1\left(s_1 - s_2\right) - \left(h_1 - h_2\right)\right) \\ -x\left(h_3 - h_e\right) \end{cases}$$

Summary

- If $\mathbf{T_1}$, $\mathbf{T_2}$, $\mathbf{T_3}$ of the system are held constant, the yield **y** of the system is a linear function of expander mass flow ratio **x**.

- The equation of **y** is valid only when $\mathbf{x+y<1}$. Beyond a certain value of **x**, a limiting value of **y** is estimated as $\mathbf{y<1-x}$.

- For a given value of $\mathbf{T_3}$, the yield **y** crosses a maxima with the increase in the value of **x**.

- Also, the maxima shifts to the right with the decrease in the value of $\mathbf{T_3}$.

Summary

- For a given value of $\mathbf{T_3}$, the $\mathbf{W/m_f}$ of the system goes through a minima with the increase in the **x**.
- Also, the position of this minima shifts to the right with the decrease in the value of $\mathbf{T_3}$.

Assignment

A. Determine **W/m_f** for a Claude Cycle with N_2 as working fluid. The system operates between 1.013 bar (1 atm) and 50.56 bar (50 atm). The expander inlet **T_3** is at 250 K. The expander flow ratio is varied between 0.1 and 0.9.

B. Repeat the above problem for **T_3** = 300 K, 275 K and 250 K. Plot the data **y**, **W/m_f** versus **x** graphically and comment on the results.

Assignment

- **Answers**

300		275		250	
x	**y**	**x**	**y**	**x**	**y**
0.10	0.07	0.10	0.06	0.10	0.06
0.20	0.12	0.20	0.11	0.20	0.10
0.30	0.16	0.30	0.15	0.30	0.14
0.40	0.21	0.40	0.19	0.40	0.17
0.50	0.26	0.50	0.23	0.50	0.21
0.60	0.31	0.60	0.27	0.60	0.25
0.66	**0.33**	**0.68**	**0.31**	**0.70**	**0.29**
0.70	0.29	0.70	0.29	0.72	0.27
0.80	0.19	0.80	0.19	0.80	0.19
0.90	0.09	0.90	0.09	0.90	0.09

Assignment

- **Answers**

300		275		250	
x	**W/m_f**	**x**	**W/m_f**	**x**	**W/m_f**
0.10	4666.1	0.10	5229.3	0.10	5605.3
0.20	2618.5	0.20	2956.9	0.20	3230.1
0.30	1744.2	0.30	1986.1	0.30	2195.9
0.40	1259.3	0.40	1447.5	0.40	1617.1
0.50	951.1	0.50	1105.1	0.50	1247.2
0.60	737.8	0.60	868.2	0.60	990.4
0.67	**632.6**	**0.69**	**719.5**	**0.70**	**801.7**
0.70	707.6	0.70	758.3	0.72	846.2
0.80	972.6	0.80	1061.1	0.80	1132.6
0.90	1826.7	0.90	2036.7	0.90	2206.7

Thank You!