

# CRYOGENIC ENGINEERING

The background is a dark, abstract collage of scientific and technical imagery. It features a large circular diagram in the upper right corner, possibly a molecular structure or a complex circuit. In the center and lower right, there are several computer workstations with monitors and towers. On the left, a microscope is visible. The overall color palette is dominated by purples, blues, and oranges, with a glowing, ethereal atmosphere.

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Lecture No - 20

## Earlier Lecture

- A mixture composition can be represented by either volume, mass or mole fractions.
- Work of separation is represented by  $W_{i,m}/n_m$ ,  $W_{i,m}/n_A$  and  $W_{i,m}/n_B$  (for **Gas A** and **B**). Additionally, it is also represented by  $W_{i,A}/n_A$ ,  $W_{i,B}/n_B$  and  $W_{i,C}/n_C$  (for **Gas A, B** and **C**).
- Ideal work of separation/mole of mixture with **N** constituents is given by 
$$\frac{-W_i}{n_m} = \mathfrak{R}T_m \sum_{j=1}^N y_j \ln \left( \frac{1}{y_j} \right)$$
- where  $y_j$  is mole fraction of  $j^{\text{th}}$  component.

## Outline of the Lecture

Topic : Gas Separation (contd)

- Gibbs Phase Rule
- Phase Equilibrium Curves
- Temperature Composition Diagrams

## Introduction

- A mixture can have various components and can exist in various phases in thermal equilibrium.
- For example, a mixture of ice and water is a 1 – component and a two phase mixture.
- If number of components and number of phases in thermal equilibrium are denoted by  $C$  and  $P$  respectively, then for above mixture  $C=1$  and  $P=2$
- Similarly, boiling **LAir** is **LOX** + **LN<sub>2</sub>** + **N<sub>2</sub>** + **O<sub>2</sub>**. the values of  $C$  and  $P$  are  $2$  and  $2$  respectively.

## Introduction

- Every mixture can be uniquely defined by a set of properties called as Thermostatic Properties.
- These properties can either be Intensive (independent of mass) or Extensive (dependent on mass) properties.
- Pressure, temperature, density are examples of Intensive properties and volume, enthalpy, entropy are few examples of Extensive properties.

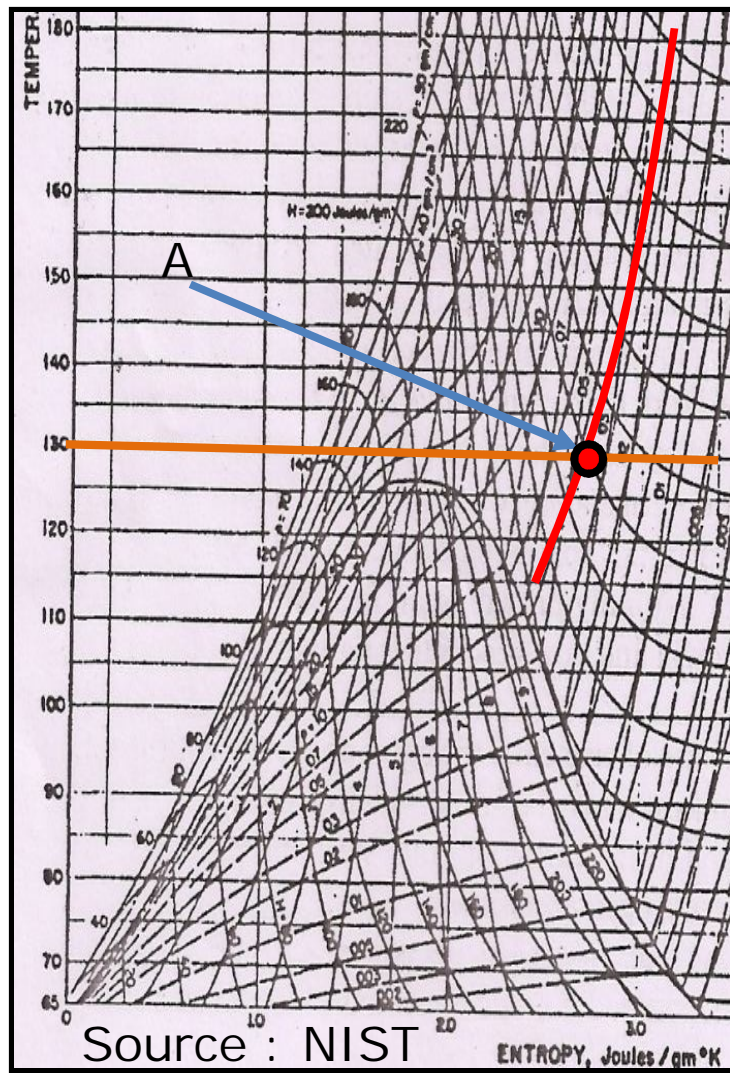
## Introduction

- For any mixture, there are certain minimum number of Intensive properties required to define the complete state.
- In other words, these properties are required to locate a unique point on  $T - s$ ,  $p - T$  etc. diagrams, corresponding to the unique state of the mixture.
- Gibbs Phase Rule is used in determining these properties/degrees of freedom, for a given mixture.

## Gibbs Phase Rule

- Gibbs Phase Rule was formulated by Josiah Willard Gibbs, an American physicist, in 1878.
- If  $F$  is the degrees of freedom/minimum number of Intensive properties required to define the thermodynamic state of a system, then by Gibbs Phase Rule  $F = C - P + 2$
- For example, for gaseous  $\text{N}_2$ , we have  $C = 1$  and  $P = 1$ . Therefore,  $F = 2$ .
- These can be pressure and temperature or pressure and specific volume.

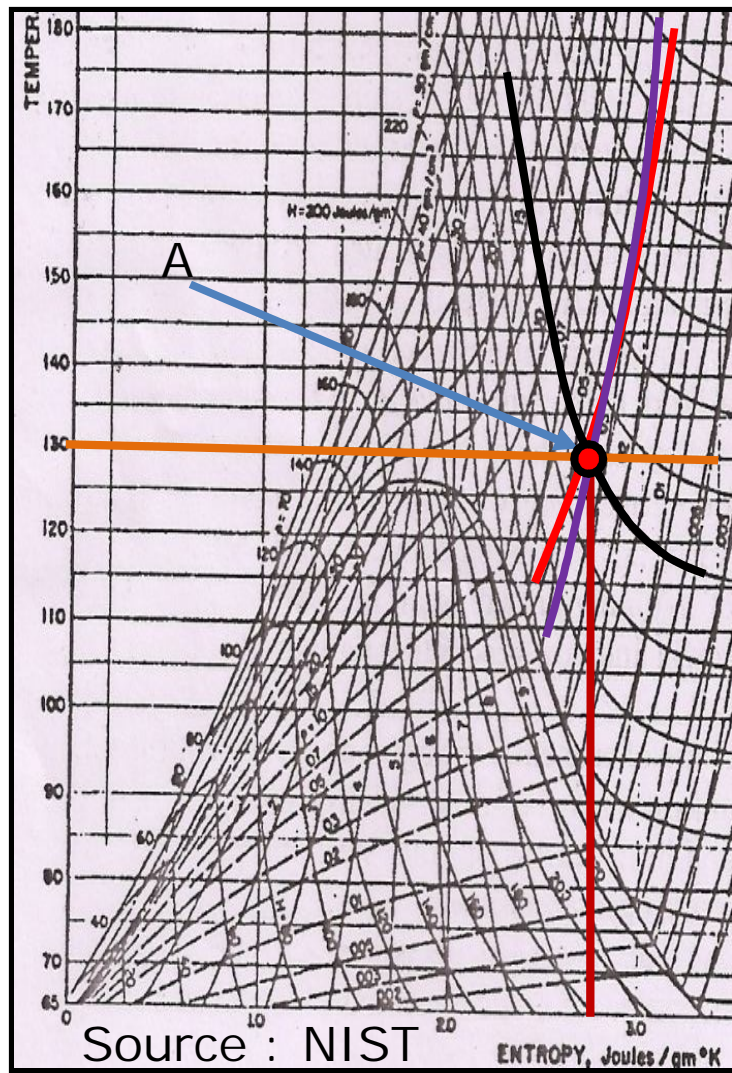
## Gibbs Phase Rule



- The T – s diagram for  $N_2$  is as shown.
- Let us say that for a known  $p$  and  $T$  lines in the gaseous region, intersect at **A** as shown in the figure.
  - Pressure
  - Temperature



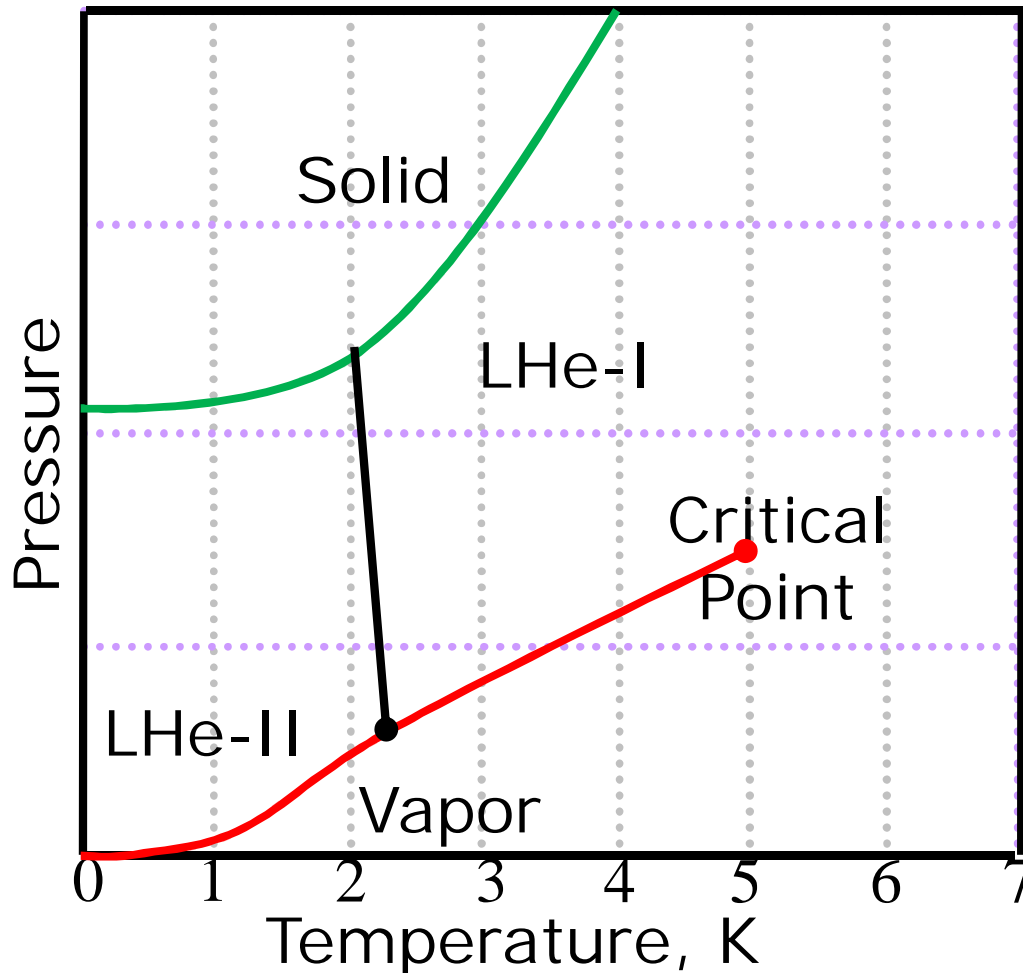
## Gibbs Phase Rule



• It is clear that, all other properties can be uniquely defined.

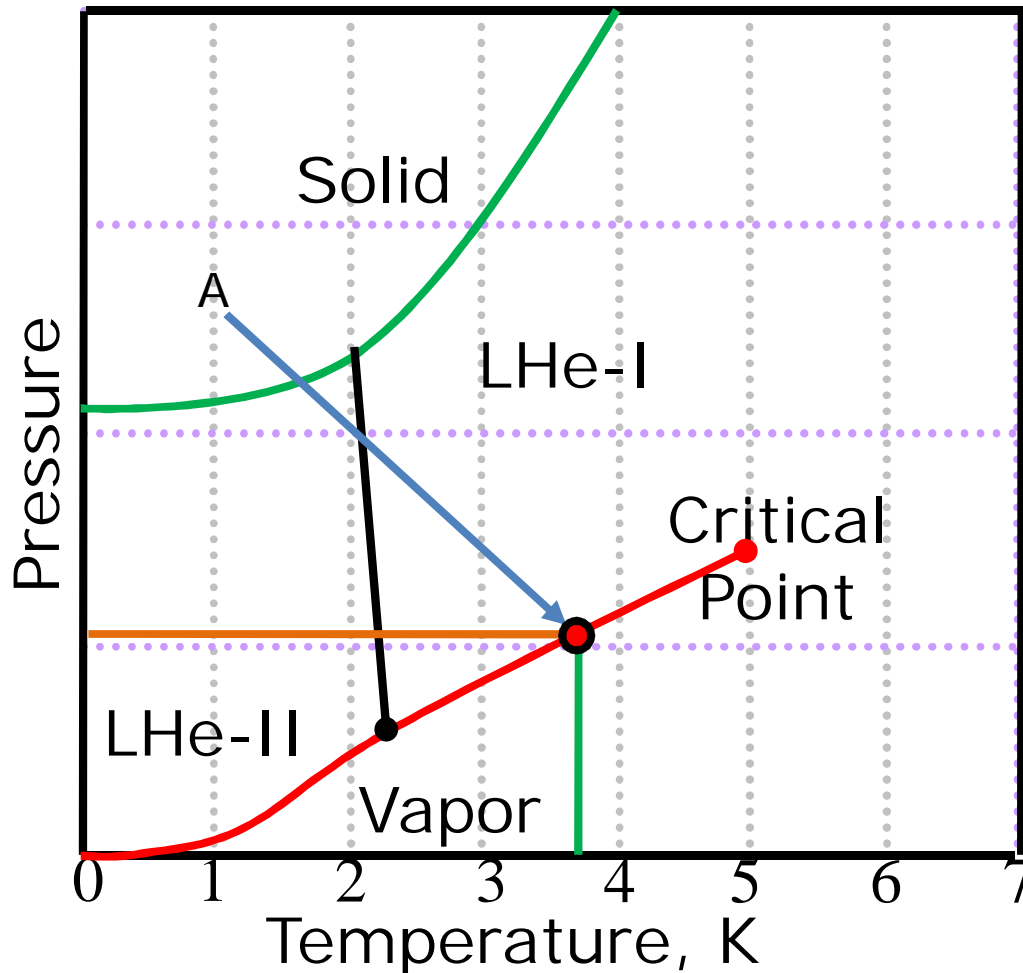
- Specific enthalpy
- Specific entropy
- Specific volume

## Gibbs Phase Rule



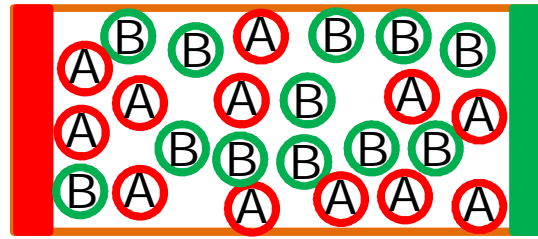
- Similarly, for a two phase mixture of Helium in thermal equilibrium, we have  $P = 2$   $C = 1$
- By Gibbs Phase Rule  $F = C - P + 2 = 1$
- This property can either be saturation temperature or saturation pressure.

## Gibbs Phase Rule



- The  $p - T$  diagram for Helium is as shown.
- Let us say, the pressure  $p$  is known, it intersects the vapor line at **A**.
- The corresponding temperature can be known and vice versa.

## Phase Equilibrium Curves

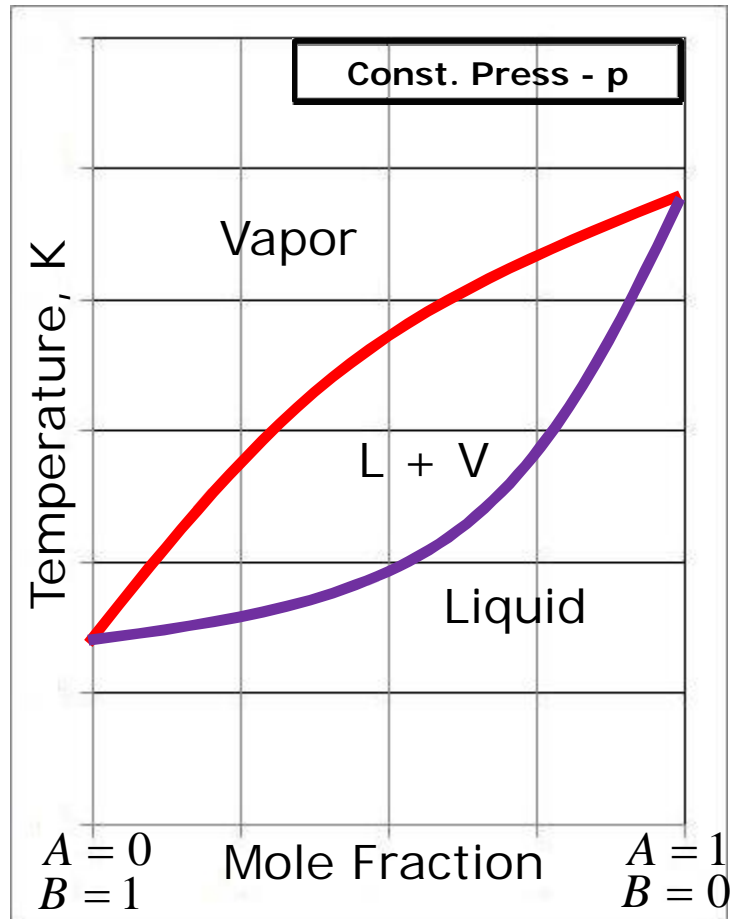


- Again, for a mixture of **Gas A** and **Gas B** in single phase, we have  $P = 1$  and  $C = 2$ .
- Using the Gibbs Phase Rule,  $F = 3$
- These properties are pressure, temperature and mole fraction of one of the components.

## Phase Equilibrium Curves

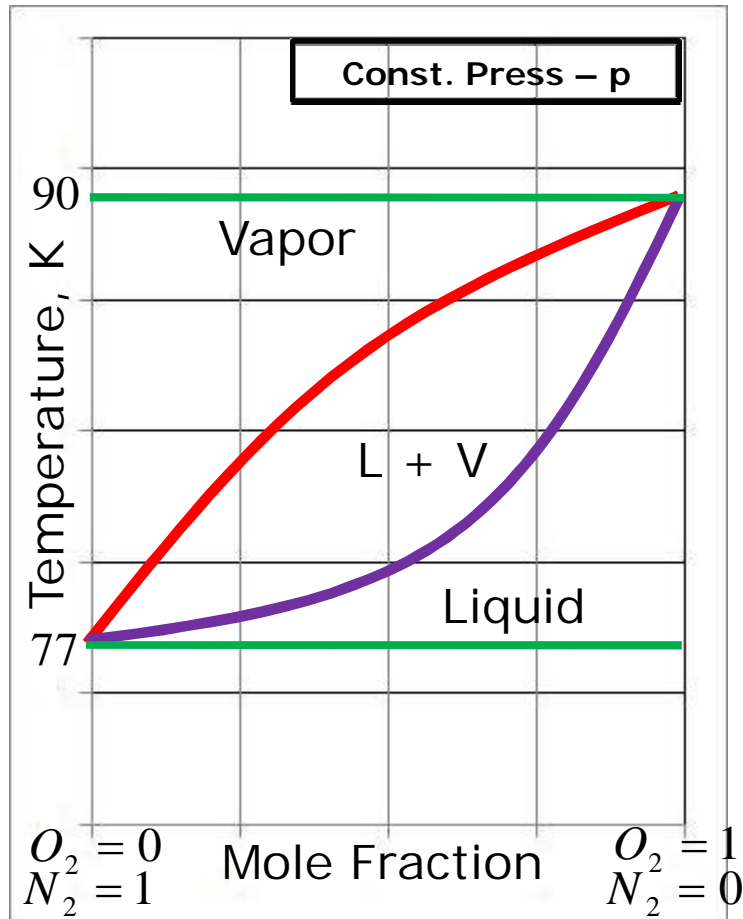
- In general, phase of a 1 – component system is governed by pressure and temperature.
- But for a 2 – component mixture at a given pressure, the mole fractions of components in vapor and liquid phases change with the temperature.
- Variation of mole fraction ( $y$ ) with temperature ( $T$ ) at a constant pressure ( $p$ ) is given by **Temperature composition diagram** or **Phase – equilibrium curve**. 3 typical curves are explained in further slides.

## Phase Equilibrium Curves



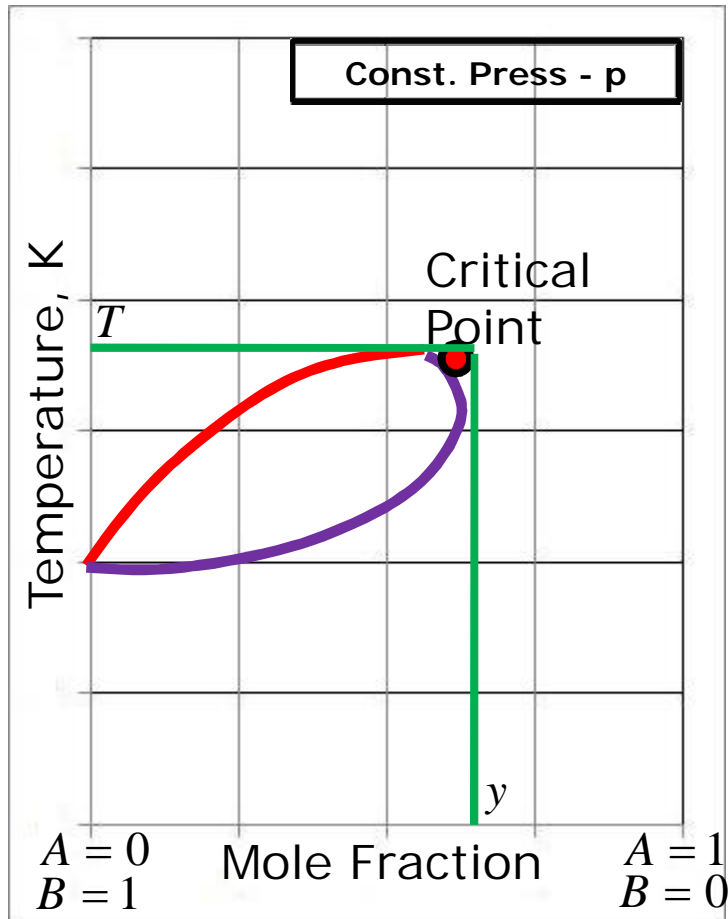
- Consider a 2 – component system (say, **A** and **B**) at some pressure **p** and temperature **T**.
- Critical Pressures ( $p_c$ ) of both these components are more than the pressure **p**.
- The plot shows the variation of mole fractions of the components with temperature.

## Phase Equilibrium Curves



- For example, consider a mixture of  $N_2$  and  $O_2$  at 1 atm.
- It is clear that, above 90 K, the mixture is in gaseous phase and below 77 K, it is in liquid phase.
- The area formed by these curve lines indicate a two phase region (Liquid + Vapor).

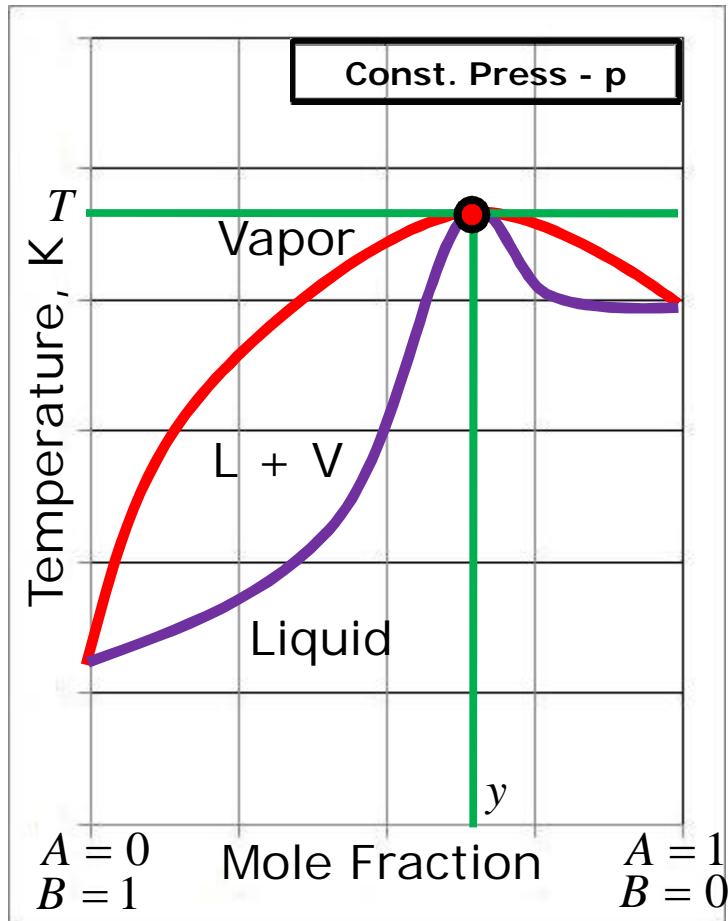
## Phase Equilibrium Curves



- Again consider a 2 – component system (say, **A** and **B**) at some pressure **p** and temperature **T**.
- The plot shows the case, in which one of the components has  $p_c$  less than the pressure **p**.
- There is no liquid phase after a certain temperature and mole fraction.

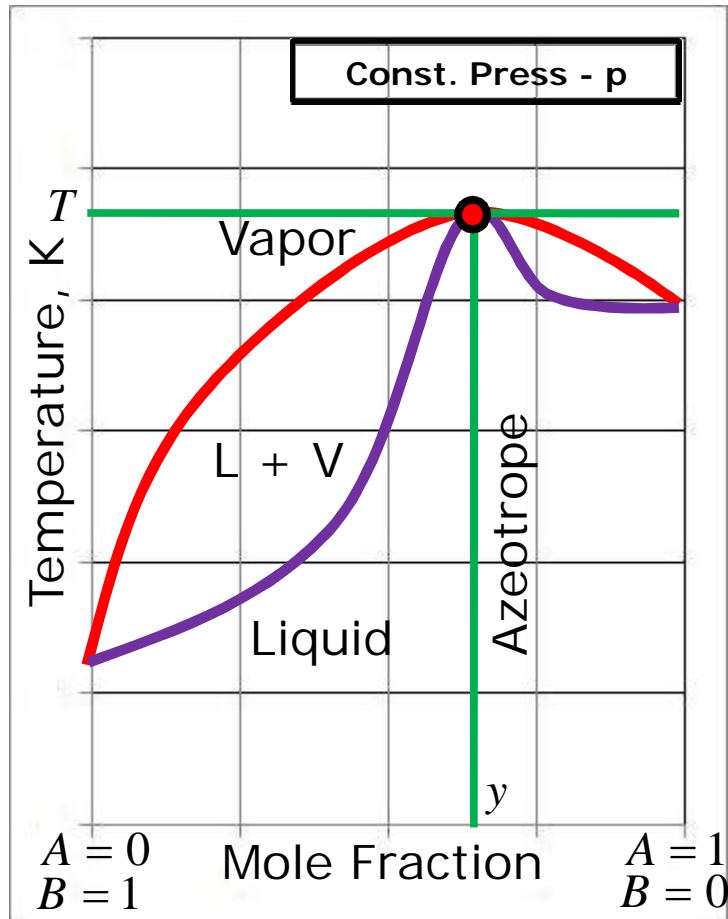


## Phase Equilibrium Curves



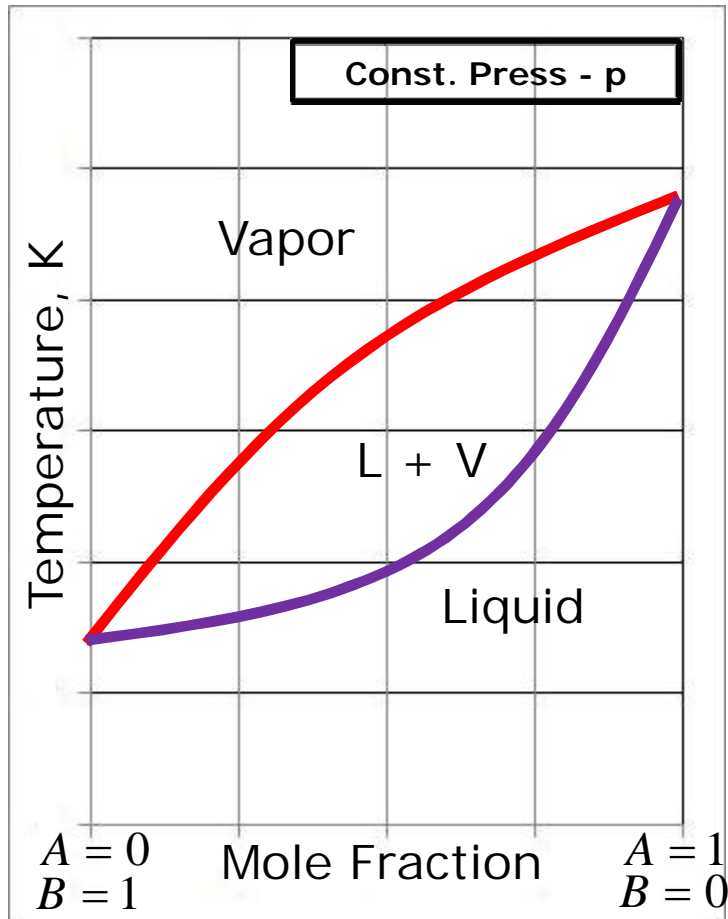
- Few substances when mixed in certain proportions, physically behave as one substance.
- For example, mixture of acetone and chloroform.
- In the figure, at  $T$  and mole fraction  $y$ , the mixture behaves as one substance.

## Phase Equilibrium Curves



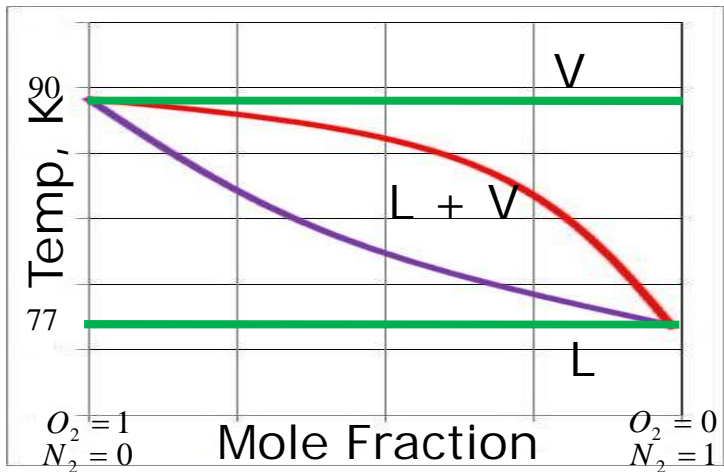
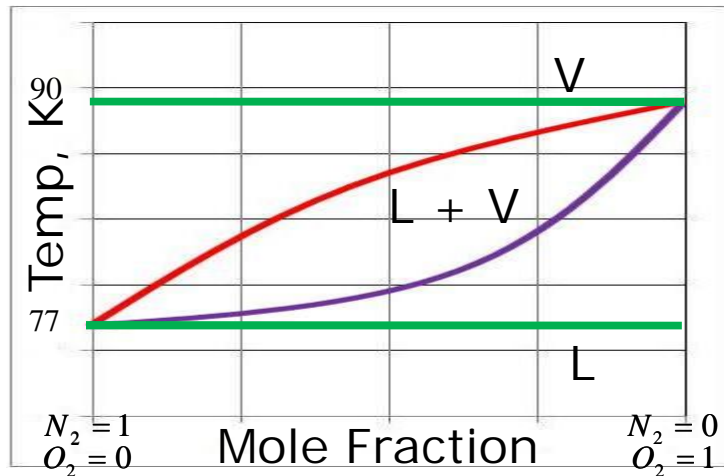
- Such mixtures are called as Azeotropic mixtures or constant boiling liquids.
- It is undesirable to separate such mixtures by rectification.
- Mixtures cannot be separated past this composition.
- Cryogenics rarely exhibit such behaviors.

## Temp. Composition Diagrams



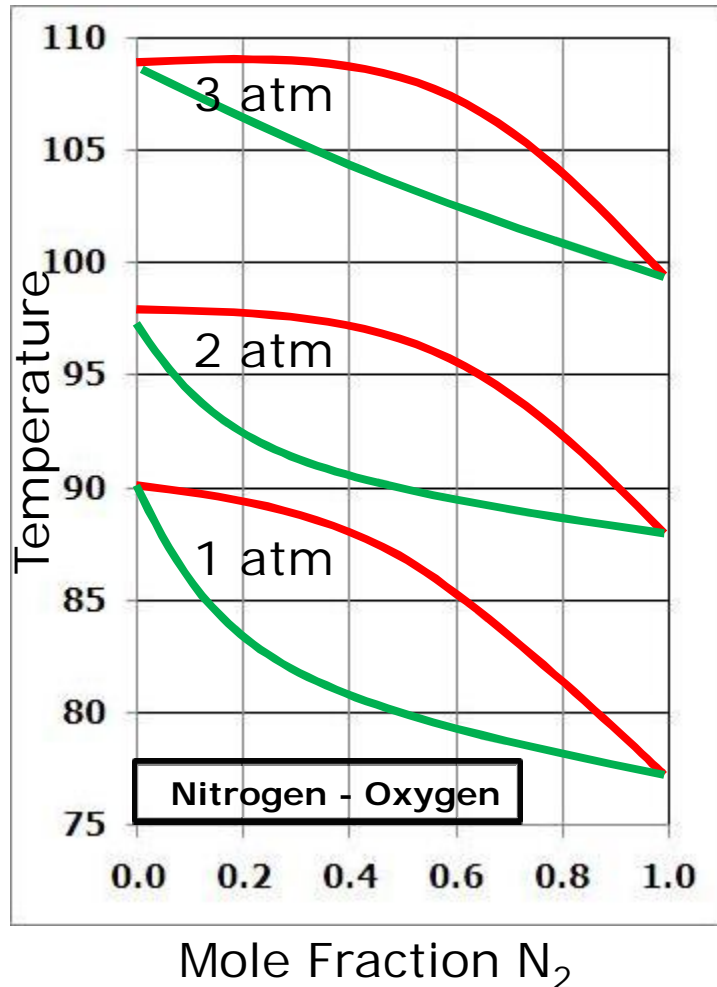
- In **Gas Separation**, the first type of diagram is of great importance.
- The process of rectification is best understood with these curves.
- Hence, it is important to study the **Temperature Composition Diagram** to estimate the composition of vapor and liquid phases.

## Temp. Composition Diagrams



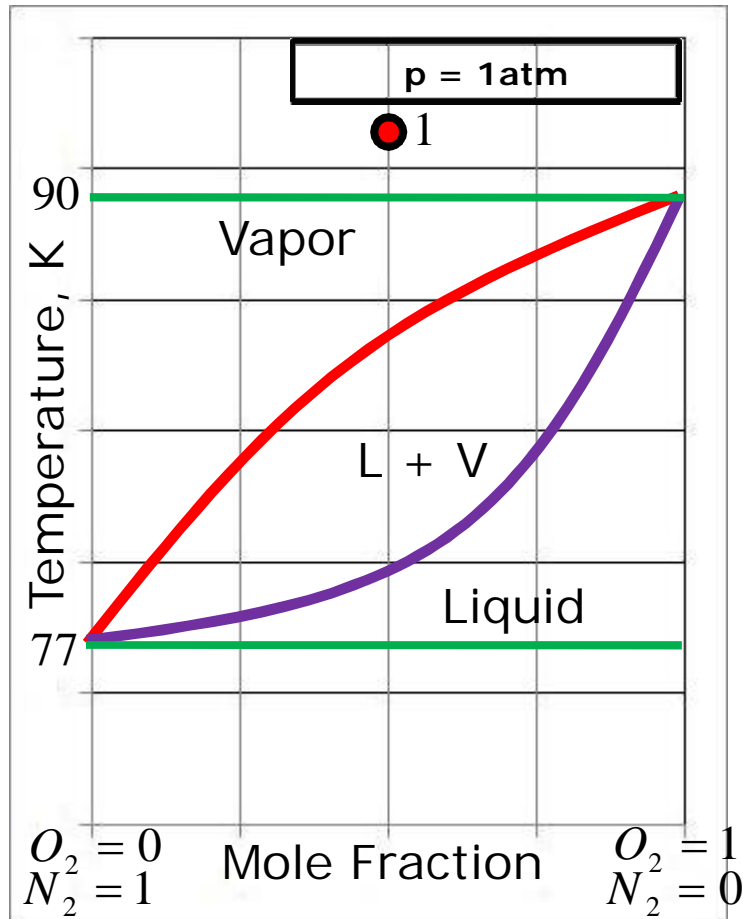
- Consider a mixture of  $O_2$  and  $N_2$  at a given constant pressure.
- If the components on the **x** – **axis** are interchanged, the diagram is as shown.
- It is important to note that, these two plots are one and the same. Either of the plots are commonly used in the literature.

## Temp. Composition Diagrams



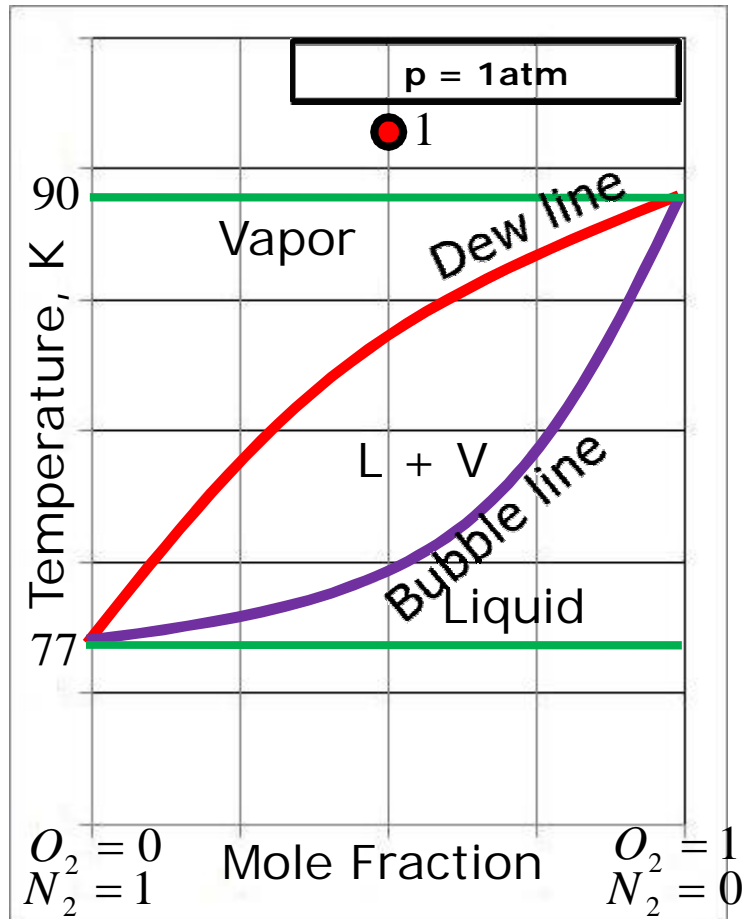
- Temperature composition diagrams for  $\text{N}_2 - \text{O}_2$  mixture for different pressures are as shown.
- These plots are obtained experimentally and are a strong function of inter-molecular forces.
- However, theoretical plots can be drawn based on some assumptions.

## Temp. Composition Diagrams



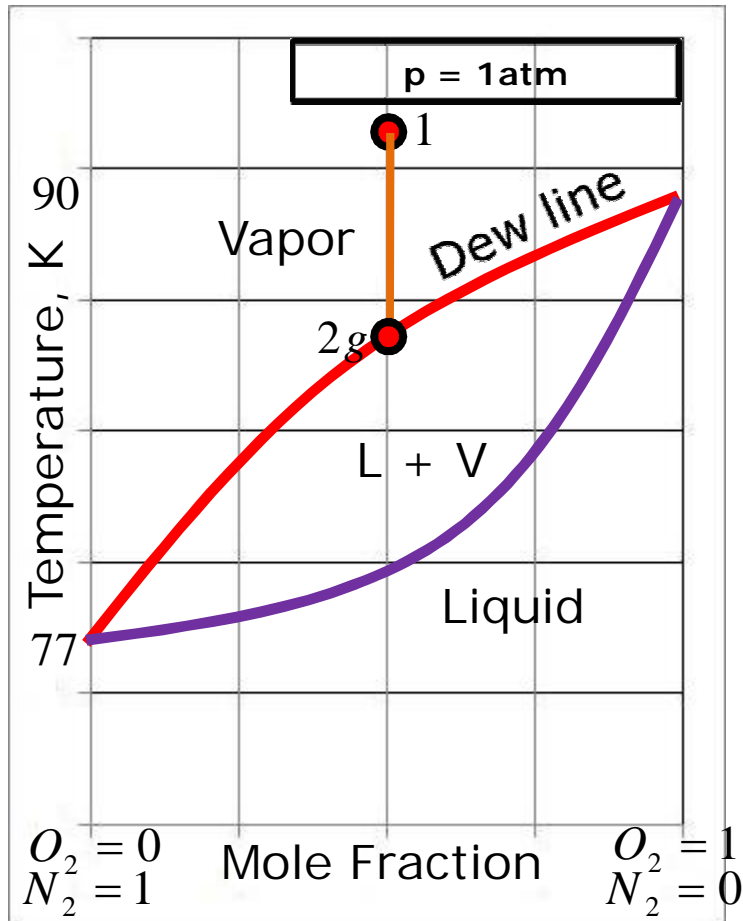
- Consider a Temperature composition diagram for a mixture of  $\text{O}_2$  and  $\text{N}_2$  at a pressure of 1 atm.
- Let the initial state of the mixture be at point **1** as shown in the figure.
- Since the temperature at point **1** is more than 90 K, the mixture exists in complete gaseous phase.

## Temp. Composition Diagrams



- The upper curve (Red) is called as the **Dew line**.
- Similarly, the lower curve (violet) is called as the **Bubble line**.
- The area formed by these curve lines indicate a two phase region.
- It means that, it has both Liquid and Vapor phases.

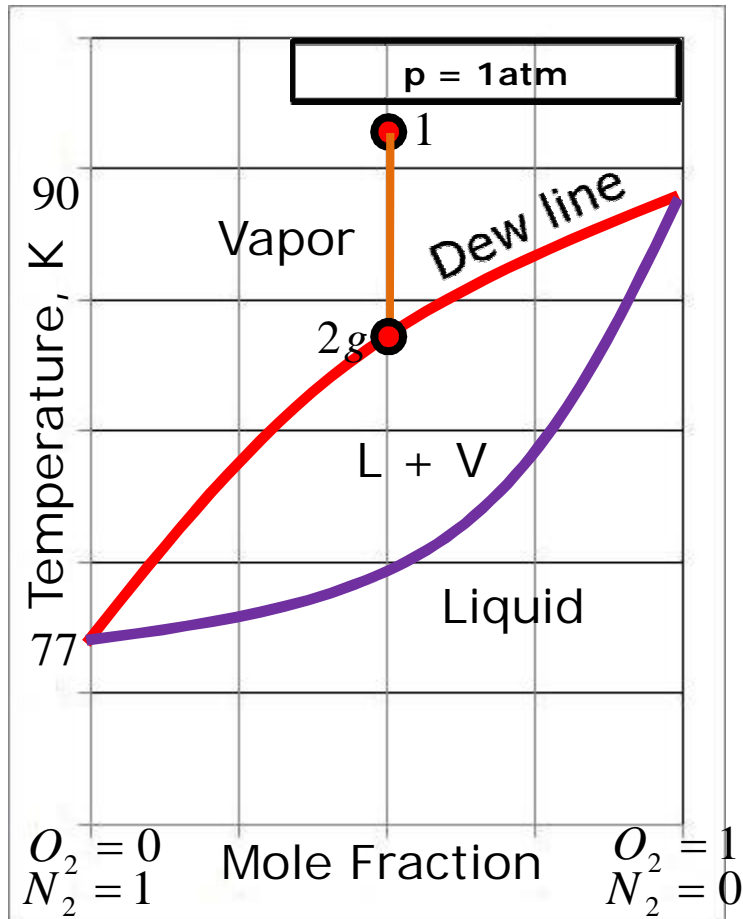
## Temp. Composition Diagrams



- Let the mixture be cooled at constant pressure.
- When the temperature of the mixture reaches point **2g**, the mixture starts condensing.
- The point **2g** lies on the dew line and the first drop or the dew appears in the mixture.

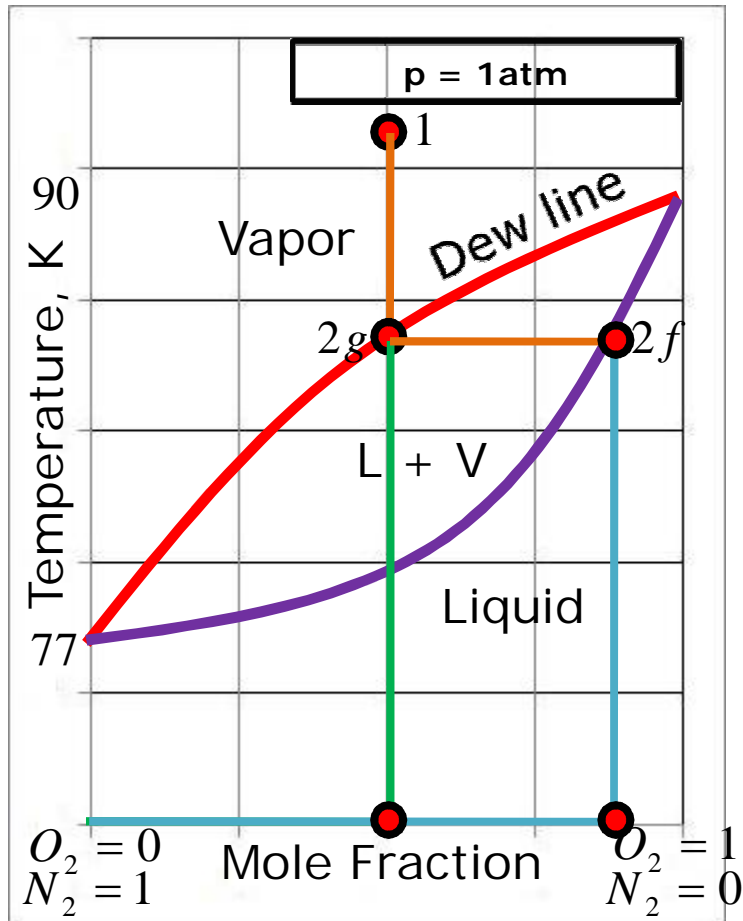


## Temp. Composition Diagrams



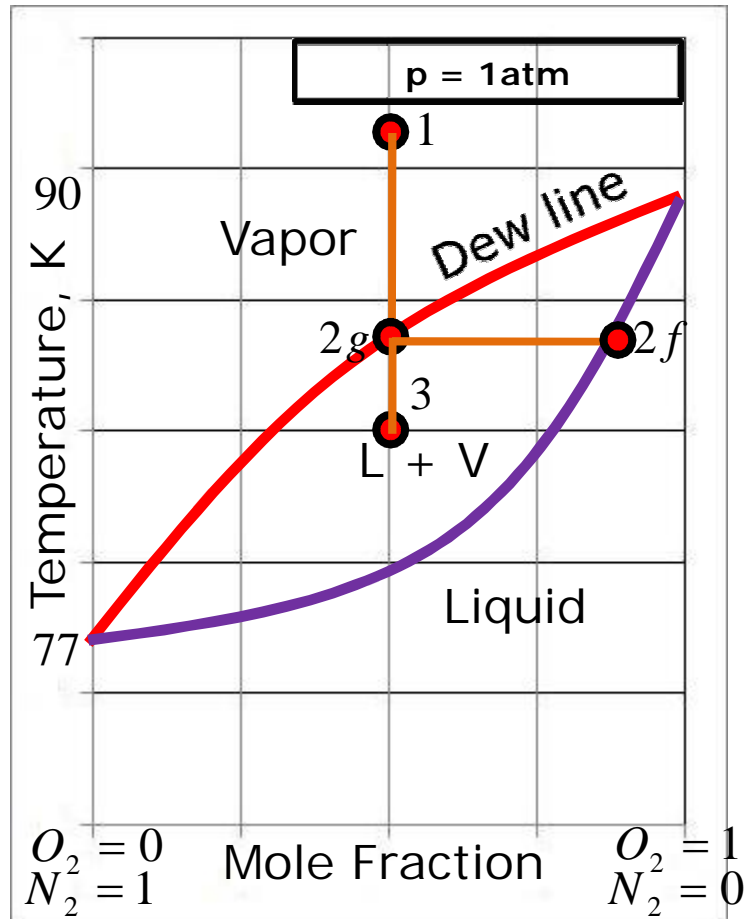
- Therefore, the mixture at point **2g** is a two – phase mixture with liquid – vapor phases in equilibrium.
- Condensate liquid has mole fractions of both high boiling and low boiling liquids.
- Liquid content is obtained by a constant temperature line about the point **2g**.

## Temp. Composition Diagrams



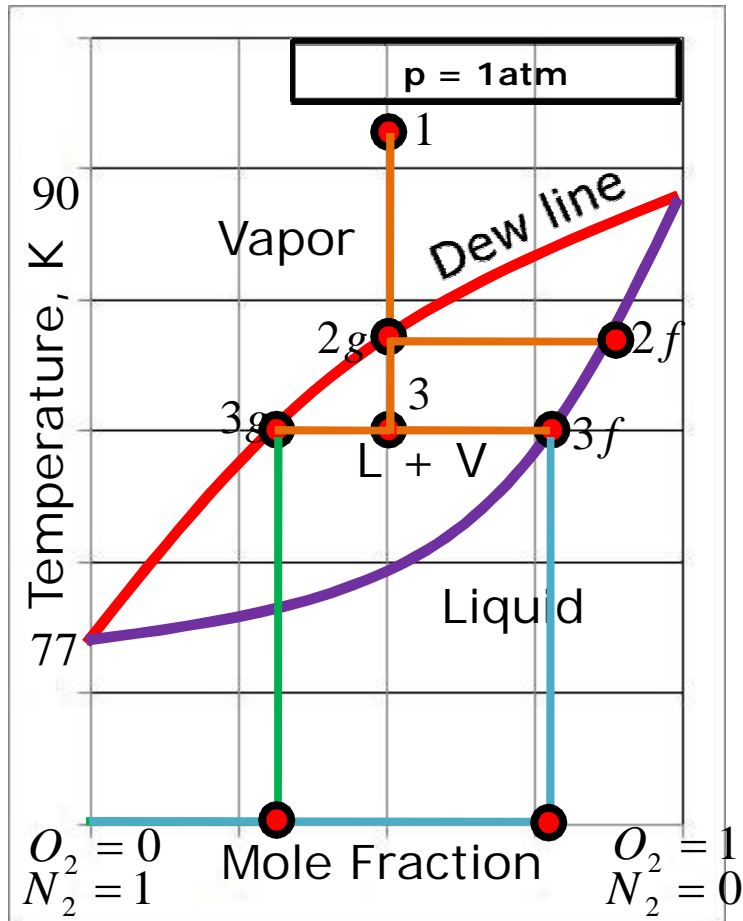
- Extending a constant temperature line about **2g**, it intersects the bubble line at point **2f**.
- **2g** and **2f** denote the mole fractions of higher BP liquid ( $O_2$ ) in gaseous and liquid phases respectively.
- The compositions are
  - $y_{\text{vap}} = \mathbf{2g}$
  - $y_{\text{liq}} = \mathbf{2f}$

## Temp. Composition Diagrams



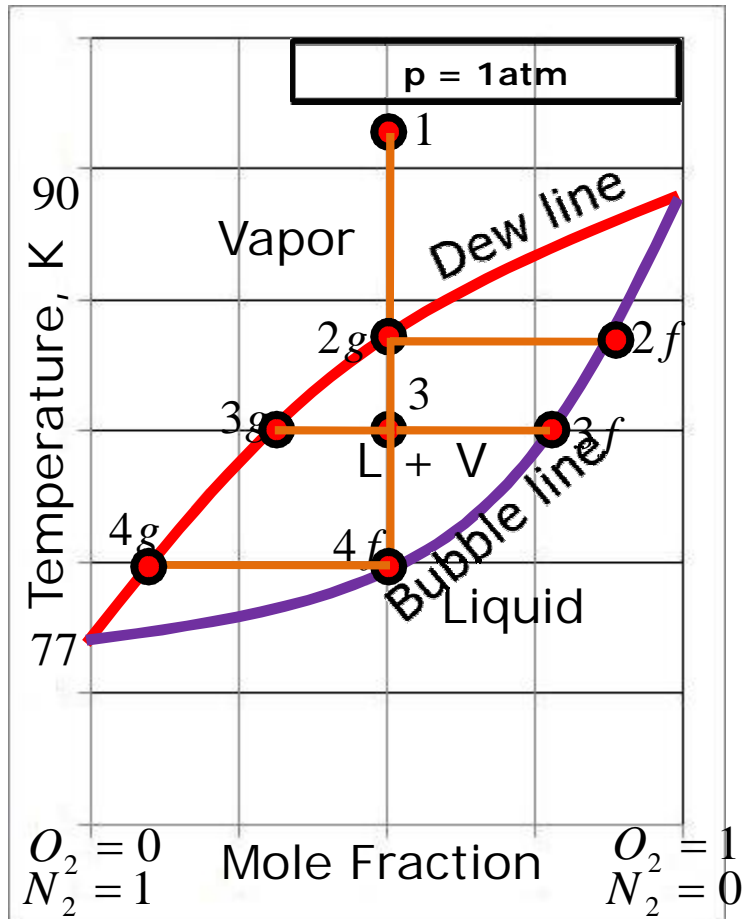
- The mixture is now cooled to a point **3** as shown in the figure.
- Again, extending a constant temperature line to the left and the right about the point **3**, we have the following.

## Temp. Composition Diagrams



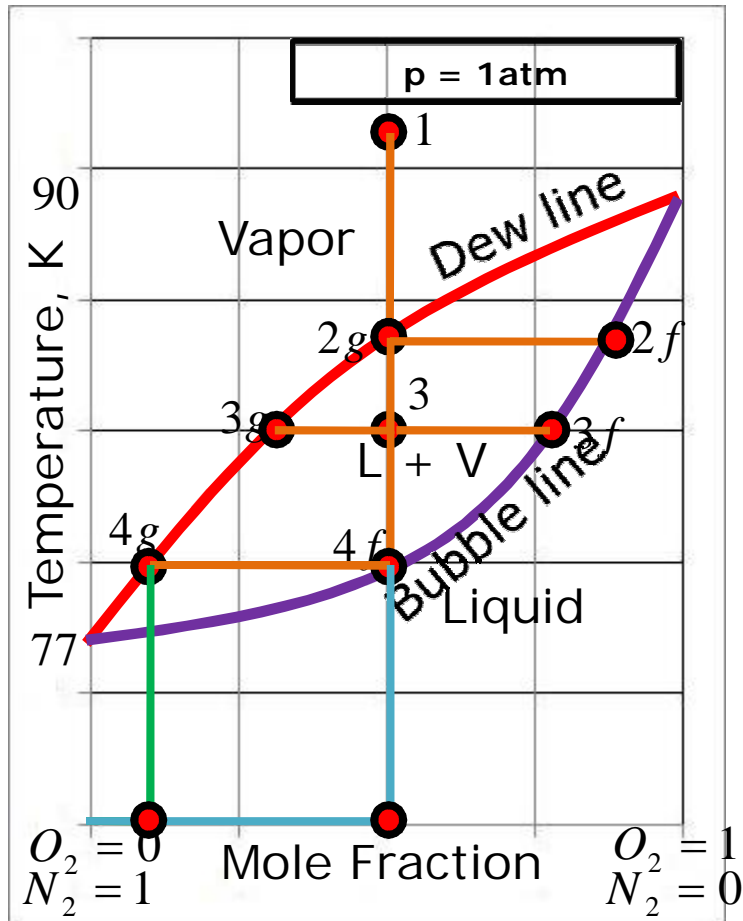
- The line extended to the left side intersects the Dew line at the point **3g**.
- And, the line extended to the right side intersects the bubble line at the point **3f**.
- The compositions of the higher boiling component at point **3** are given by
  - $y_{\text{vap}} = \mathbf{3g}$
  - $y_{\text{liq}} = \mathbf{3f}$

## Temp. Composition Diagrams



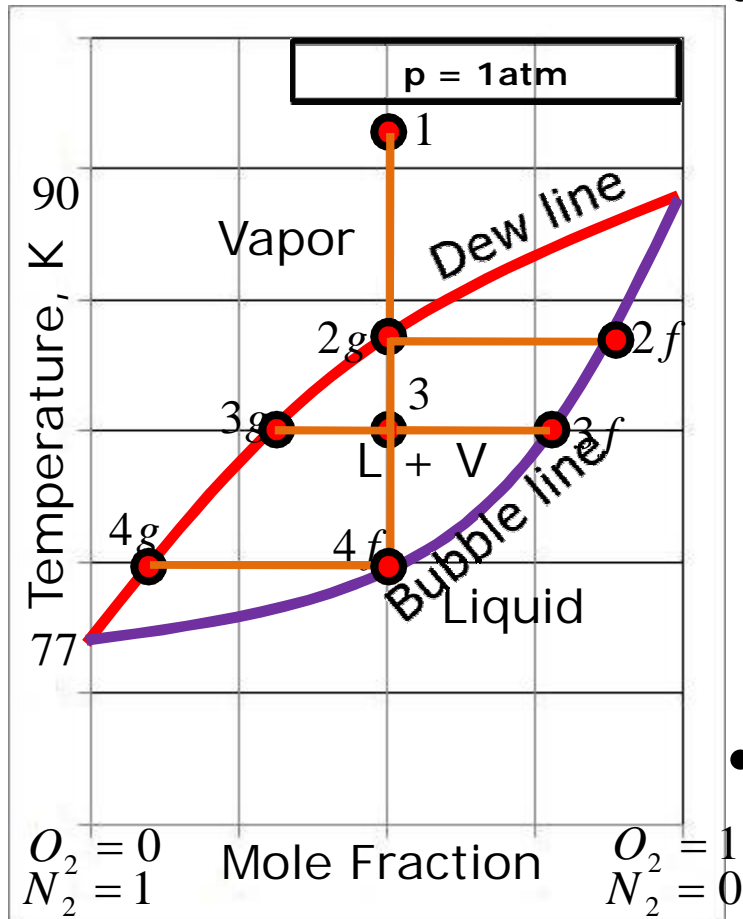
- On further cooling of the mixture, the temperature reaches a point **4f** as shown in the figure.
- At this point, most of the high boiling component of the vapor is condensed.
- Extending a constant temperature line about the point **4f** to left, we have **4g**.

## Temp. Composition Diagrams



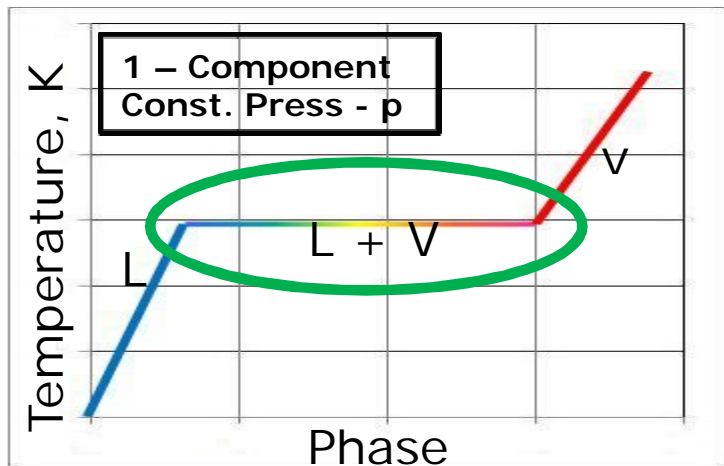
- As mentioned earlier, this curve is called as **Bubble line** because when the liquid mixture is heated, the first bubble or vapor appears on this line.
- The compositions of the higher boiling liquid are given by
  - $y_{\text{vap}} = 4g$
  - $y_{\text{liq}} = 4f$

## Temp. Composition Diagrams



- By cooling of the mixture
  - The % of low BP component has increased
  - The % of high BP component has decreased in vapor and liquid phases.
- The mole fraction of mixture is unchanged, but the mole fractions in vapor and liquid phases have changed.

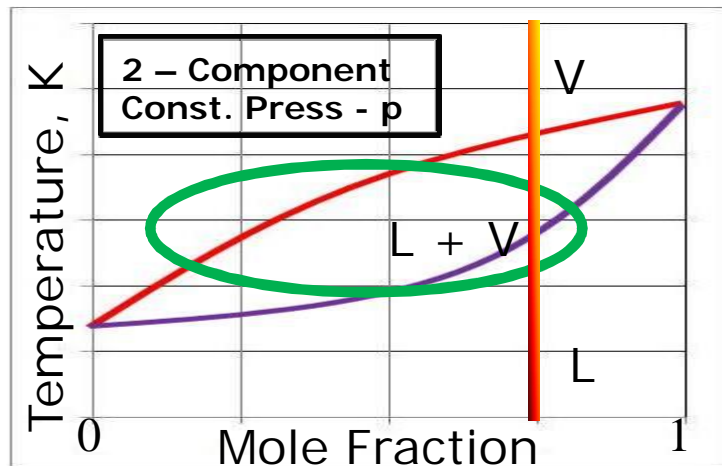
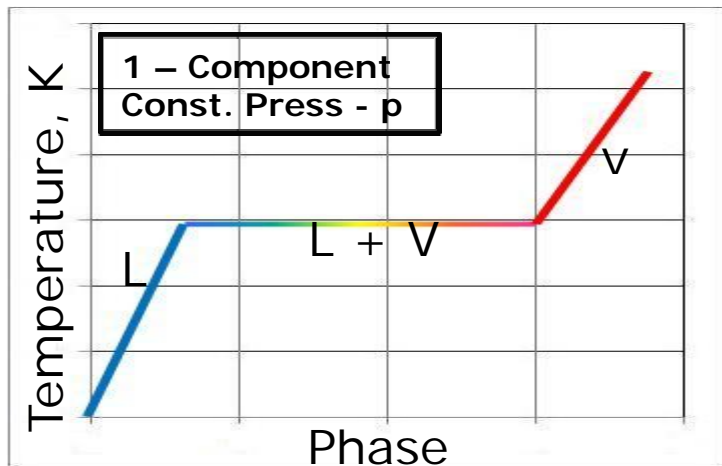
## Temp. Composition Diagrams



- The temperature – phase diagram for a 1 – component system is as shown.
- It is clear that, during the phase change the temperature remains constant.
- It is an isothermal process.

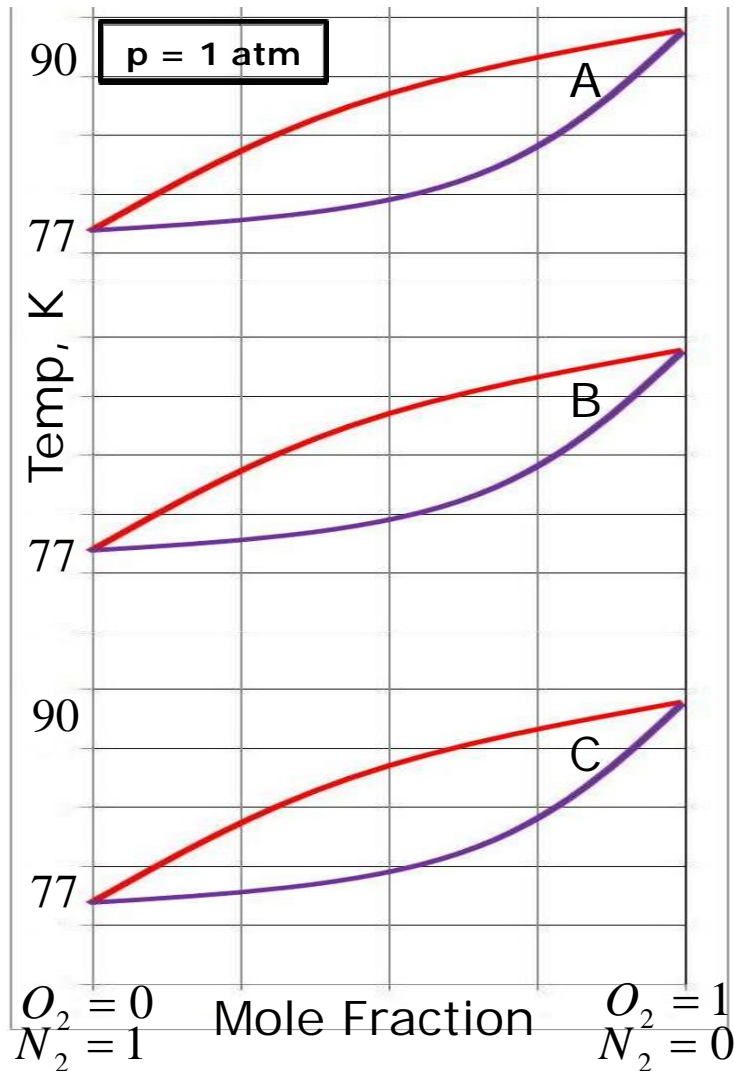


## Temp. Composition Diagrams



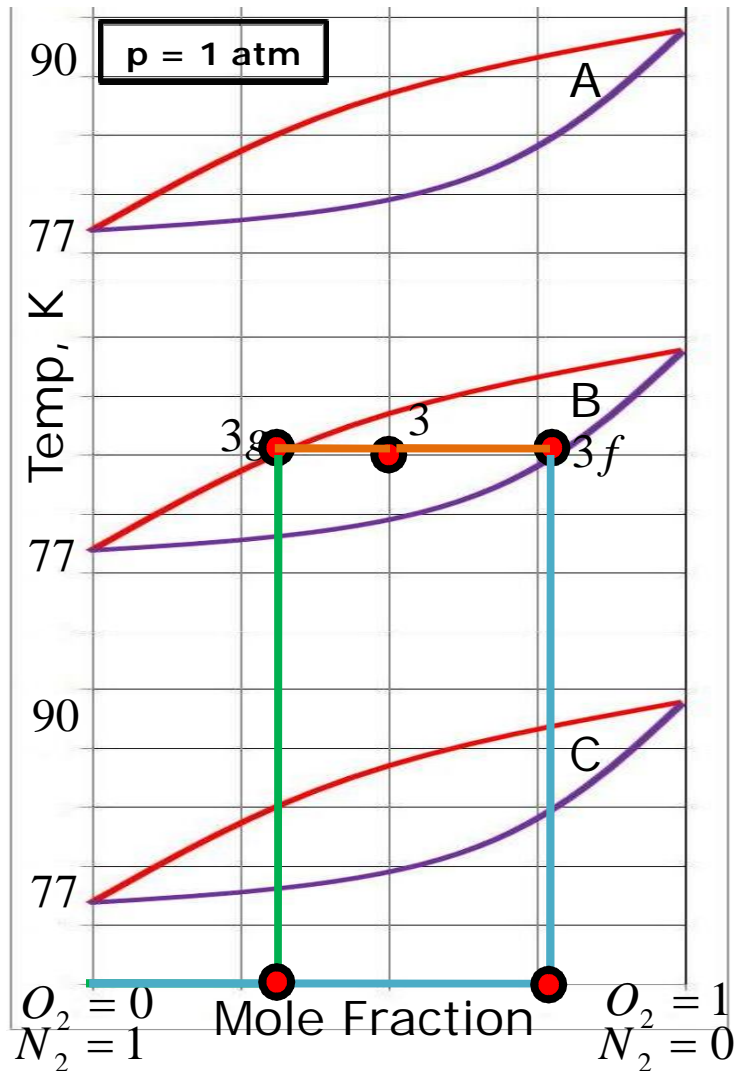
- Similarly, the temperature composition diagram for a 2 – component mixture is as shown.
- There is a change in the temperature when the mixture condenses or boils.
- Therefore, the phase change is a non – isothermal process for mixtures.

## Temp. Composition Diagrams



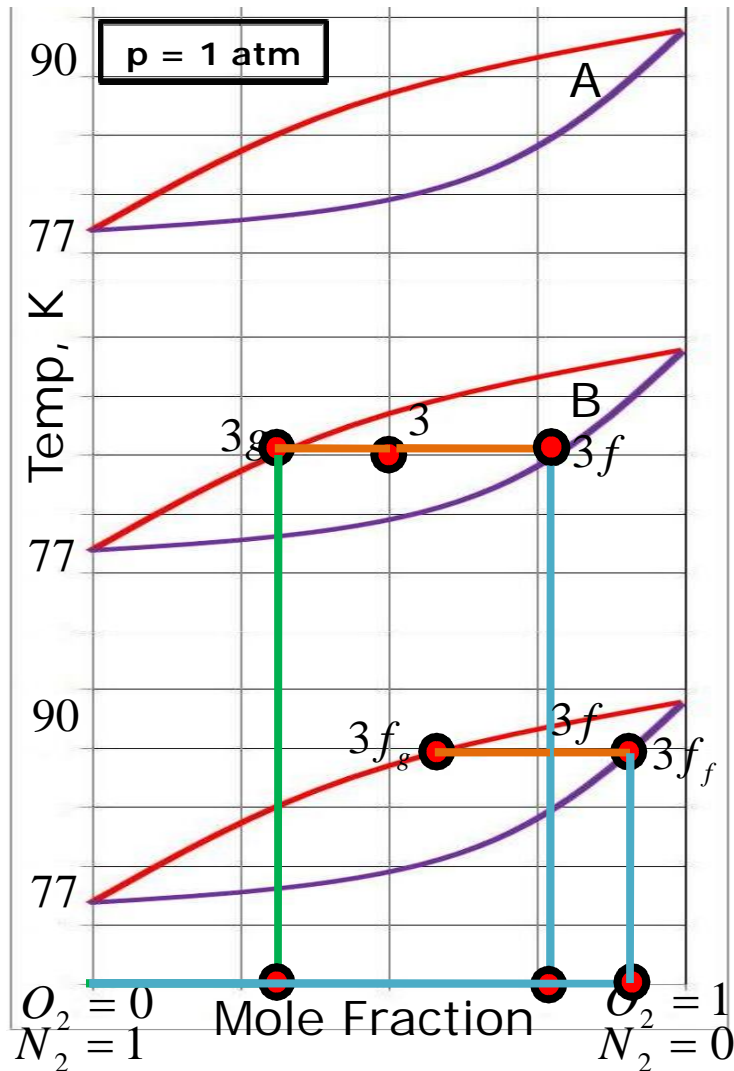
- Mixtures are separated by rectification and this is explained using the adjacent diagram.
- Consider a mixture of  $N_2$  and  $O_2$  at 1 atm.
- The figure has three diagrams **A**, **B** and **C**. They all are same but are placed one over the other for the ease of understanding.

## Temp. Composition Diagrams



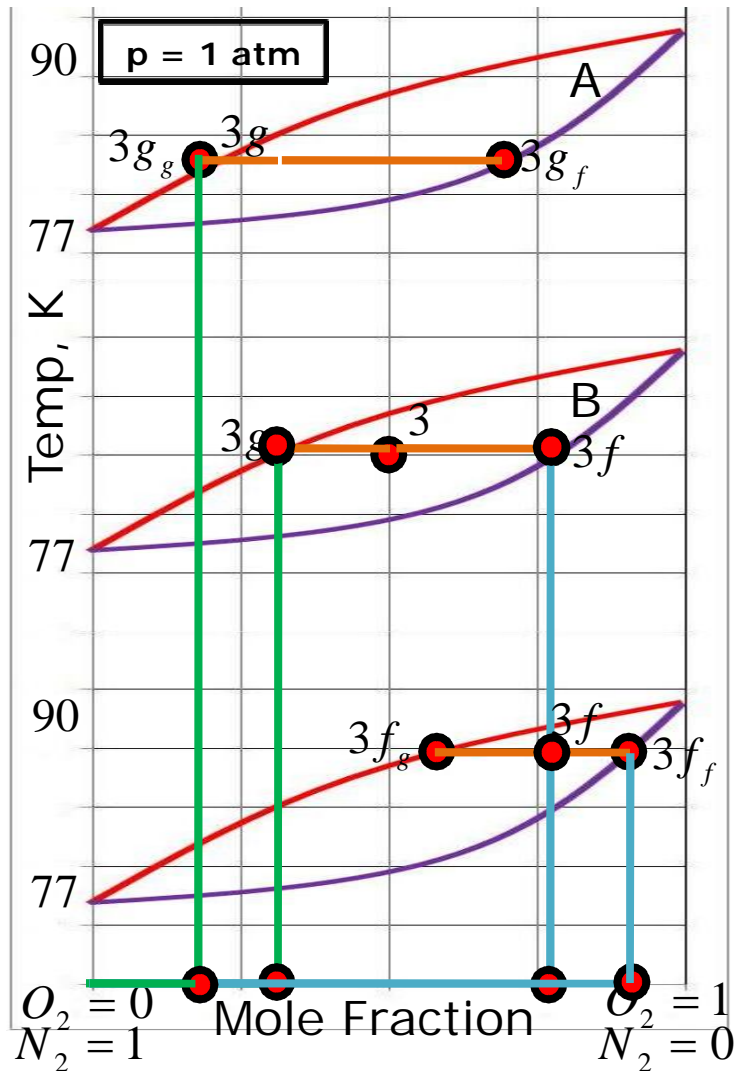
- Let the initial condition of the mixture be at point **3** as shown in the figure.
- The compositions of the higher boiling liquid at point **3** are given by
  - $y_{\text{vap}} = \mathbf{3g}$
  - $y_{\text{liq}} = \mathbf{3f}$

## Temp. Composition Diagrams



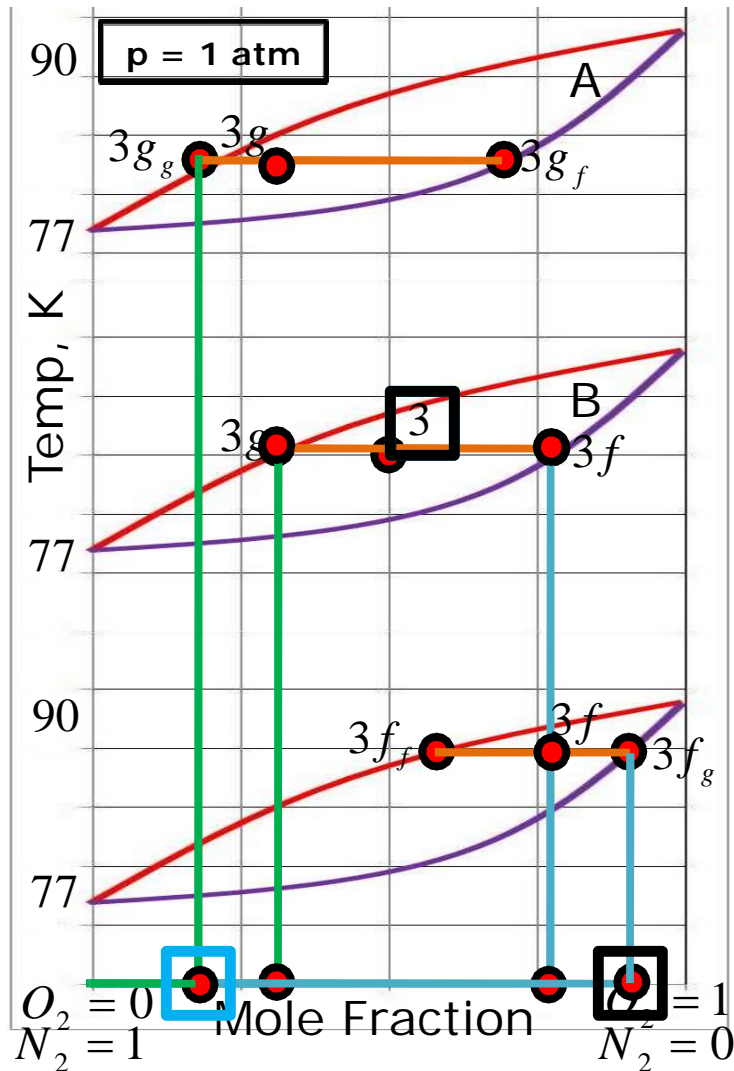
- Now, consider the rectification of mixture with composition at point **3f**.
- Again, extending the constant temperature lines about point **3f**, we have **3f<sub>f</sub>** and **3f<sub>g</sub>** respectively.
- The liquid composition of higher boiling liquid at point **3f<sub>f</sub>** is given by
  - $y_{\text{liq}} = \mathbf{3f_f}$

## Temp. Composition Diagrams



- Consider the rectification of mixture with composition at point  $3g$ .
- Again, extending the constant temperature lines about point  $3g$ , we have  $3g_f$  and  $3g_g$  respectively.
- The vapor composition of lower boiling component at point  $3g_g$  is given by
  - $y_{\text{vap}} = 3g_g$

## Temp. Composition Diagrams



- Thus, the rectification of mixture at point **3**, the vapor is enriched in the lower boiling component (here,  $N_2$ ).
- Similarly, the liquid is enriched in high boiling component (here,  $O_2$ ).
- This process forms the fundamental step for the rectification column.

## Summary

- If number of components, number of phases and degrees of freedom for a mixture in thermal equilibrium are denoted by  $C$  ,  $P$  and  $F$  respectively, then the Gibbs Phase Rule

$$F = C - P + 2$$

- The variation of mole fraction ( $y$ ) with temperature ( $T$ ) at a constant pressure ( $p$ ) is given by **Temperature composition diagram** or **Phase – equilibrium curve**.
- Condensation or boiling of a mixture is a non – isothermal process.

## Summary

- Repeated rectification of a mixture enriches the liquid and vapor phases with high and low boiling components respectively.



- A self assessment exercise is given after this slide.
- Kindly asses yourself for this lecture.

## Self Assessment

1. According to Gibbs Phase Rule \_\_\_\_\_.
2. For a two phase mixture of argon,  $F =$  \_\_\_\_.
3. Mixtures which behave as one substance at certain **T** and **y** are \_\_\_\_\_.
4. Temperature composition diagrams are a strong function of \_\_\_\_\_ forces.
5. Area enclosed by dew and bubble lines is a \_\_\_\_\_ region.
6. Condensation or boiling of mixture is \_\_\_ process.

## Answers

1.  $F = C - P + 2$
2.  $F = 1$
3. Azeotropic mixtures.
4. Inter - molecular
5. Two – phase
6. Non – isothermal

**Thank You!**