CRYOGENIC ENGINEERING

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Lecture No - 23

Earlier Lecture

- Earlier, we have studied the Temperature composition diagrams, the Enthalpy composition diagrams and their importance in Gas separation.
- The separation of a mixture is more effective when the difference in the boiling points is more.
- In this column, Low and High Boiling components are collected at top and bottom respectively.
- **Murphree efficiency** is the ratio of actual change in mole fraction to the maximum possible change that can occur.

Outline of the Lecture

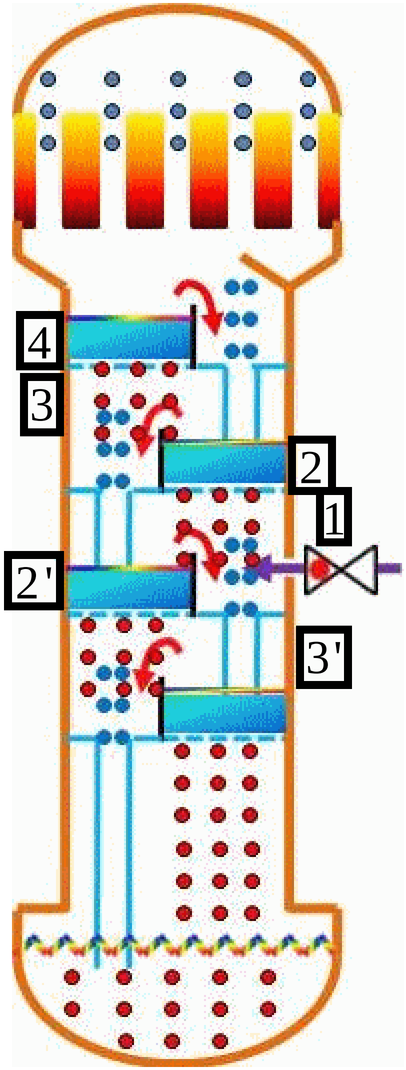
Topic : Gas Separation (contd)

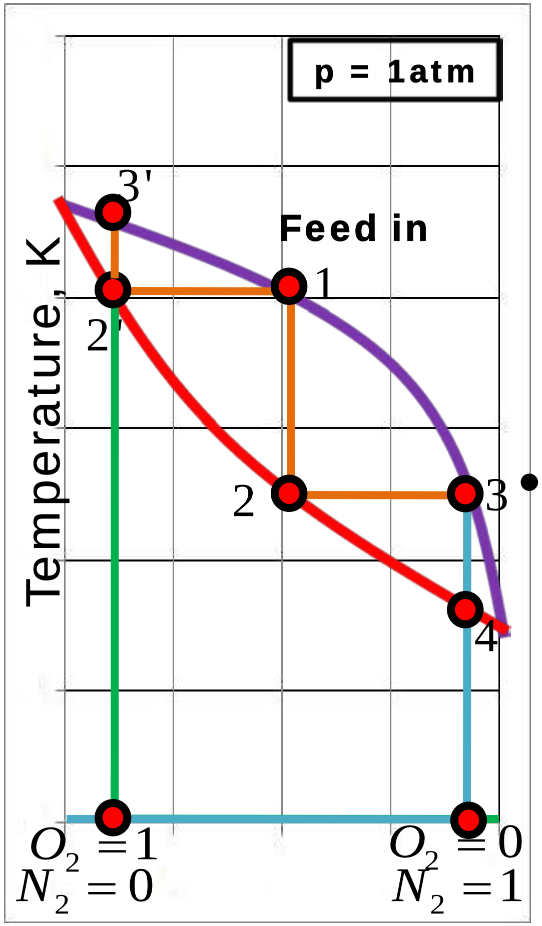
- Understanding of Rectification Column using an Animation
- Theoretical Plate Calculations

Rectification Column

- Animation

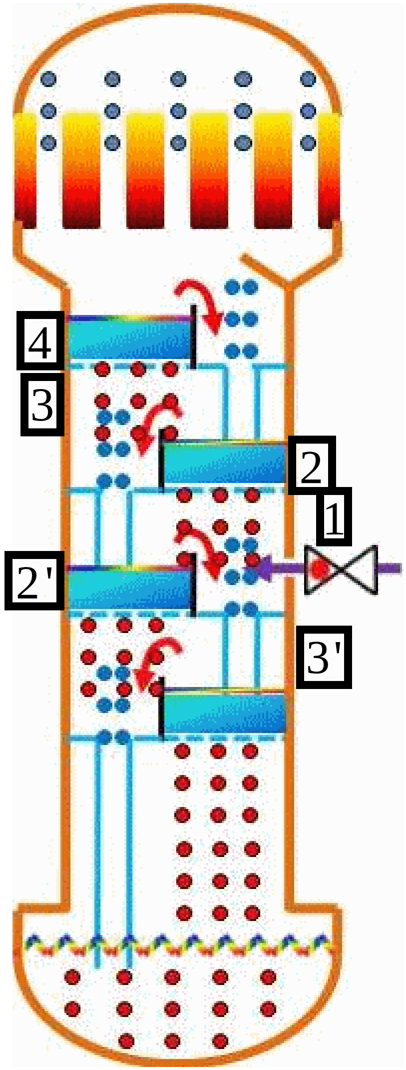
Rectification Column

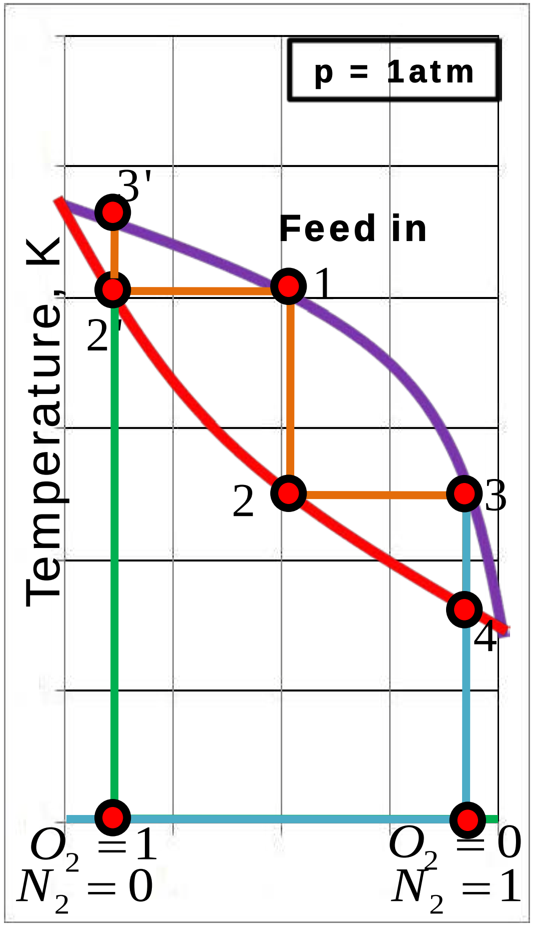




- As seen earlier in a rectification column, the liquid moving down is enriched in high boiling point component (**O_2**).
- On the other hand, the vapor moving up is enriched in low boiling point component (**N_2**).

Rectification Column





- For getting 100% pure products, infinite number of rectification processes – plates, would be required.
- But in reality, the size and the cost of the column limit the number of rectification processes and hence the purity.

Rectification Column

- In the past, researchers have developed various mathematical procedures to calculate the required number of rectification processes – plates, to obtain a desired purity.

- These procedures require the following data.
  - Number of components
  - Phase diagrams of the mixtures
  - Property data of mixture
  - Heat transfer correlations

Theoretical Plate Calculations

- The methods of calculation that are used for theoretical plate calculations are
  - Method of Ponchon and Savarit.
  - Method of McCabe and Thiele.
  - Numerical Methods.

Theoretical Plate Calculations

- **Ponchon – Savarit** method is an exact method for plate calculations.

- It is applicable to any number of components and this method requires a detailed data of enthalpy composition diagram(s) of the mixture.

Theoretical Plate Calculations

- **McCabe – Thiele** method was proposed by two American scientists, Warren McCabe and Ernest Thiele, in the year 1925.

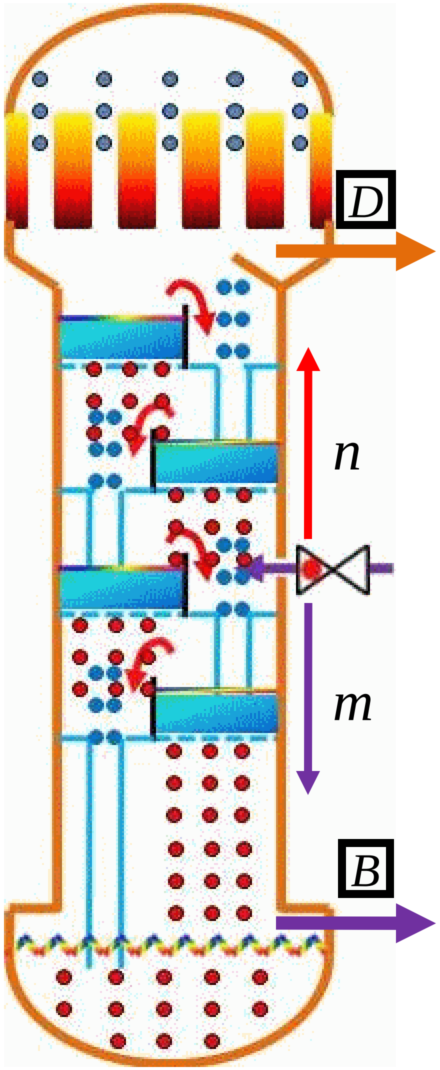
- This method is less general and is the simplest technique. It is widely used for binary mixtures at cryogenic temperatures.

Theoretical Plate Calculations

- **Numerical** methods are the latest techniques, which are tedious, time consuming and computer intensive methods.

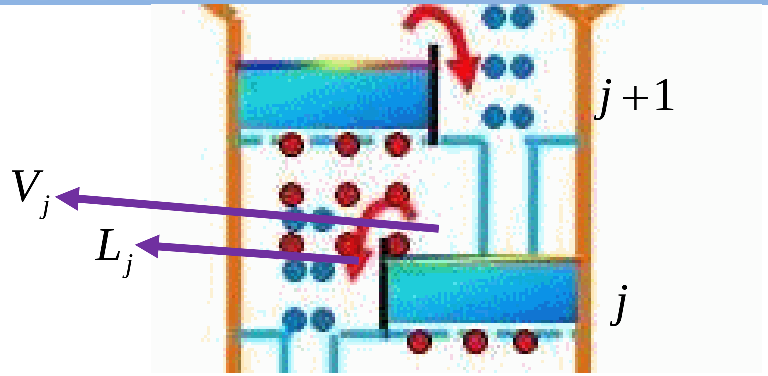
- For the sake of understanding and simplicity, only **McCabe – Thiele** method will be explained in this topic.

McCabe – Thiele Method



- This method calculates liquid and vapor fractions of each component at every plate and also the number of plates.
- For the sake of understanding, let the plates above the feed be denoted by subscript **n**.
- Similarly, the plates below the feed be denoted by subscript **m**.
- Let the total mole flow rate of top and bottom product be **D** and **B** respectively

McCabe – Thiele Method

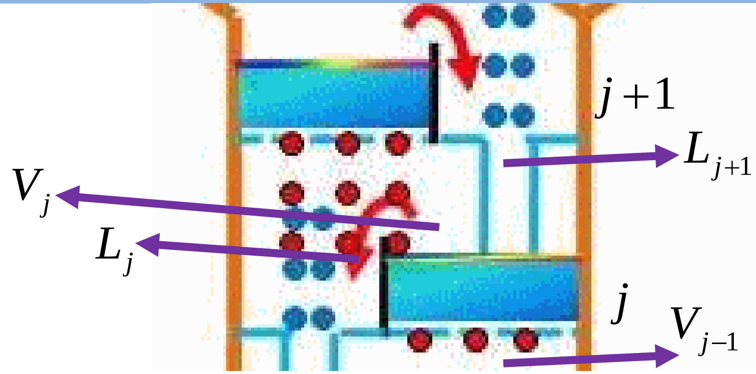


- It is important to understand the indexing pattern of the plate and its corresponding liquid and vapor.

- Let **j^th^** and **(j+1)^th^** plate be any intermediate plate as shown in the figure.

- The **liquid** and **vapor** leaving from top of the **j^th^** plate are **L_j** and **V_j** respectively.

McCabe – Thiele Method

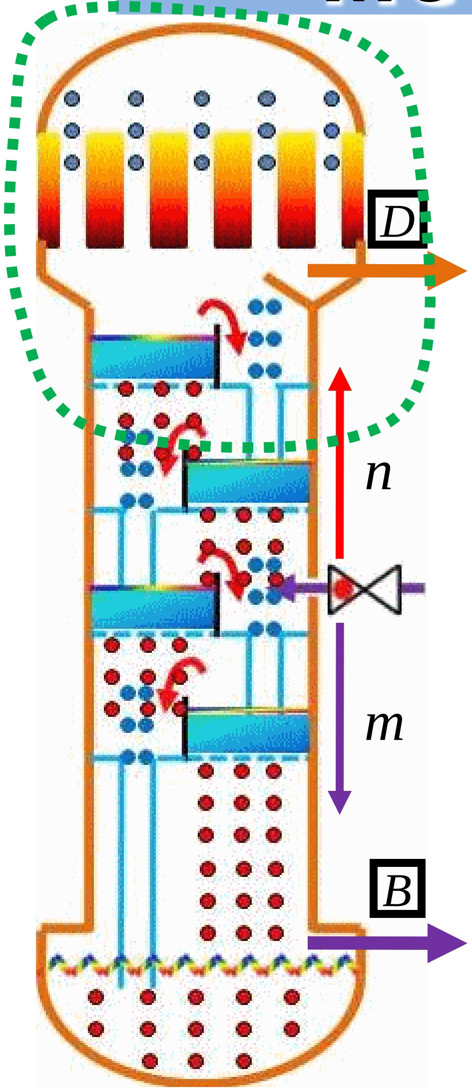


- Similarly, the liquid coming to the **j^th^** plate is from **(j+1)^th^** plate, therefore it is $\mathbf{L_{j+1}}$.

- Also, the vapor coming to **j^th^** plate from bottom is vapor leaving the **(j-1)^th^** plate. It is therefore, $\mathbf{V_{j-1}}$.

- The vapor and liquid on any plate, $\mathbf{L_j}$ and $\mathbf{V_j}$, are in thermal equilibrium.

McCabe – Thiele Method



- Consider a control volume enclosing the condenser and the top section of the **n**[**th**] plate as shown in the figure.

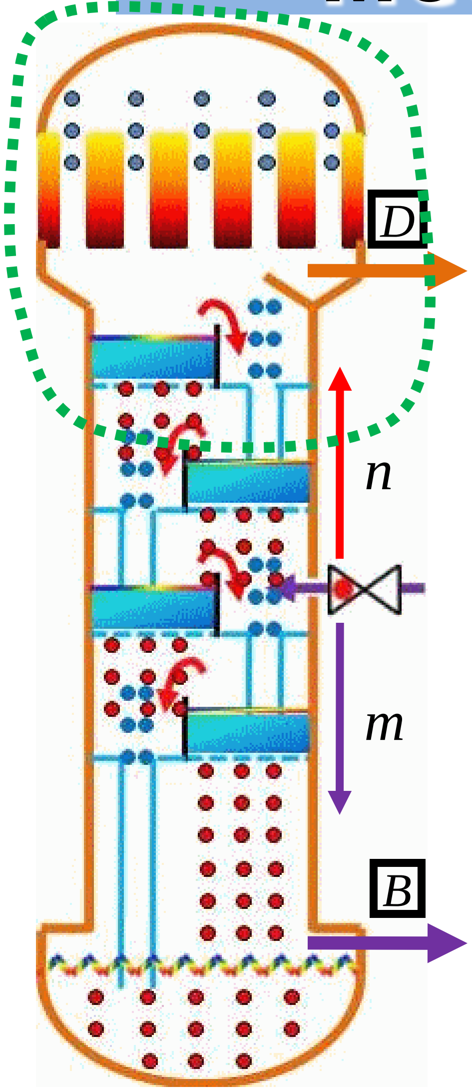
- As explained earlier, for this **n**[**th**] plate, the vapor leaving is $\mathbf{V_n}$ and the liquid added is $\mathbf{L_{n+1}}$.

- Applying the mole balance across the control volume per unit time, we have

IN	OUT
V_n	L_{n+1}
	D

$$V_n = L_{n+1} + D$$

McCabe – Thiele Method



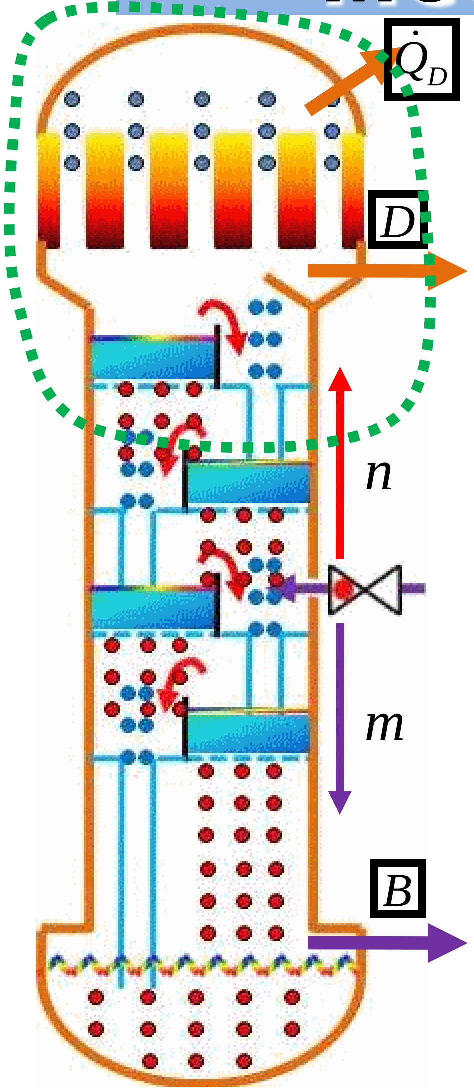
- Multiplying the mole balance equation with mole fraction of a particular component in a mixture, we get mole balance for that component as

$$y_n V_n = x_{n+1} L_{n+1} + x_D D$$

- Where,
  - $\mathbf{y_n}$, $\mathbf{x_{n+1}}$ and $\mathbf{x_D}$ are mole fractions of a particular component in vapor, liquid and top product respectively.

  - It automatically means that $\mathbf{x_D}$ (mole fraction) is the desired purity of the top product.

McCabe – Thiele Method



- For control volume taking into account $\mathbf{Q_D}$ (watts) as the heat rejected by the condenser, the enthalpy balance is given by

$$H_n V_n = h_{n+1} L_{n+1} + h_D D + \dot{Q}_D$$

- Dividing the above equation by **D**, we have

$$\frac{H_n V_n}{D} = h_{n+1} \frac{L_{n+1}}{D} + h_D + \frac{\dot{Q}_D}{D}$$

- Rearranging the total mole balance equation, we have

$$L_{n+1} = V_n - D$$

$$\frac{L_{n+1}}{D} = \frac{V_n}{D} - 1$$

McCabe – Thiele Method

- Eliminating $\mathbf{L_{n+1}/D}$ from the earlier equations, we get

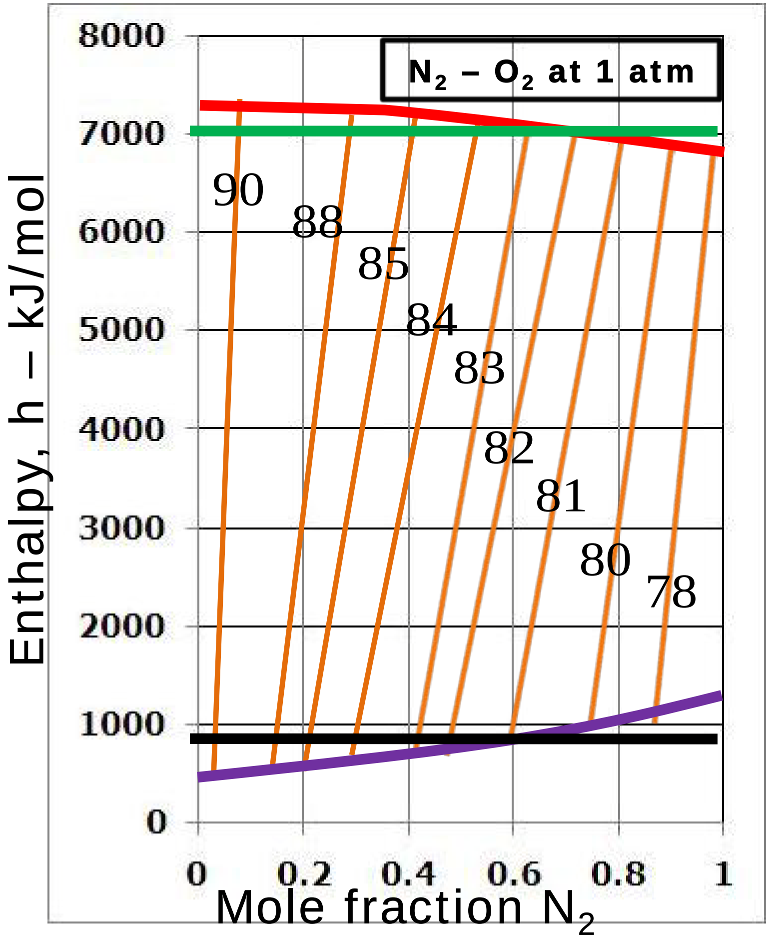
$$\frac{H_n V_n}{D} = h_{n+1}\left(\frac{V_n}{D} - 1\right) + h_D + \frac{\dot{Q}_D}{D}$$

$$\left(H_n - h_{n+1}\right)\frac{V_n}{D} = \frac{\dot{Q}_D}{D} + h_D - h_{n+1}$$

- Rearranging as a ratio of **D** and $\mathbf{V_n}$, we have

$$\frac{D}{V_n} = \frac{H_n - h_{n+1}}{\frac{\dot{Q}_D}{D} + h_D - h_{n+1}}$$

McCabe – Thiele Method



- The enthalpy composition diagram for a mixture of $\mathbf{N_2}$ and $\mathbf{O_2}$ is as shown.

- If we neglect the enthalpy variation with the mole fraction, the bubble and dew lines can be taken as horizontal.

McCabe – Thiele Method

$$\frac{D}{V_n} = \frac{H_n - h_{n+1}}{\frac{\dot{Q}_D}{D} + h_D - h_{n+1}} \qquad \frac{L_{n+1}}{V_n} = 1 - \frac{D}{V_n}$$

- These arguments lead to the fact that liquid **(h)** and vapor **(H)** enthalpies are constant. Hence, $\mathbf{D/V_n}$ and $\mathbf{L_{n+1}/V_n}$ are constant.

- Rearranging the molar balance for a component as

$$y_n = \left(\frac{L_{n+1}}{V_n}\right) x_{n+1} + \left(\frac{D}{V_n}\right) x_D$$

- The above equation represents a straight line and is called as **Operating Line** for stripping section.

McCabe – Thiele Method

$$y_n = \left(\frac{L_{n+1}}{V_n}\right) x_{n+1} + \left(\frac{D}{V_n}\right) x_D$$

- For the top or upper most plate near the condenser, $\mathbf{x_{n+1} = x_D}$.
- Substituting,

$$y_n = \left(\frac{L_{n+1}}{V_n}\right) x_D + \left(\frac{D}{V_n}\right) x_D$$

$$y_n = \left(\frac{L_{n+1}}{V_n} + \frac{D}{V_n}\right) x_D$$

1

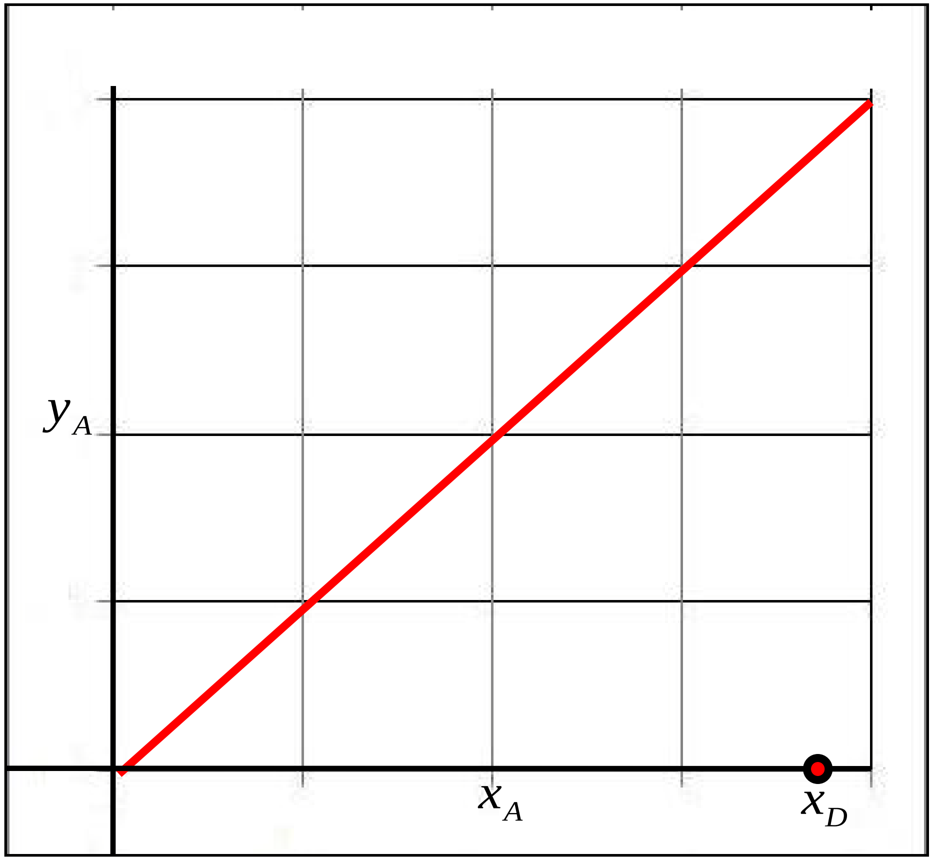
$$y_n = x_D$$

- For y – intercept, $\mathbf{x_{n+1} = 0}$.

$$y_n = \left(\frac{D}{V_n}\right) x_D$$

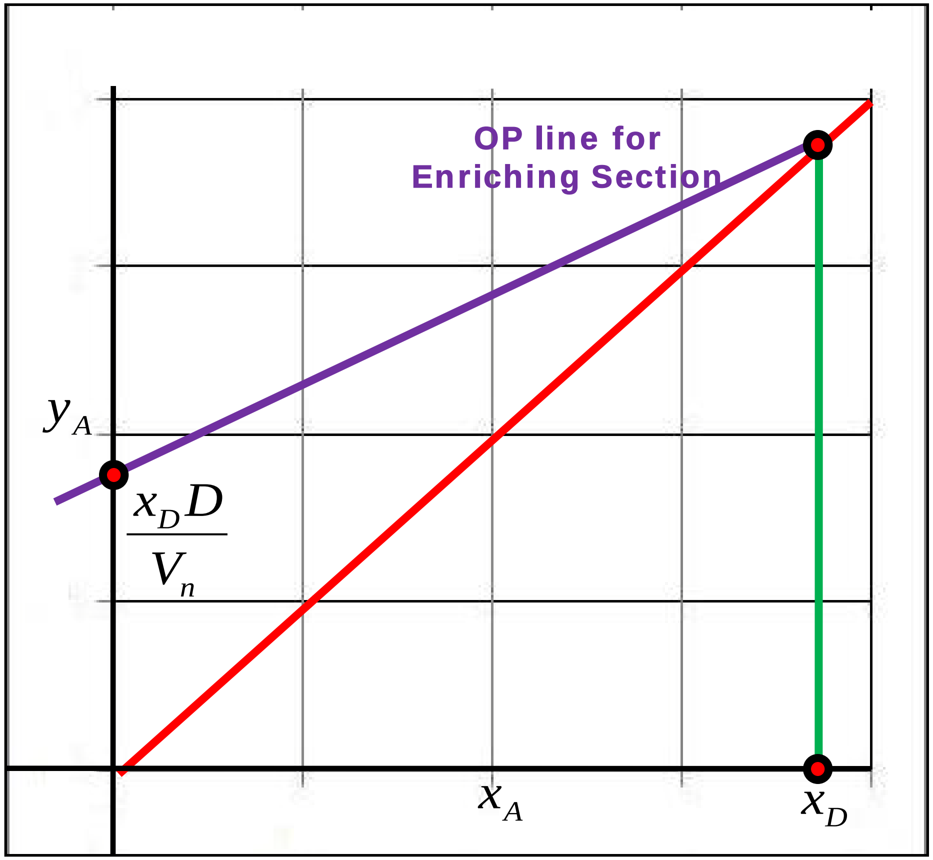
Two Points
$y_n = x_D$ @ $x_{n+1} = x_D$
$y_n = (D/V_n) x_D$ @ $x_{n+1} = 0$

McCabe – Thiele Method



- A plot of vapor versus liquid mole fractions for a particular component, say **A**, is as shown in the figure.
- Let **45°** diagonal or **y=x** line be as shown.
- The desired purity of this component **A**, in the top product is $\mathbf{x_D}$ as shown in the figure.

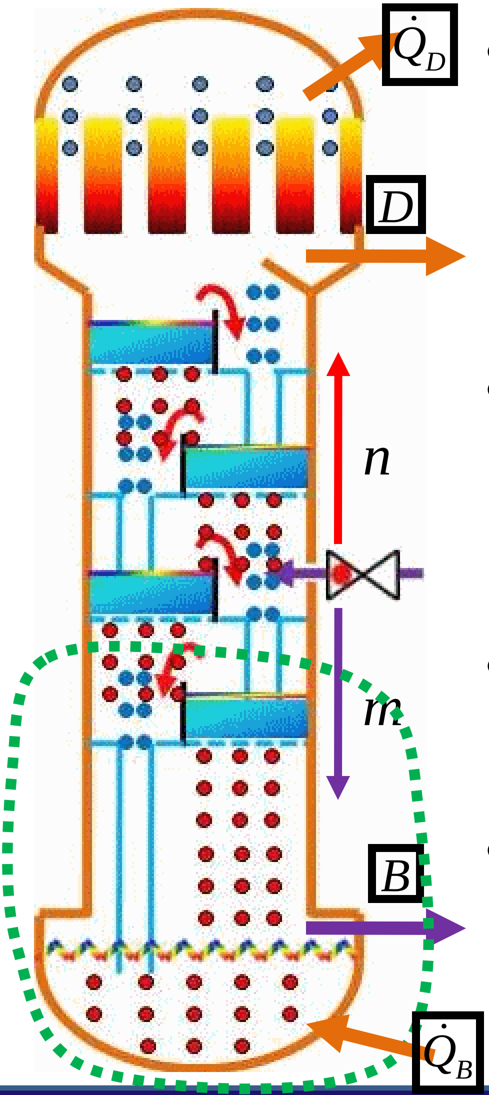
McCabe – Thiele Method



$$y_n = \left(\frac{L_{n+1}}{V_n}\right) x_{n+1} + \left(\frac{D}{V_n}\right) x_D$$

- The y – intercept of the straight line is $\mathbf{(D / V_n) x_D}$.
- Similarly, the slope of the operating line is given by $\mathbf{L_{n+1} / V_n}$, as shown in the above equation.

McCabe – Thiele Method



- Similarly, for the analysis of **m^th** plate and boiler in the lower part, we have the following equations.

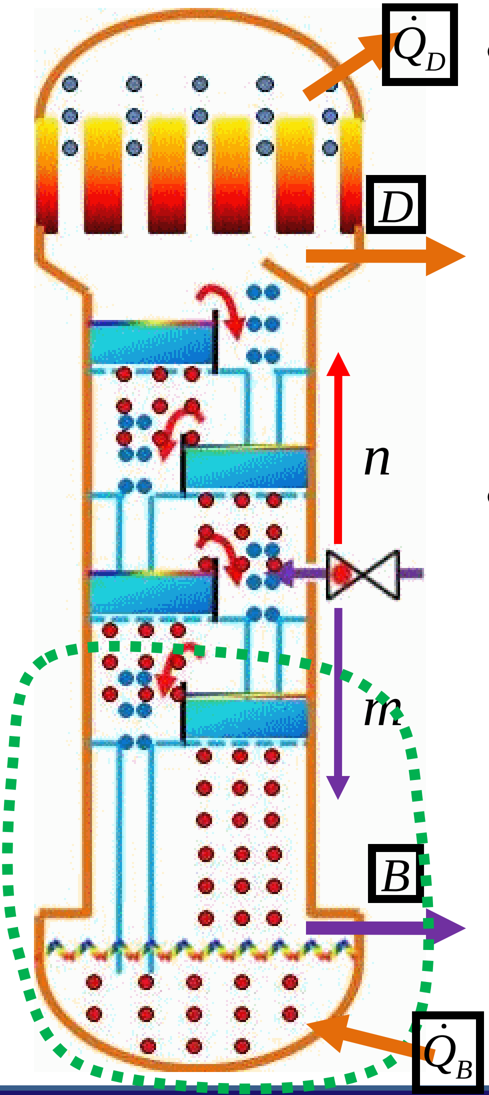
- Mole Balance: $L_{m+1} = V_m + B$

$$x_{m+1}L_{m+1} = y_m V_m + x_B B$$

- Energy Balance: $h_{m+1}L_{m+1} + \dot{Q}_B = H_m V_m + h_B B$

- where, **B** and **Q_B** are mole flow rate out at the bottom and heat input to the boiler respectively.

McCabe – Thiele Method



- Rearranging the above equations, we have the following.

$$\frac{B}{V_m} = \frac{H_m - h_{m+1}}{\frac{\dot{Q}_m}{B} - h_B + h_{m+1}}$$

$$\frac{L_{m+1}}{V_m} = 1 + \frac{B}{V_m}$$

- Applying the assumption, we have $\mathbf{H_m}$ and $\mathbf{h_{m+1}}$ as constant, implies $\mathbf{B/V_m}$ and $\mathbf{L_{m+1}/V_m}$ are constant. The operating line for stripping section is

$$y_m = \left(\frac{L_{m+1}}{V_m}\right) x_{m+1} - \left(\frac{B}{V_m}\right) x_B$$

McCabe – Thiele Method

$$y_m = \left(\frac{L_{m+1}}{V_m}\right)x_{m+1} - \left(\frac{B}{V_m}\right)x_B$$

- For the bottom or lower most plate near the boiler, $\mathbf{x_{m+1} = x_B}$.

- Substituting,

$$y_m = \left(\frac{L_{m+1}}{V_m}\right)x_B - \left(\frac{B}{V_m}\right)x_B$$

$$y_m = \left(\frac{L_{m+1}}{V_m} - \frac{B}{V_m}\right)x_B$$

1

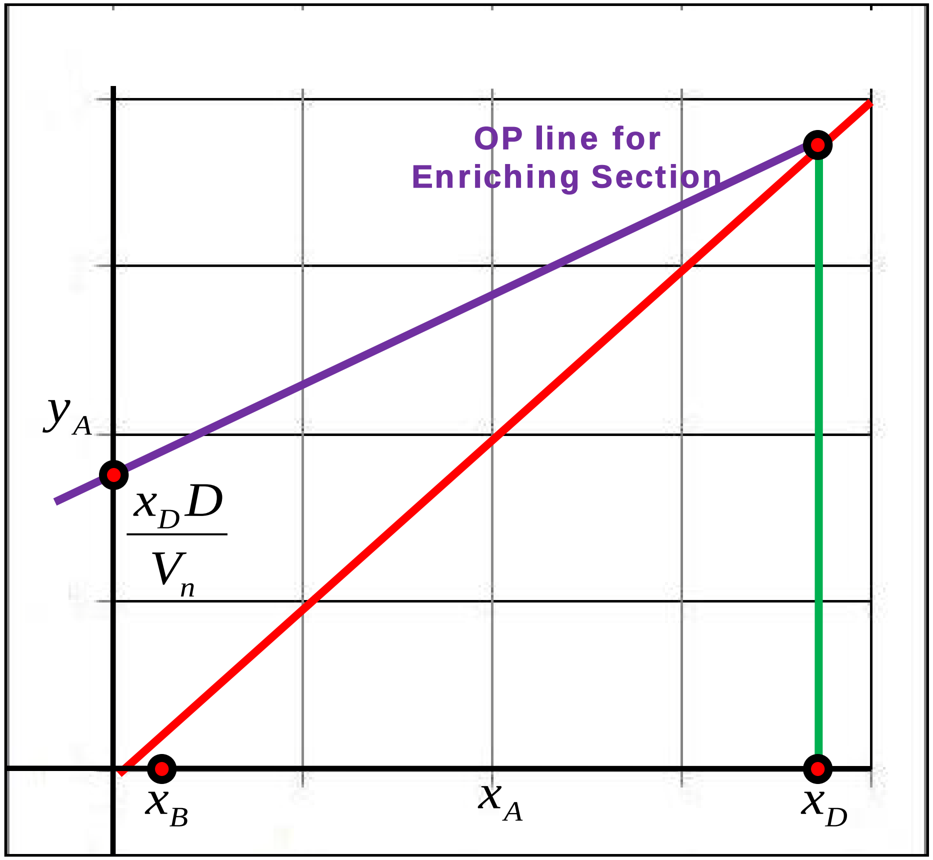
$$y_m = x_B$$

- For y – intercept, $\mathbf{x_{m+1} = 0}$.

$$y_m = -\left(\frac{B}{V_m}\right)x_B$$

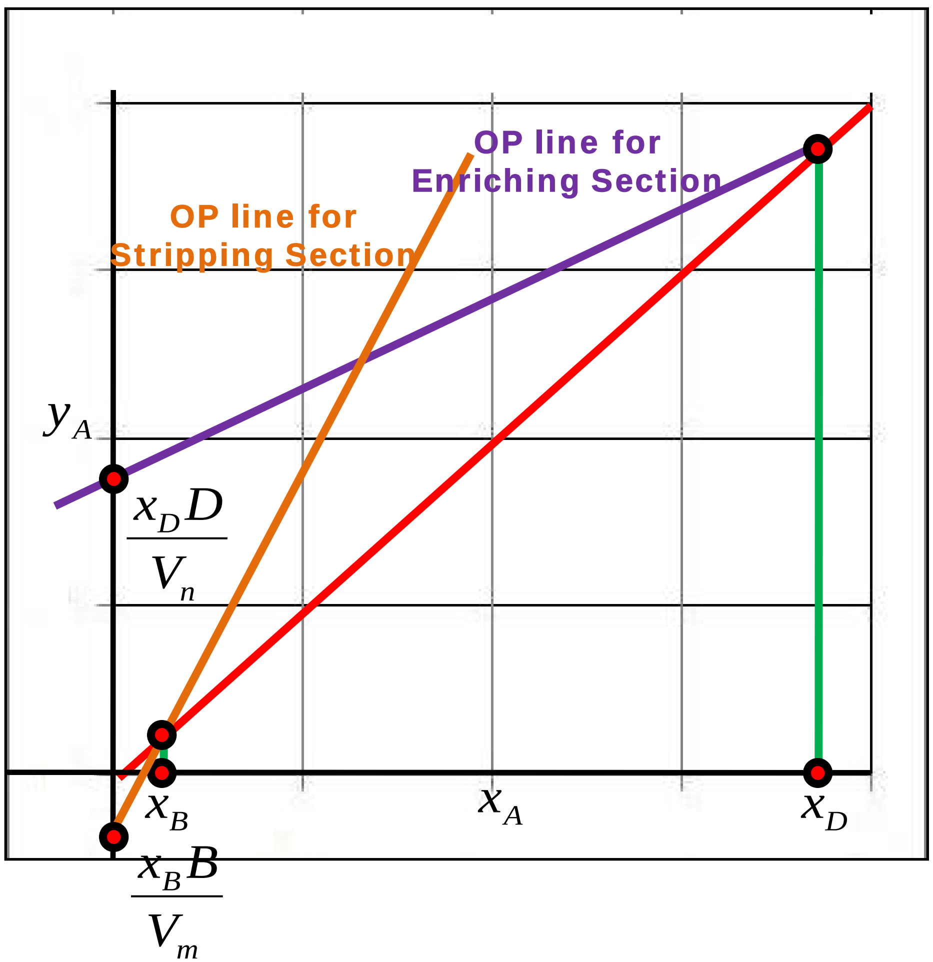
Two Points
$y_m = x_B$ @ $x_{m+1} = x_B$
$y_m = -(B/V_m)x_B$ @ $x_{m+1} = 0$

McCabe – Thiele Method



- The plot of vapor versus liquid mole fractions for a component **A** with operating line and **45°** diagonal be as shown.
- The purity of component **A** in the bottom product is $\mathbf{x_B}$ as shown in the figure.

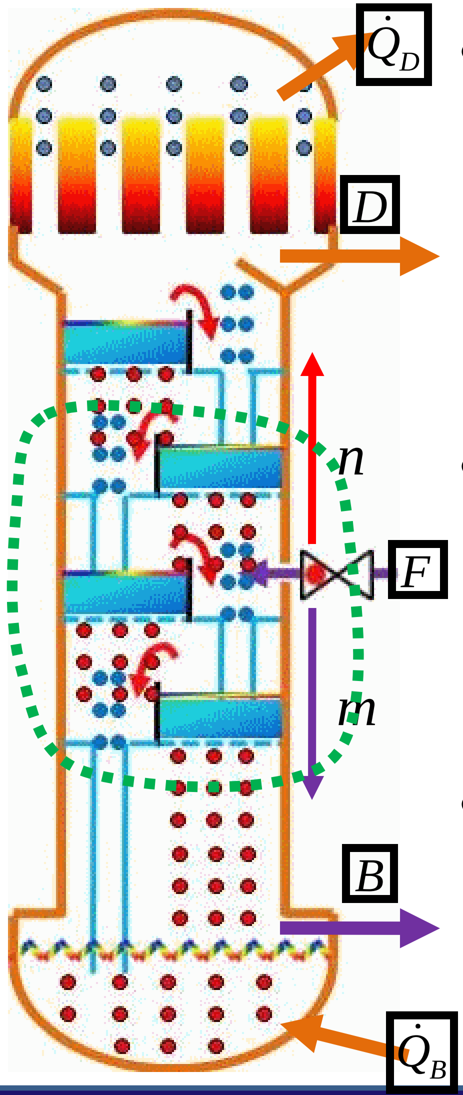
McCabe – Thiele Method



$$y_m = \left(\frac{L_{m+1}}{V_m}\right) x_{m+1} - \left(\frac{B}{V_m}\right) x_B$$

- The y – intercept of the straight line is **$(-(B/V_m)x_B)$**.
- The slope of the operating line is given by **L_{m+1}/V_m** as shown in the above equation.

McCabe – Thiele Method

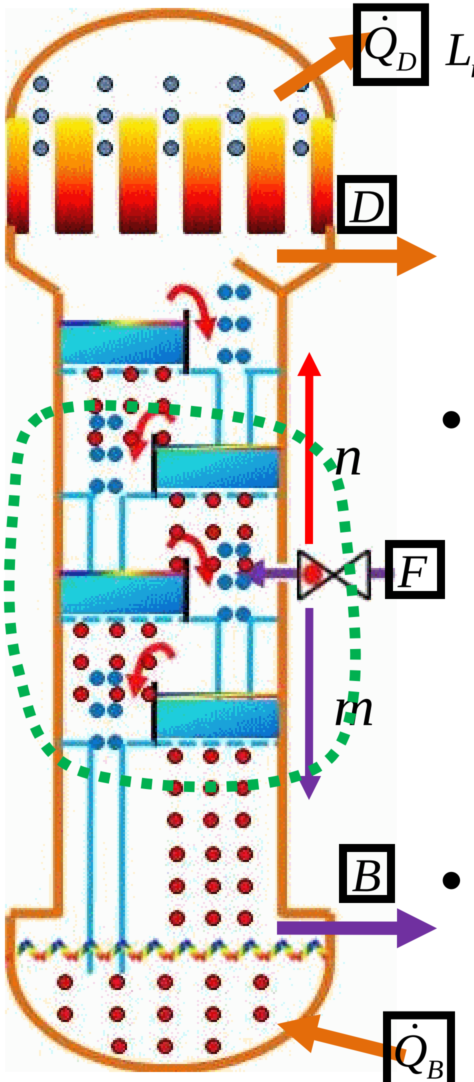


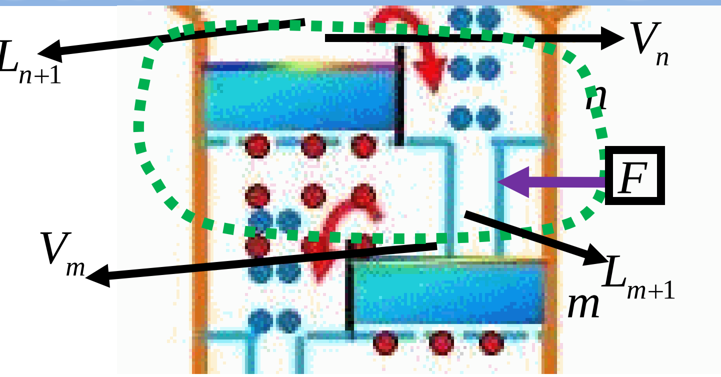
- The mixture that is to be separated is called as Feed. It is introduced into the column through an opening called as Feed inlet as shown in the figure.

- Consider a control volume enclosing the **n**th and **m**th plates and feed inlet as shown.

- Let **F** be the total number of moles in the Feed.

McCabe – Thiele Method





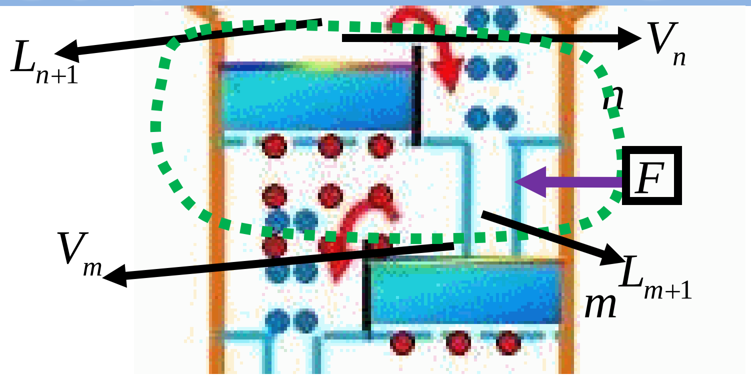
- For the above control volume, we have

IN	OUT
F	
V_m	V_n
L_{n+1}	L_{m+1}

- Applying the molar balance

$$F = V_n - V_m + L_{m+1} - L_{n+1}$$

McCabe – Thiele Method

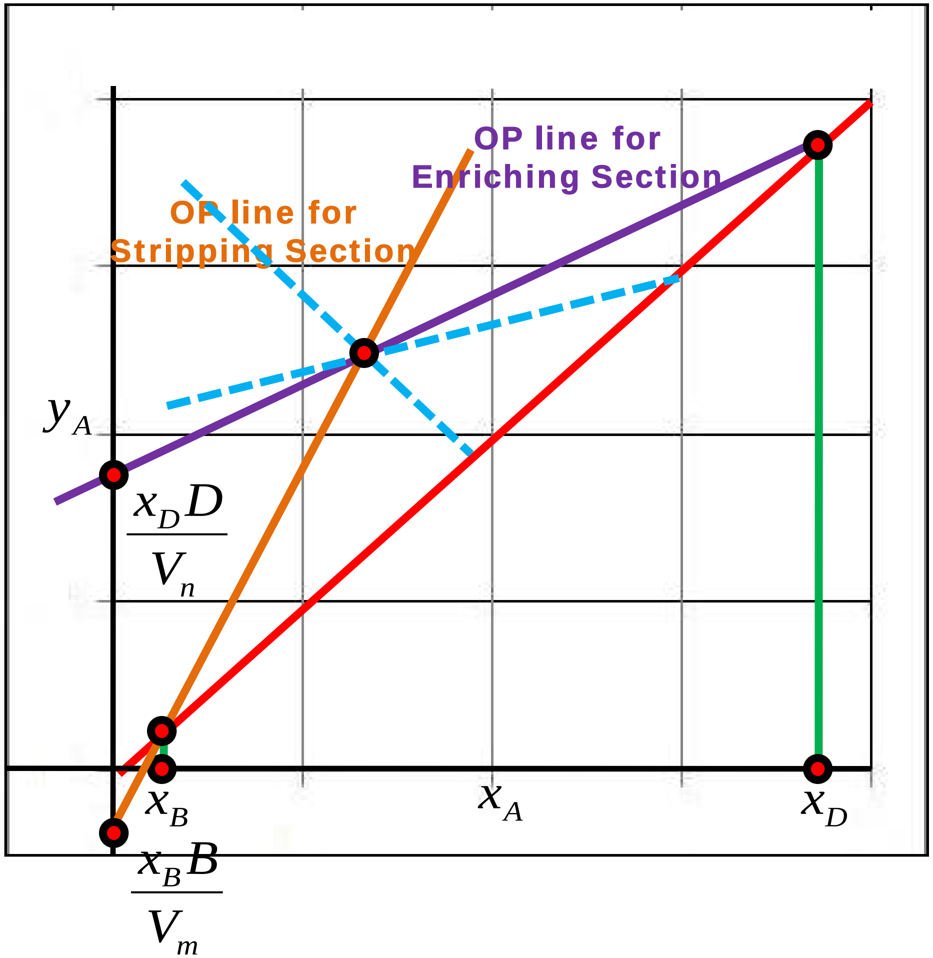


- We define a parameter **q** as the ratio of liquid moles in the feed to the total number moles in the feed.

- Mathematically, $$q = \frac{(L_{m+1} - L_{n+1})}{F}$$

- That is for **q=0**, feed is totally vapor and for **q=1**, it is totally liquid.

McCabe – Thiele Method



- From the earlier slides, we know the equations for both the sections.
- The locus of intersection of these operating lines denotes the feed condition.
- The condition of the feed is vital to determine the number of plates.

McCabe – Thiele Method

- Based on feed equation and **q** definition, we have

$$F = V_n - V_m + L_{m+1} - L_{n+1}$$

$$q = \frac{\left(L_{m+1} - L_{n+1}\right)}{F}$$

$$V_n - V_m = \left(1 - q\right)F$$

- Again, from the operating lines of upper and lower sections, we can rearrange to give $\mathbf{V_n}$ and $\mathbf{V_m}$ as

$$V_n = \left(\frac{L_{n+1}}{y_n}\right)x_{n+1} + \left(\frac{D}{y_n}\right)x_D$$

$$V_m = \left(\frac{L_{m+1}}{y_m}\right)x_{m+1} - \left(\frac{B}{y_m}\right)x_B$$

- It is important to note that $\mathbf{V_n - V_m}$ is the vapor content in the feed.

McCabe – Thiele Method

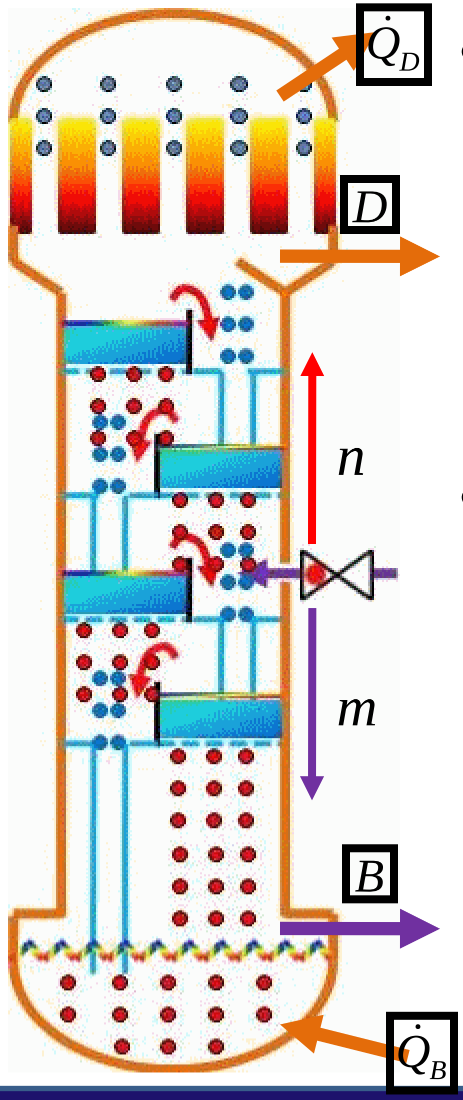
- In the calculation of point of intersection of operating lines, we choose a common point to both these lines as **(x,y)**.

- Hence, $\mathbf{x_{n+1}}$, $\mathbf{x_{m+1}}$, $\mathbf{y_m}$ and $\mathbf{y_n}$ are replaced with this point as shown in the following equation.

$$V_n - V_m = \frac{\left(L_{n+1} - L_{m+1}\right)x}{y} + \frac{\left(x_D D + x_B B\right)}{y} = \left(1-q\right)F$$

- The locus of this point of intersection is the feed line or **q** line and is calculated as explained in the next slide.

McCabe – Thiele Method



- For a column as a whole, using the mass balance, we can write

$$x_F F = x_D D + x_B B$$

$$q = \frac{\left(L_{m+1} - L_{n+1}\right)}{F}$$

- Rearranging the following equations, we have

$$\frac{\left(L_{n+1} - L_{m+1}\right)x}{y} + \frac{\left(x_D D + x_B B\right)}{y} = \left(1-q\right)F$$

$$-qF\frac{x}{y} + \frac{x_F F}{y} = \left(1-q\right)F$$

McCabe – Thiele Method

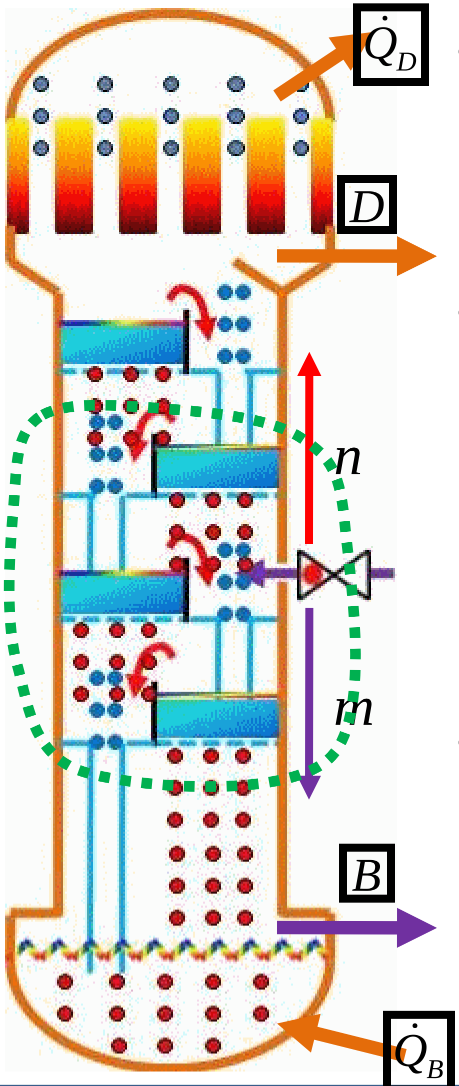
- Rearranging,

$$y = \left(\frac{q}{q-1}\right)x + \frac{x_F}{1-q}$$

- The above equation represents a straight line with **q / (q-1)** and **x_F / (1-q)** as slope and y – intercept respectively.

- More importantly, it is the locus of point of intersection of operating lines. This line is called as **Feed** line or **q** line.

McCabe – Thiele Method



- It is clear that the value of parameter **q** is yet to be determined.
- Applying energy balance to the control volume as shown in figure, we have

$$h_F F = V_n H_n - V_m H_m + L_{m+1} h_{m+1} - L_{n+1} h_{n+1}$$

- Mathematically, McCabe – Thiele assumption is

$$H_n = H_m = H, h_{m+1} = h_{n+1} = h$$

McCabe – Thiele Method

- Upon substitution, we have

$$h_F F = V_n H_n - V_m H_m + L_{m+1} h_{m+1} - L_{n+1} h_{n+1}$$

$$h_F F = \left(V_n - V_m\right) H + \left(L_{m+1} - L_{n+1}\right) h$$

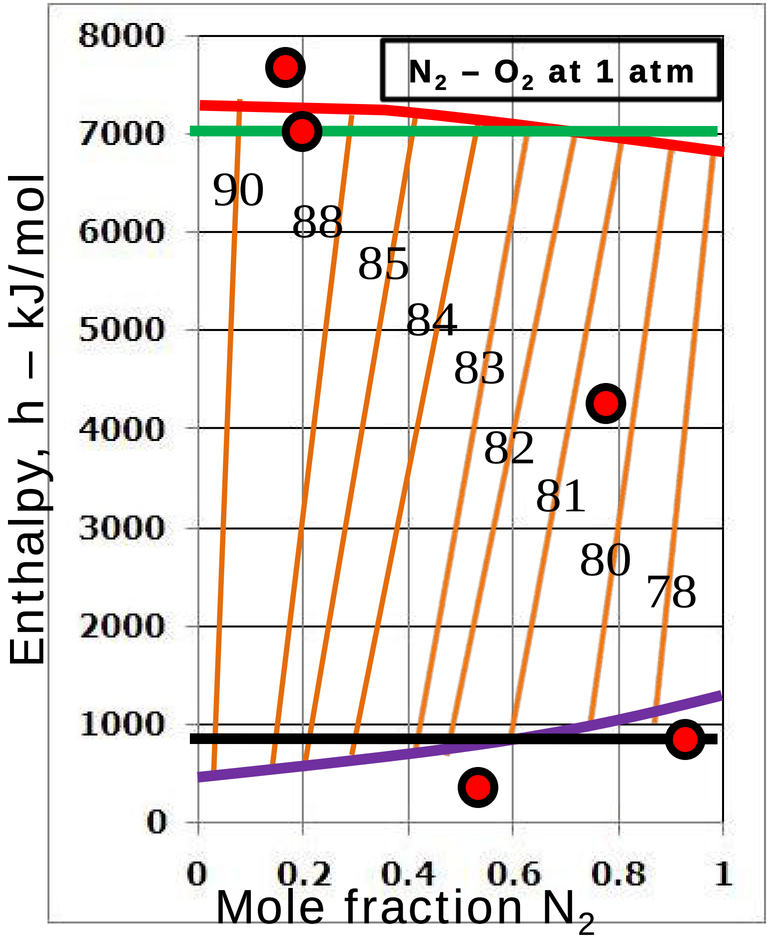
- Also, we have the following equations.

$$V_n - V_m = \left(1 - q\right) F \qquad q = \frac{\left(L_{m+1} - L_{n+1}\right)}{F}$$

- Combining the above equations and rearranging, we have

$$q = \frac{H - h_F}{H - h}$$

McCabe – Thiele Method



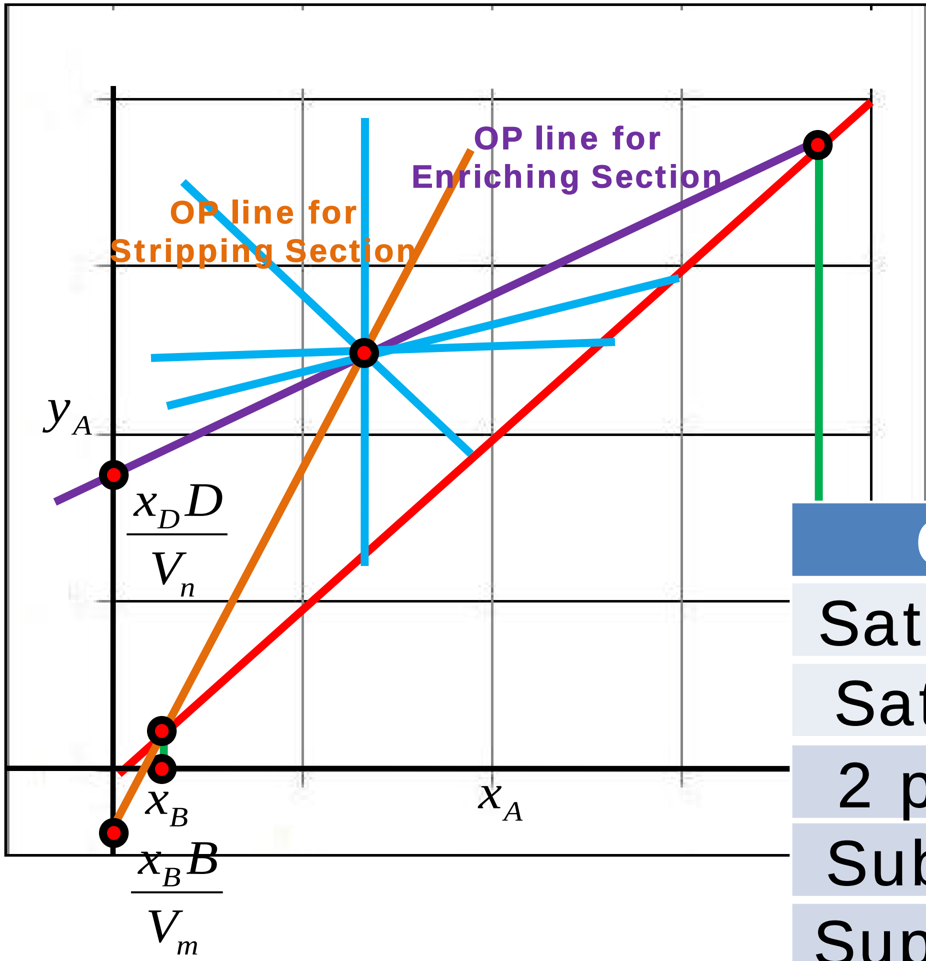
$$q = \frac{H - h_F}{H - h}$$

$$slp = \left(\frac{q}{q-1}\right)$$

- Depending on the feed condition, **q** can take any value.

Condition	q	Slp
Sat. Vap. ($h_F = H$)	$q = 0$	0
Sat. Liq. ($h_F = h$)	$q = 1$	∞
2 ph. ($H < h_F < h$)	$0 < q < 1$	-ve
Sub. Liq. ($h_F < h$)	$q > 1$	+ve
Sup. Vap. ($h_F > h$)	$q < 0$	+ve

McCabe – Thiele Method

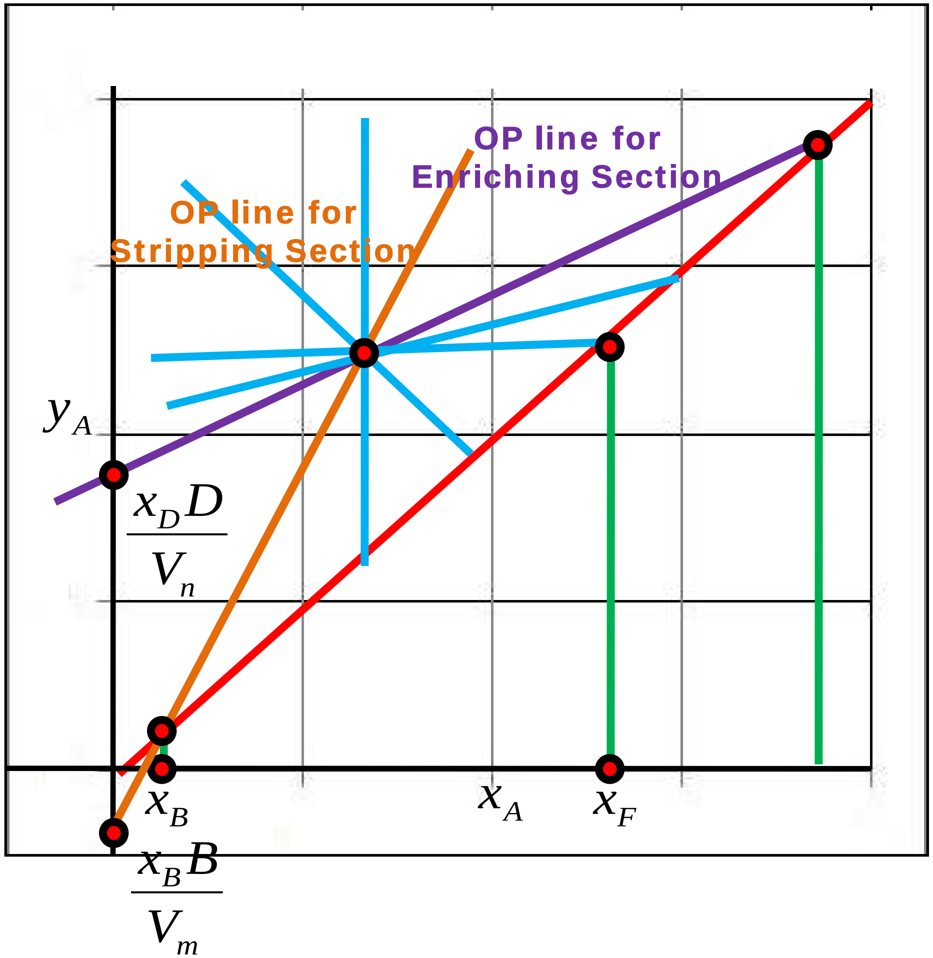


- The equation of feed line is

$$y = \left(\frac{q}{q-1}\right)x + \frac{x_F}{1-q}$$

Condition	q	Slp
Sat. Vap. (h_F=H)	q=0	0
Sat. Liq. (h_F=h)	q=1	∞
2 ph. (H<h_F<h)	0<q<1	-ve
Sub. Liq. (h_F<h)	q>1	+ve
Sup. Vap. (h_F>h)	q<0	+ve

McCabe – Thiele Method



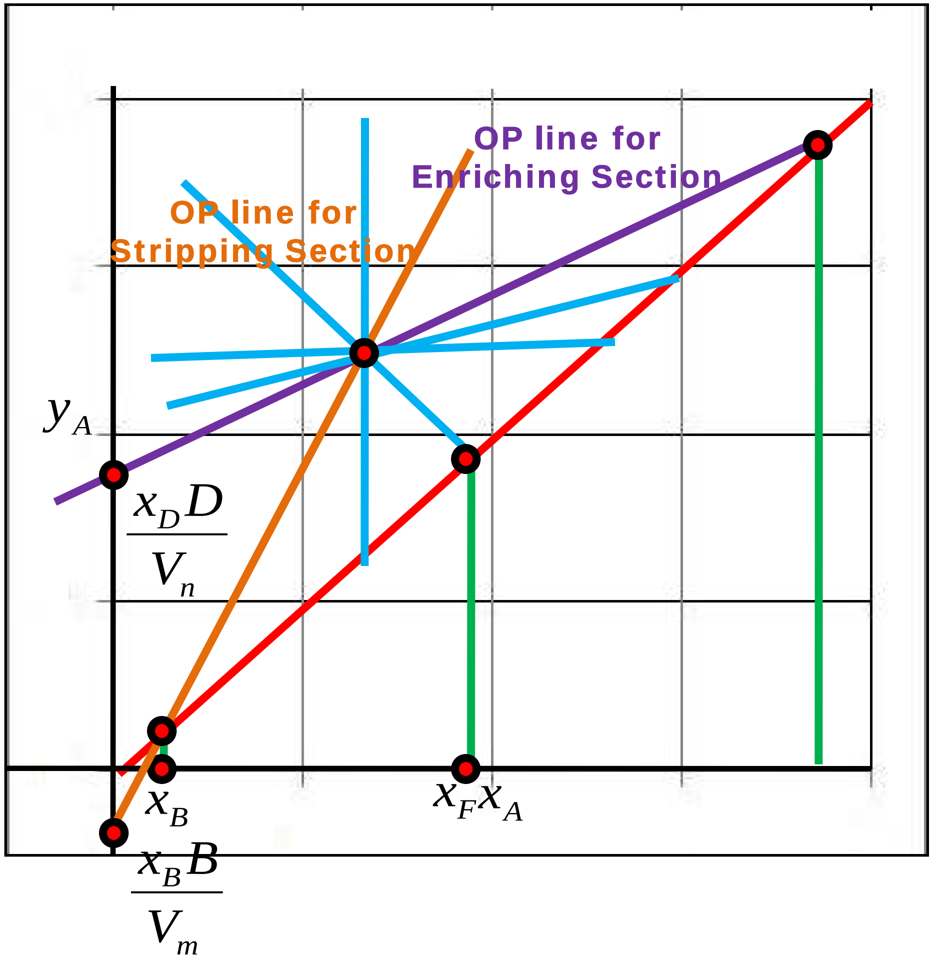
- The point of intersection of feed line or **q** line and **y=x** gives the content of the component **A** in feed, $\mathbf{x_F}$.

- It is calculated by substituting **y=x** in the feed line as shown.

$$x = \left(\frac{q}{q-1}\right)x + \frac{x_F}{1-q}$$

$$x = x_F$$

McCabe – Thiele Method



- Graphically, it is easier to draw a line through two given points rather than using a given slope and a point.
- This intersection point is used to draw the feed line as shown in the figure.

Summary

- Plate calculation procedures require the data like number of components, phase diagrams, property data of the mixtures, heat transfer correlations.
- **McCabe – Thiele** method is less general and is widely used for binary mixtures at cryogenic temperatures.
- The major assumption in this method is that the liquid and vapor enthalpies are independent of mole fraction.

Summary

- The equations of operating lines for striping and enriching sections are

$$y_n = \left(\frac{L_{n+1}}{V_n}\right) x_{n+1} + \left(\frac{D}{V_n}\right) x_D \qquad y_m = \left(\frac{L_{m+1}}{V_m}\right) x_{m+1} - \left(\frac{B}{V_m}\right) x_B$$

- The locus of intersection of these operating lines denotes the feed condition. It is given as

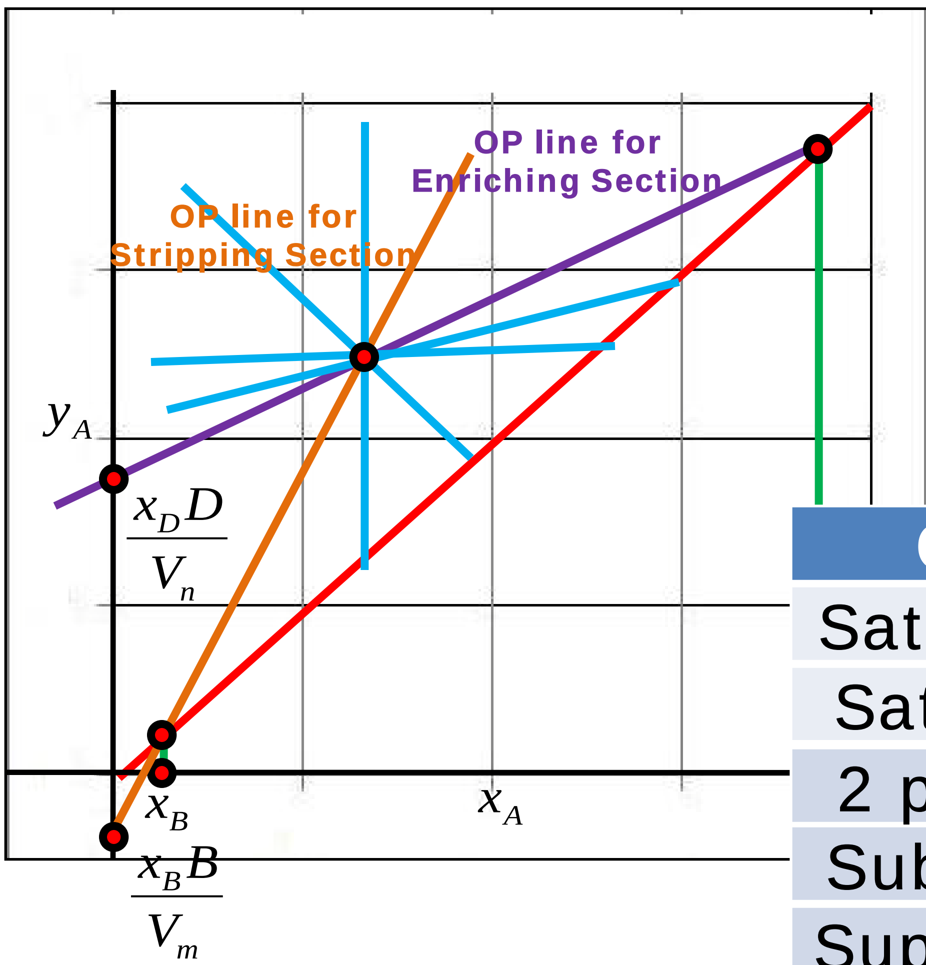
$$y = \left(\frac{q}{q-1}\right) x + \frac{x_F}{1-q}$$

- The point of intersection of feed line or **q** line and **y=x** gives the content of a component in the feed, $\mathbf{x_F}$.

Summary

- The slope of **q** line is given by

$$\left(\frac{q}{q-1}\right)$$



Condition	q	Slp
Sat. Vap. (h_F=H)	q=0	0
Sat. Liq. (h_F=h)	q=1	∞
2 ph. (H<h_F<h)	0<q<1	-ve
Sub. Liq. (h_F<h)	q>1	+ve
Sup. Vap. (h_F>h)	q<0	+ve

- A self assessment exercise is given after this slide.

- Kindly asses yourself for this lecture.

Self Assessment

1. McCabe – Thiele method calculates ______ & ______ of each component at every plate.
2. For a **jth** plate, the liquid an vapor leaving from top are denoted by ____ and ____ respectively.
3. The vapor and liquid on any plate are assumed to be in ______ equilibrium.
4. In McCabe – Thiele method, liquid and vapor enthalpies are assumed to be ________.
5. The slope of operating line for stripping section is given by _______.
6. The y – intercept of operating line for enriching section is given by _______.
7. Mixture that is to be separated is called as ____.

Self Assessment

8. q=0 when the feed is totally ______.
9. _______ and ______ are the slope and the y – intercept of q line respectively.
10. Fill the following table.

Condition	q	Slp
Sat. Vap. (h_F=H)	q=0	
Sat. Liq. (h_F=h)		∞
2 ph. (H<h_F<h)	0<q<1	-ve
Sub. Liq. (h_F<h)		+ve
	q<0	+ve

Answers

1. Vapor fraction, liquid fraction
2. L_j and V_j
3. Thermal
4. Constant
5. L_{n+1}/V_n
6. $(-(B/V_m)x_B)$
7. Feed
8. Vapor
9. $q/(q-1)$ and $x_F/(1-q)$

Condition	q	Slp
Sat. Vap. ($h_F=H$)	$q=0$	0
Sat. Liq. ($h_F=h$)	$q=1$	∞
2 ph. ($H<h_F<h$)	$0<q<1$	-ve
Sub. Liq. ($h_F<h$)	$q>1$	+ve
Sup. Vap. ($h_F>h$)	$q<0$	+ve

Thank You!