
CRYOGENIC ENGINEERING
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Lecture No - 24

Earlier Lecture

- In the earlier lecture, we have seen the working of rectification column with the help of animation.
- **Ponchon & Savarit, McCabe & Thiele** and **Numerical** techniques are used to calculate the theoretical number of plates.
- **McCabe & Thiele** method is less general and is widely used for binary mixtures.
- The major assumption is that the saturated vapor and saturated liquid enthalpies are independent of the mole fraction.

Earlier Lecture

- In the earlier lecture, the equations of operating lines for striping and enriching sections are derived.

- The locus of intersection of these operating lines denotes the feed condition.

- The point of intersection of feed line or **q** line and **y = x** gives the content of a component in the feed, $\mathbf{x_F}$.

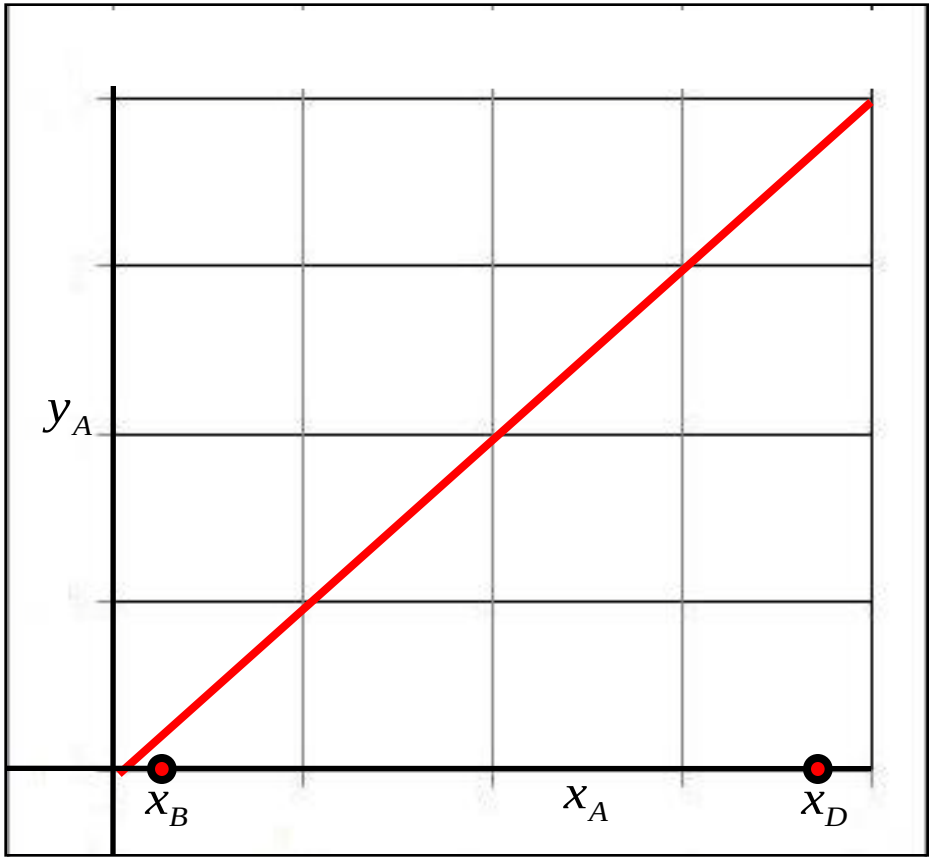
Outline of the Lecture

Topic : Gas Separation (contd)

- Graphical solution for column design using McCabe – Thiele method

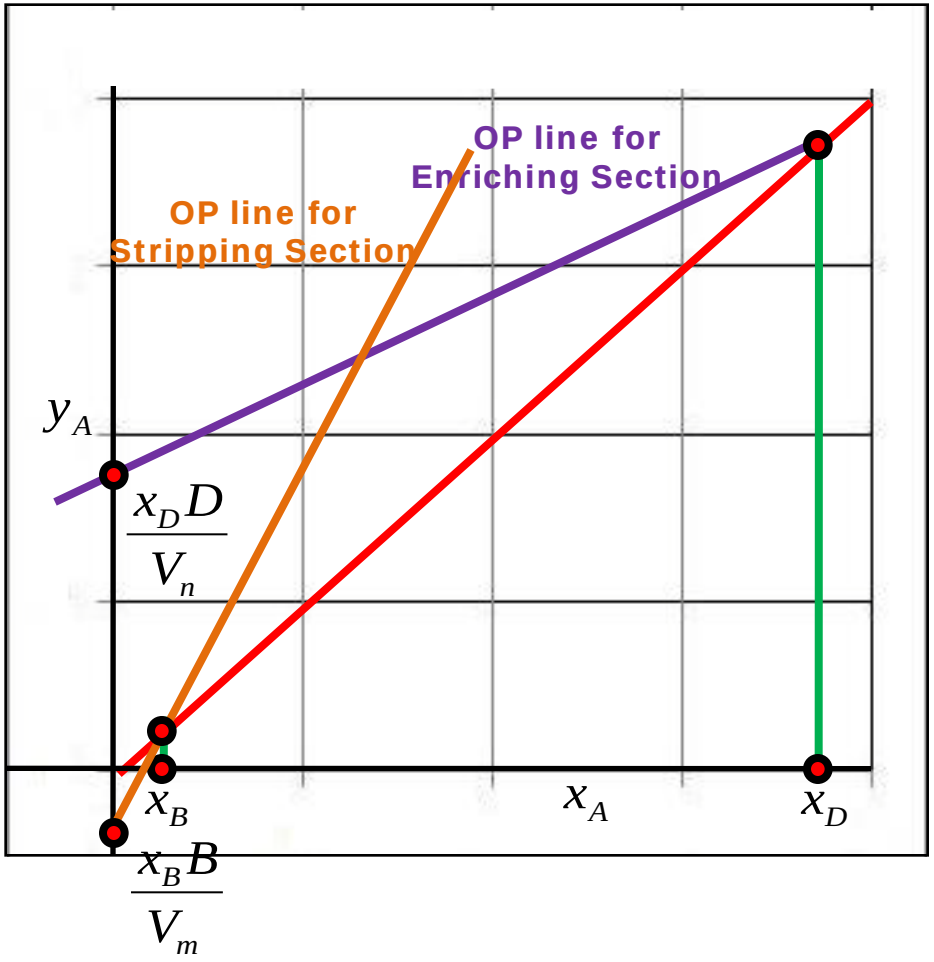
- Tutorial

McCabe – Thiele Method



- Plot of vapor and liquid mole fractions for a particular component for OP line is shown.
- Let **45º** diagonal or **y = x** line be as shown.
- The desired purity and impurity of this component in top and bottom products are $\mathbf{x_D}$ and $\mathbf{x_B}$ respectively.

McCabe – Thiele Method



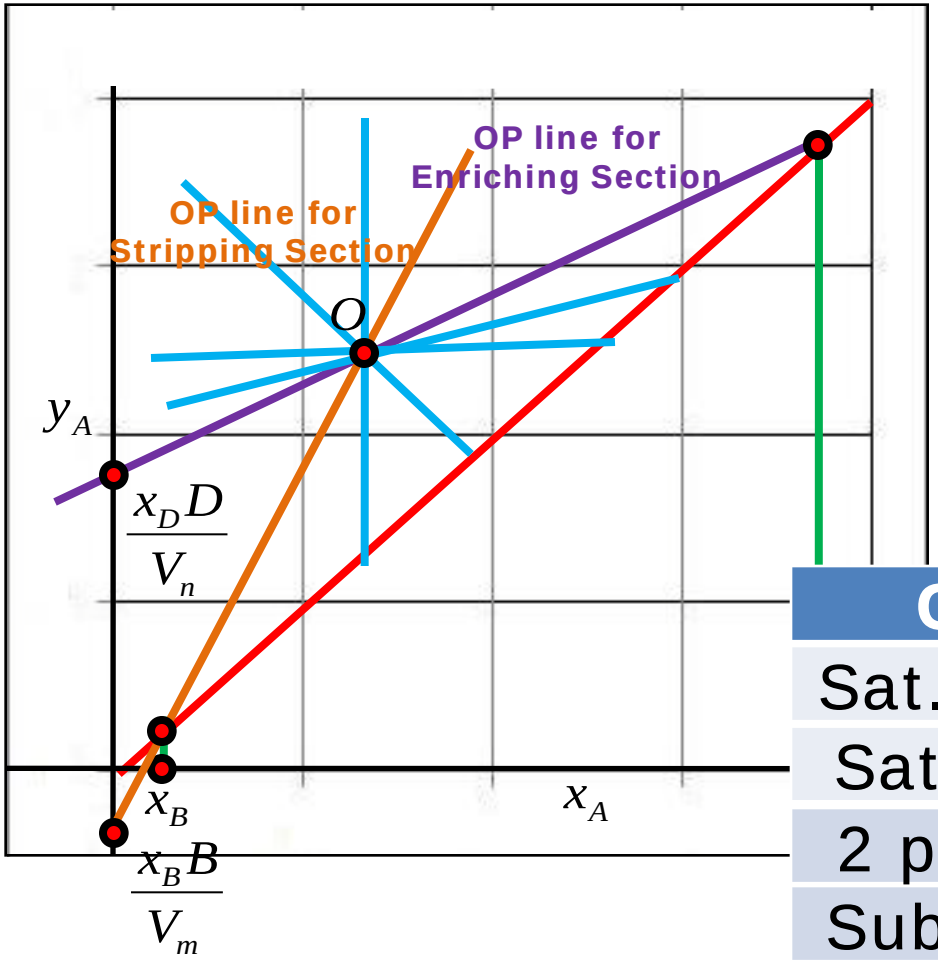
- The equation of operating line for stripping section is

$$y_m = \left(\frac{L_{m+1}}{V_m}\right) x_{m+1} - \left(\frac{B}{V_m}\right) x_B$$

- The equation for operating line in enriching section is

$$y_n = \left(\frac{L_{n+1}}{V_n}\right) x_{n+1} + \left(\frac{D}{V_n}\right) x_D$$

McCabe – Thiele Method

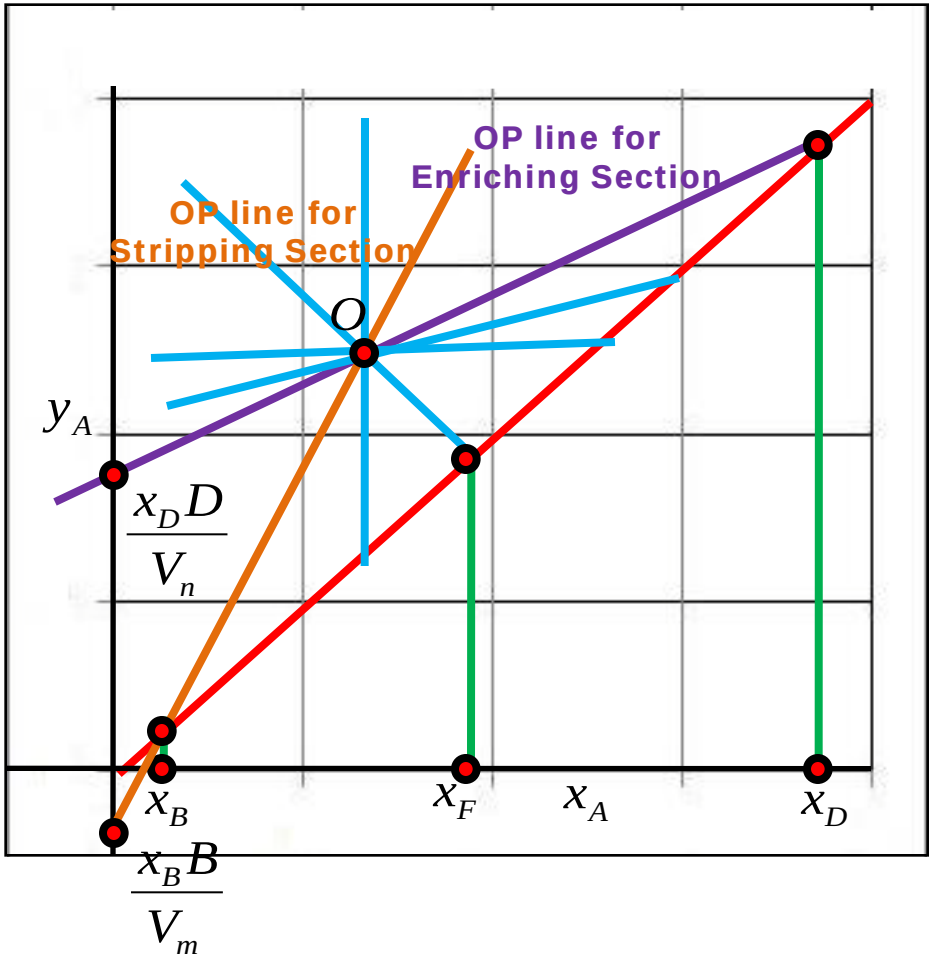


- Let this point of intersection be **O**.
- The feed line equation is

$$y = \left(\frac{q}{q-1}\right)x + \frac{x_F}{1-q}$$

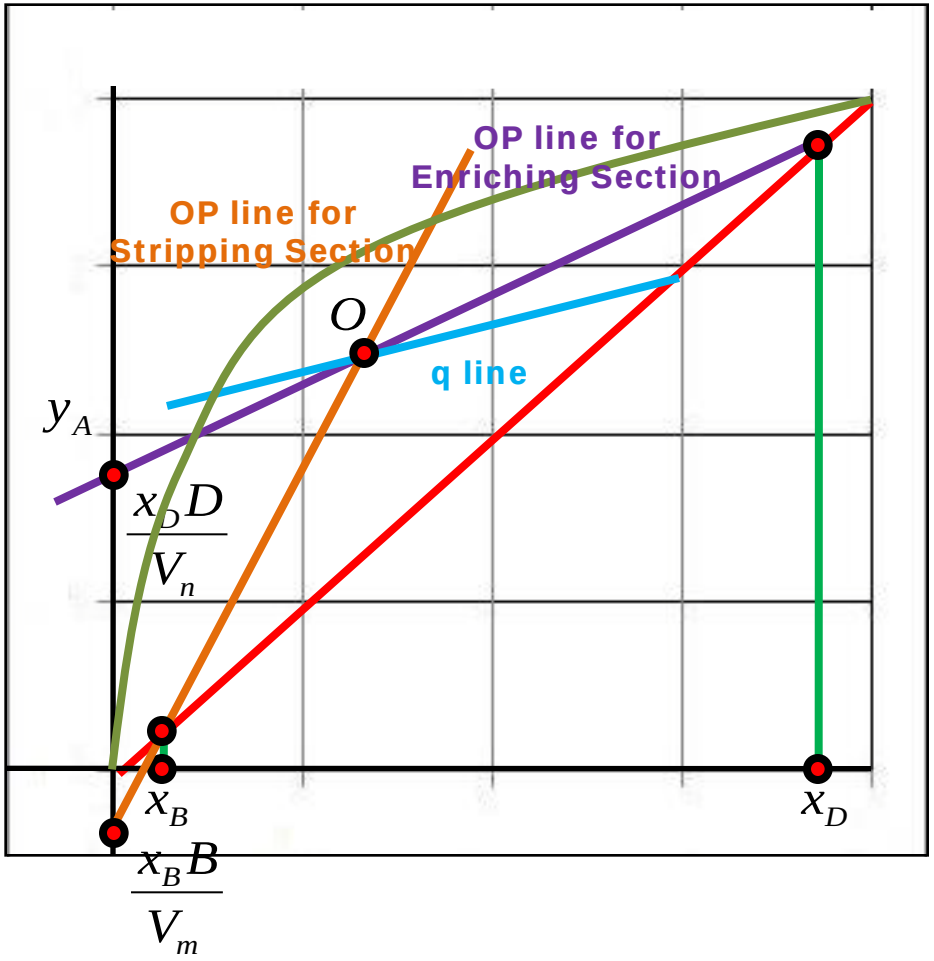
Condition	q	Slp
Sat. Vap. ($h_F = H$)	$q = 0$	0
Sat. Liq. ($h_F = h$)	$q = 1$	∞
2 ph. ($H < h_F < h$)	$0 < q < 1$	-ve
Sub. Liq. ($h_F < h$)	$q > 1$	+ve
Sup. Vap. ($h_F > h$)	$q < 0$	+ve

McCabe – Thiele Method



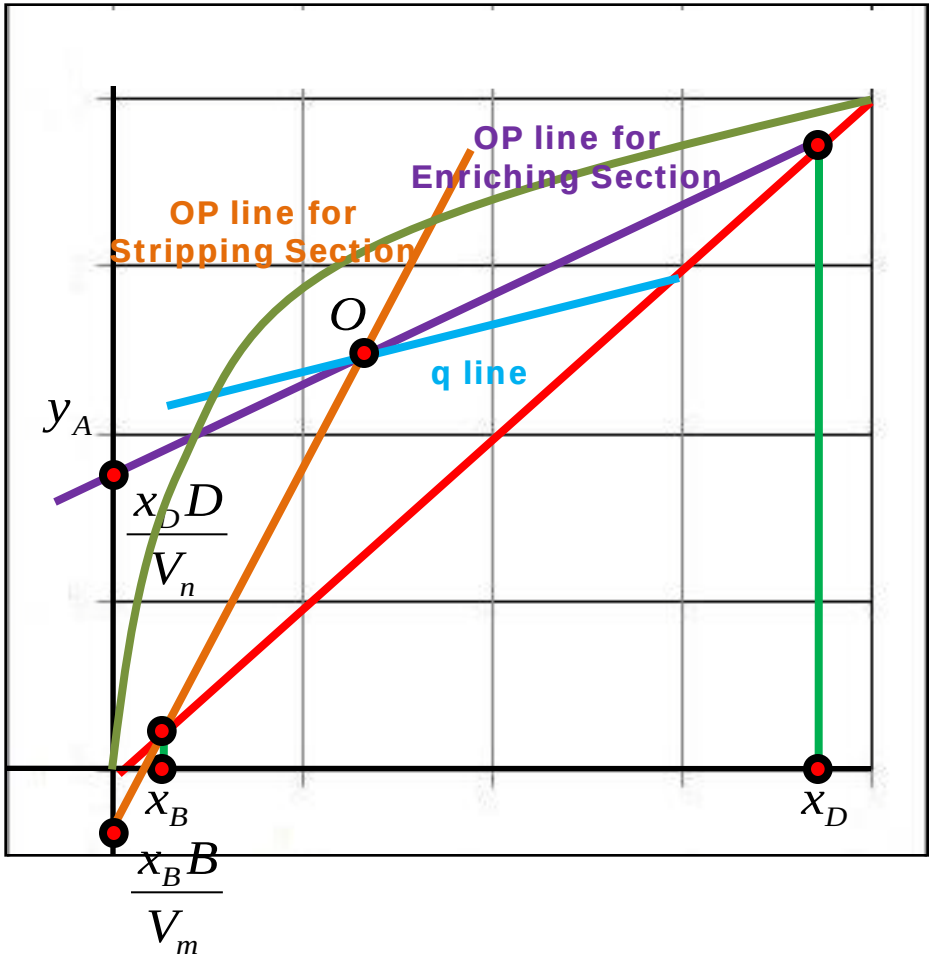
- The point of intersection of feed line or **q** line and **y = x**, gives the content of the component **A** in feed, $\mathbf{x_F}$.
- This intersection point is used to draw the feed line as shown in the figure.

McCabe – Thiele Method



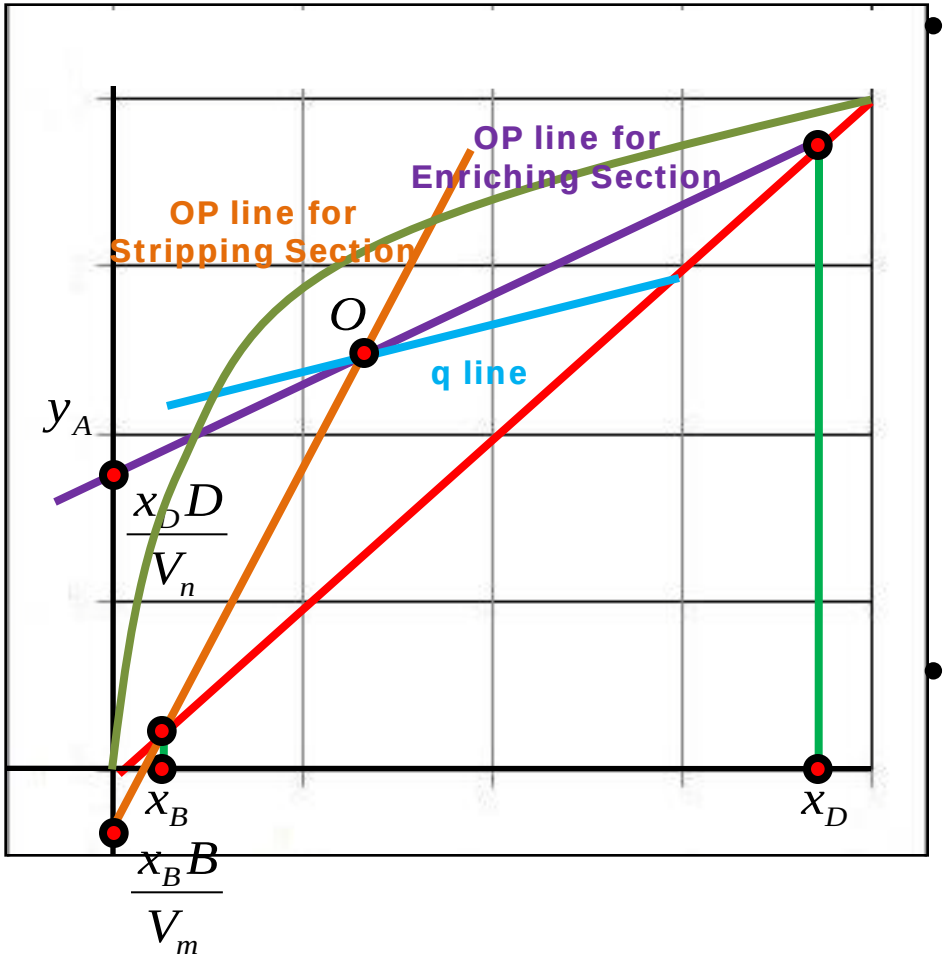
- The variation of equilibrium vapor and liquid fractions of a particular component (here, **A**), is called as equilibrium curve.
- It means that at any point on this curve, the vapor and liquid of this component are at same temperature.

McCabe – Thiele Method



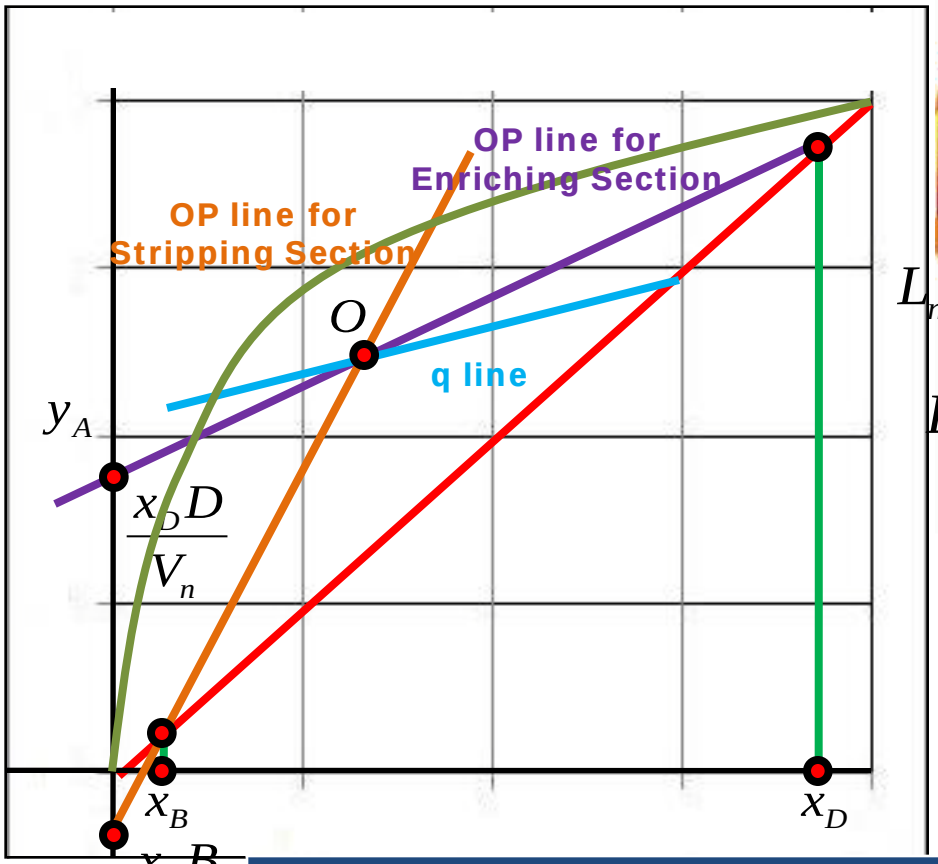
- On each plate, vapor and liquid are in thermal equilibrium. Therefore, the plate condition lies on the equilibrium line.
- That is, equilibrium curve gives the relation between liquid composition **(x_n)** and vapor composition **(y_n)** on the same plate.

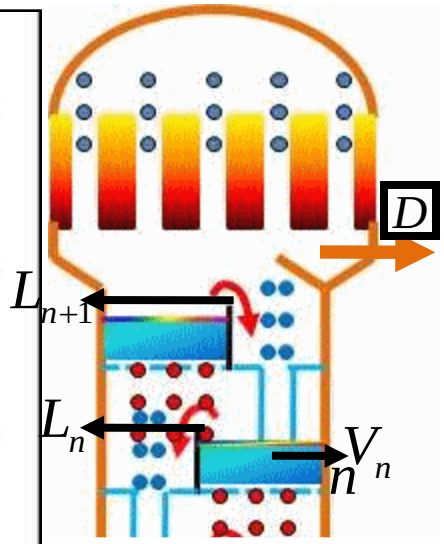
McCabe – Thiele Method



- Since, the top and the bottom products have different boiling points, there is a gradual variation of temperature across the length of the column.
- The OP lines relate the variation of liquid and vapor mole fractions of a particular component across the length of the column.

McCabe – Thiele Method

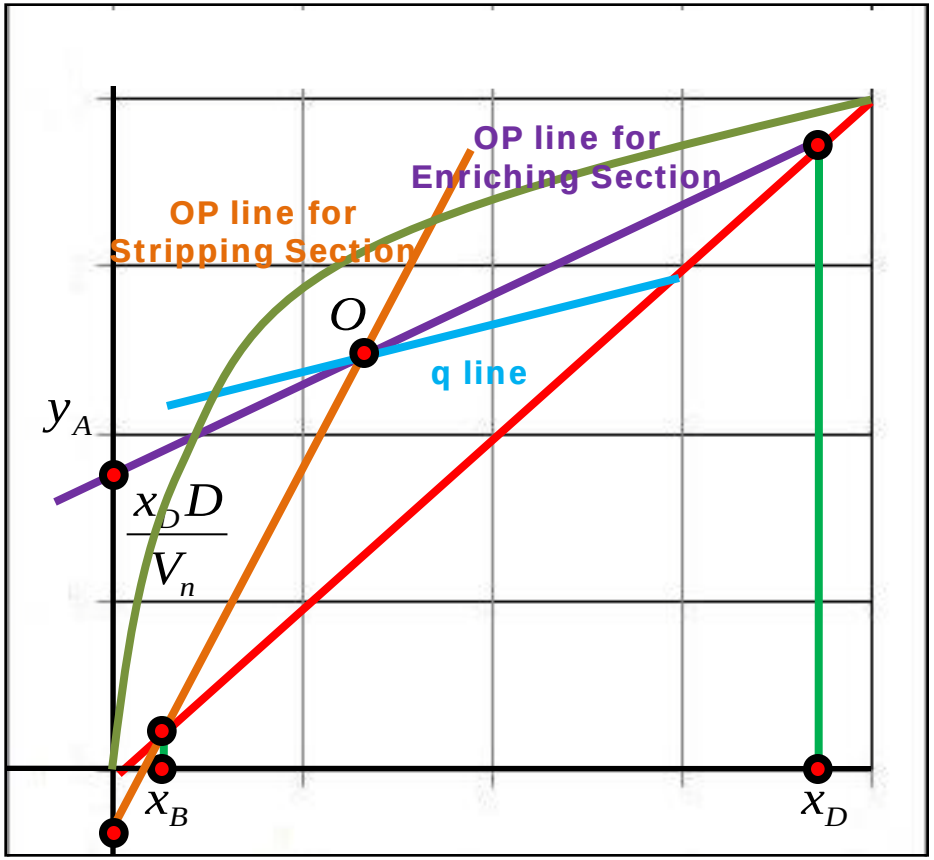




$$y_n = \left(\frac{L_{n+1}}{V_n}\right) x_{n+1} + \left(\frac{D}{V_n}\right) x_D$$

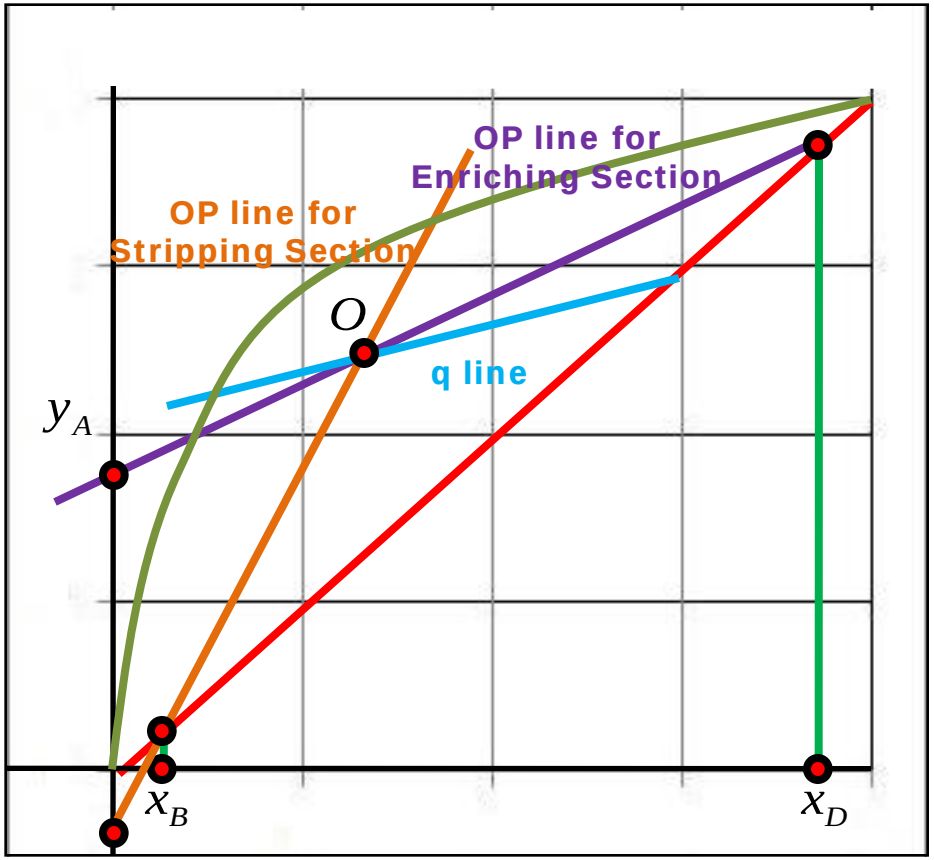
- Reviewing the OP line equation, say for the top section, it is clear that this equation relates $\mathbf{x_{n+1}}$ and $\mathbf{y_n}$, for this component, **A**.

McCabe – Thiele Method



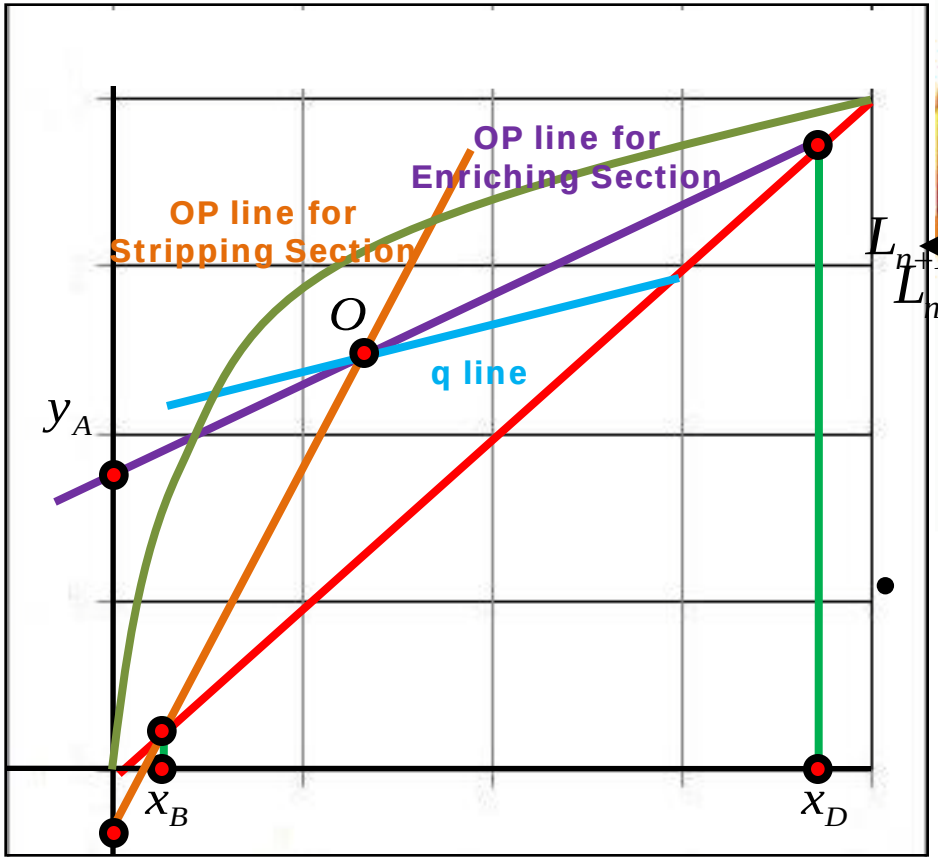
- All these curves and lines are vital in calculating the number of plates using McCabe – Thiele method.
- In view of this, the slopes of operating and **q** lines, equilibrium curve and purity requirement form the basis to determine the number of plates.

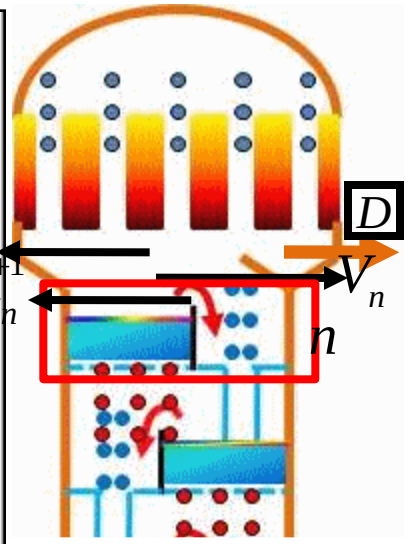
McCabe – Thiele Method



- The plate calculation method involves a stair casing method as explained below.
- The condensate **D** collected at the top, has a mole fraction of component **A** as $\mathbf{x_D}$.

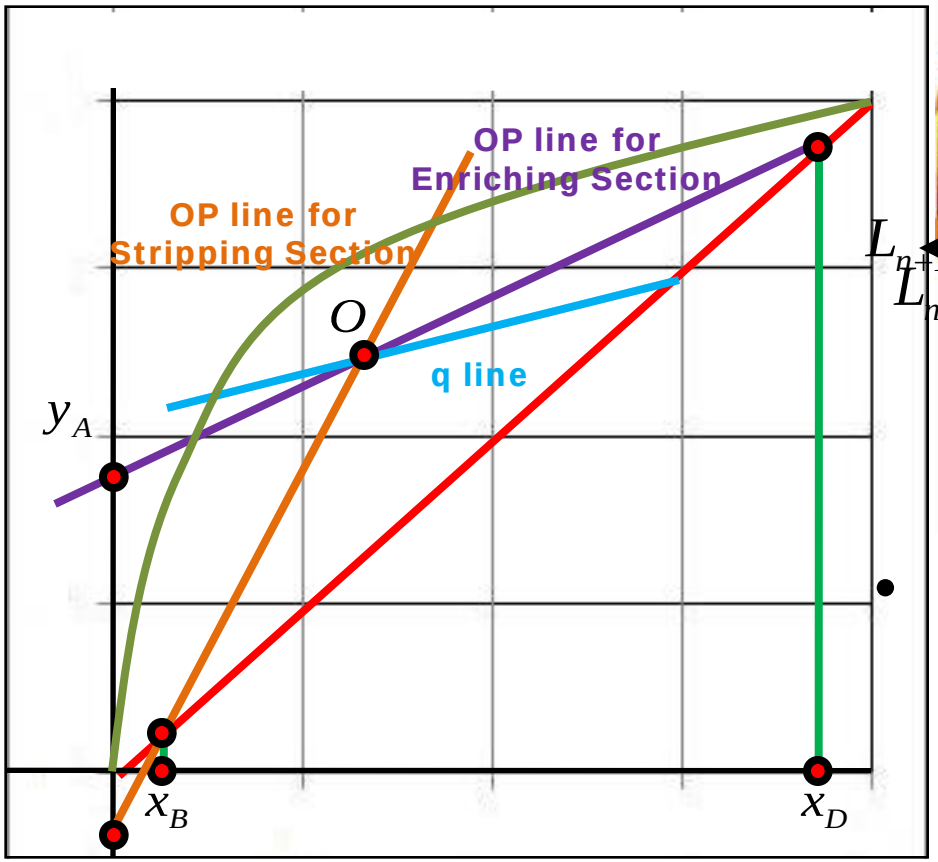
McCabe – Thiele Method

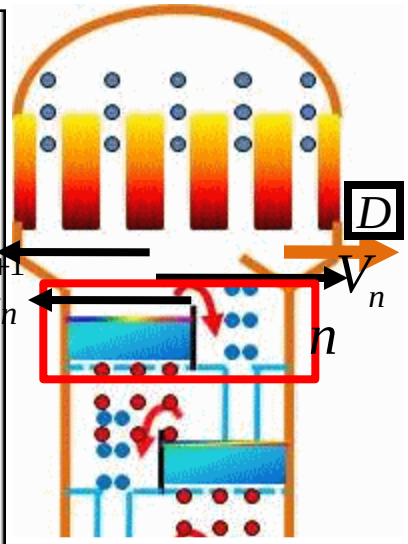




- The liquid coming onto the top plate from the condenser has a mole fraction of x_D. Hence, the point x_D lies on the OP line as shown in the figure.

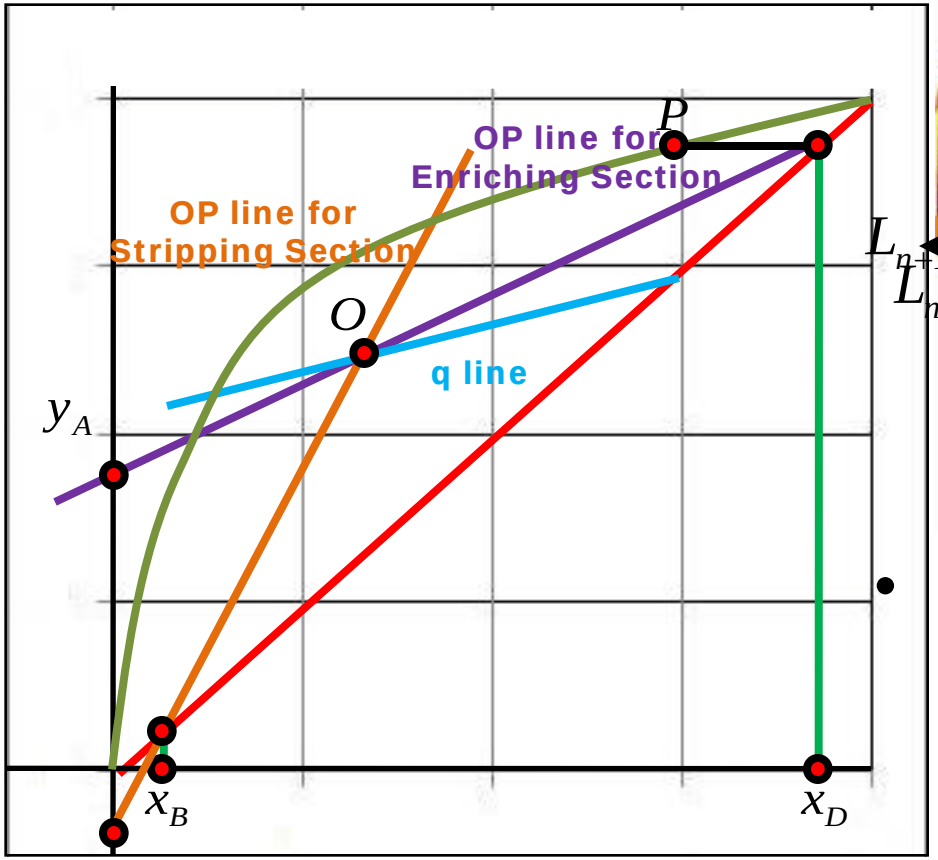
McCabe – Thiele Method

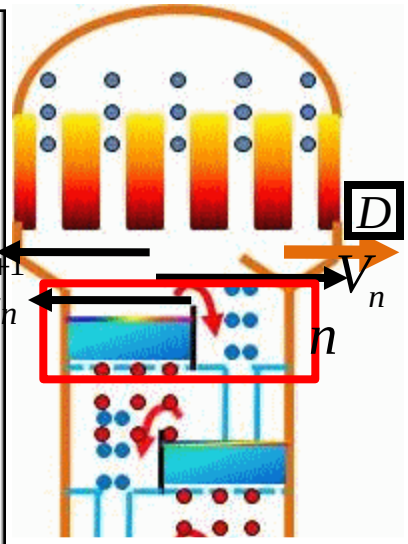




- For this plate, liquid and vapor are in thermal equilibrium. Therefore, the corresponding liquid fraction $\mathbf{x_n}$, lies on the equilibrium curve.

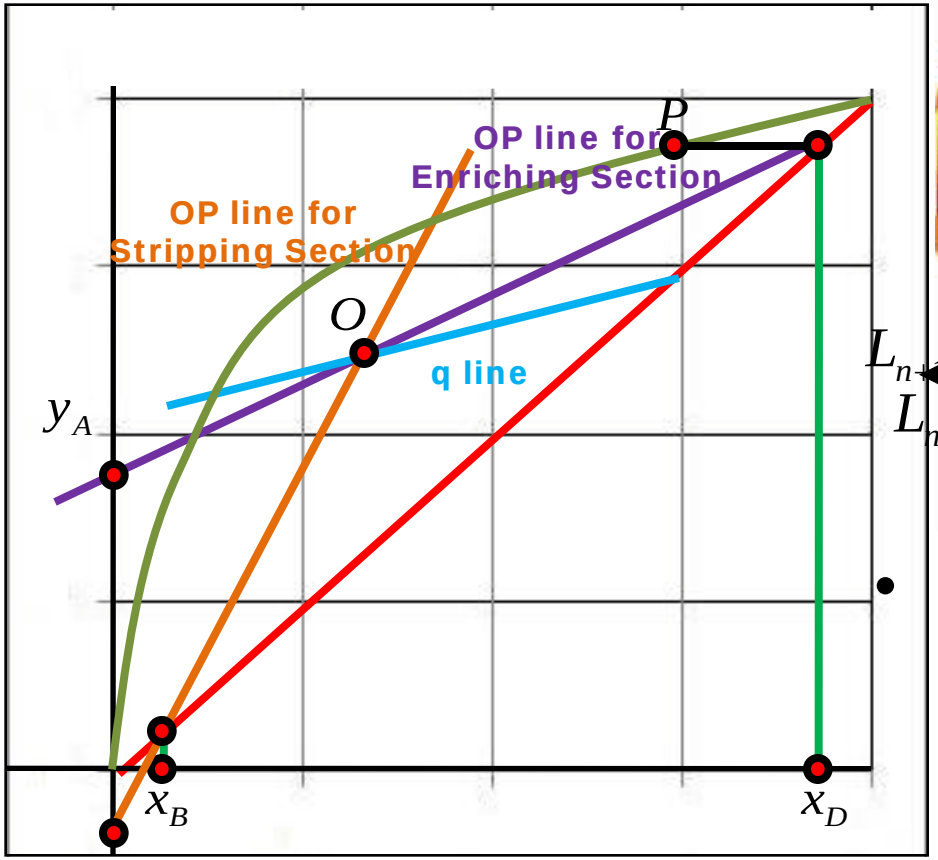
McCabe – Thiele Method

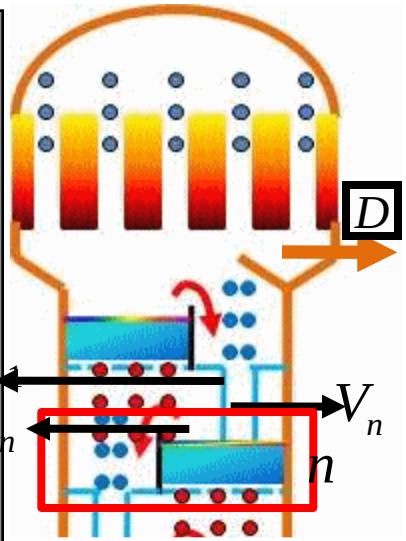




- This equilibrium point is found by extending a horizontal line from $\mathbf{x_D}$ to the equilibrium curve. Let this point be denoted by **P**.

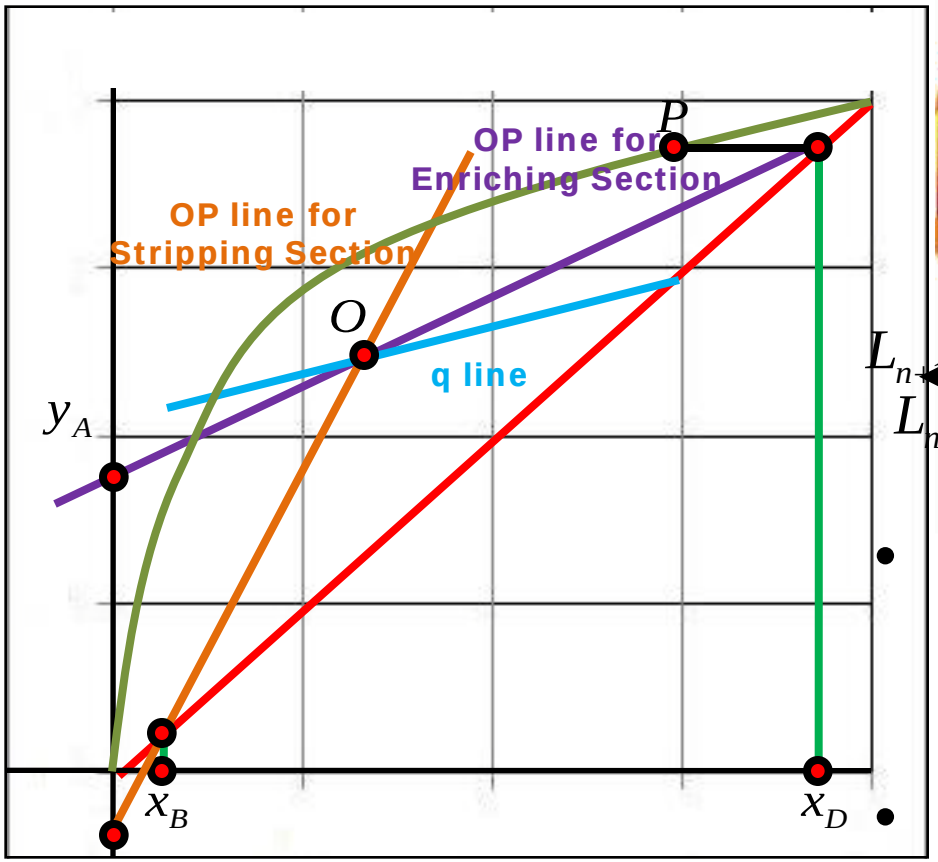
McCabe – Thiele Method

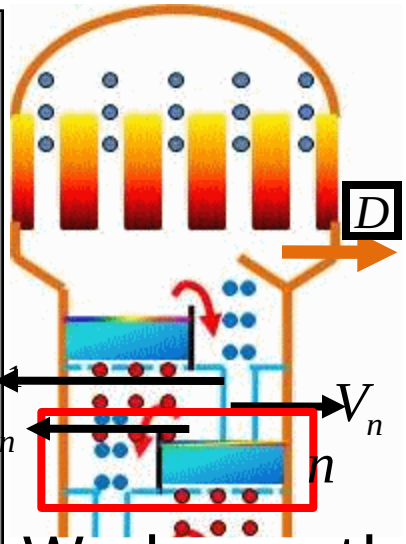




- The liquid from this plate, flows over the weir and exchanges heat with the vapor coming from the top of the lower plate.

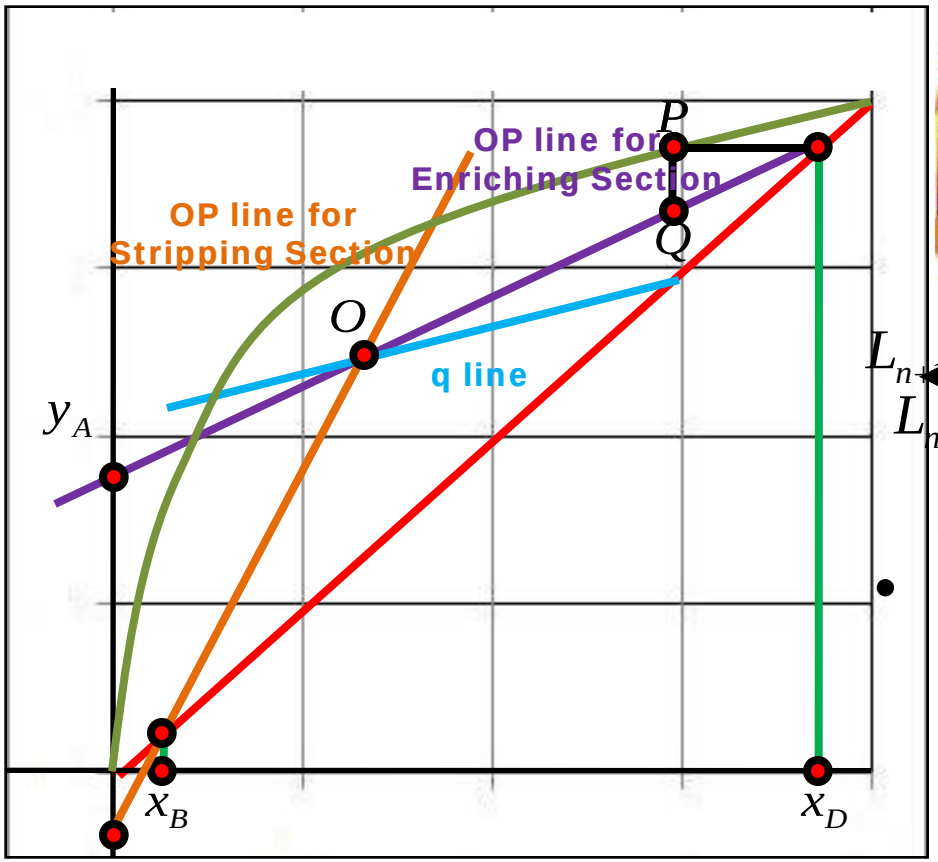
McCabe – Thiele Method

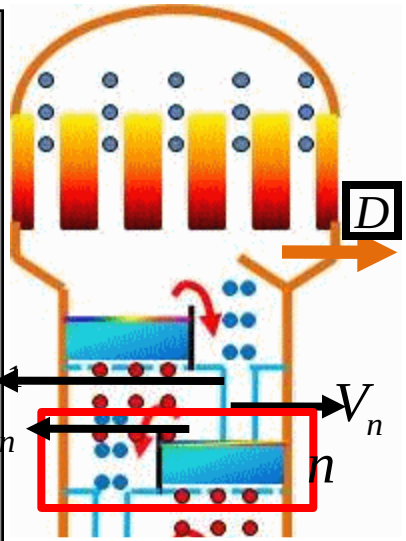




- We know that, an OP line relates $\mathbf{x_{n+1}}$ with $\mathbf{y_n}$.
- Hence for this plate, the corresponding vapor fraction lies on the OP line.

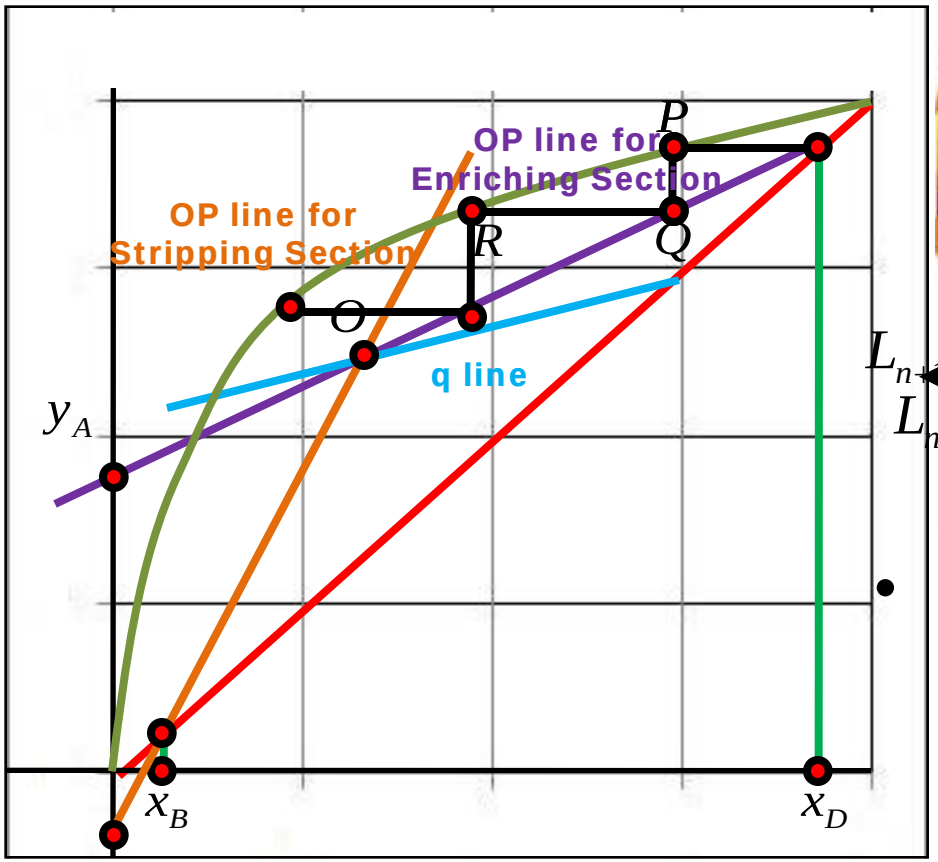
McCabe – Thiele Method

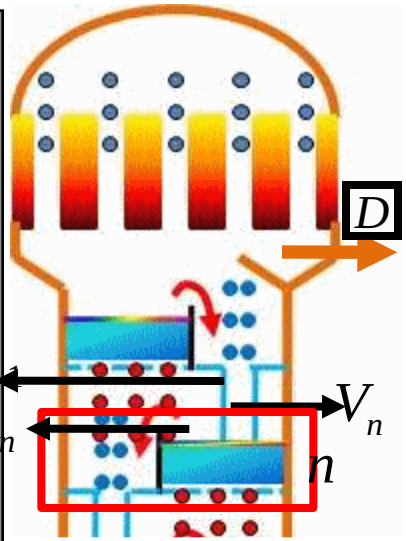




- The vapor composition, $\mathbf{y_n}$, for this plate is found by extending a vertical line from **P** onto the OP line. Let this point be denoted by **Q**.

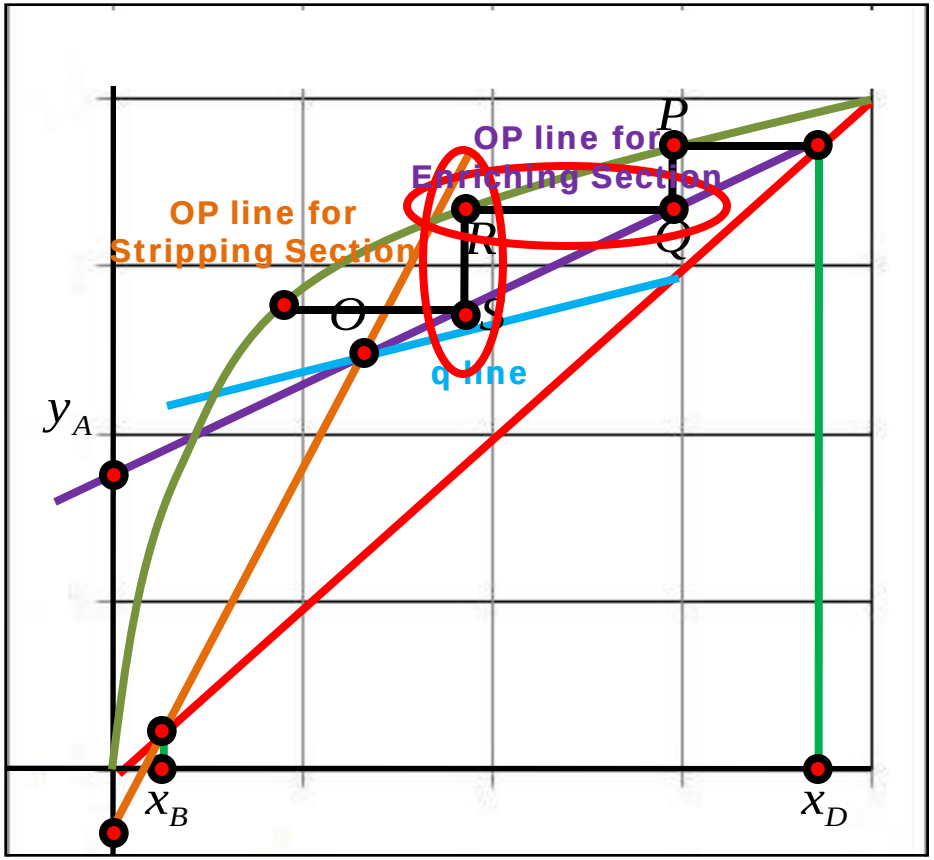
McCabe – Thiele Method





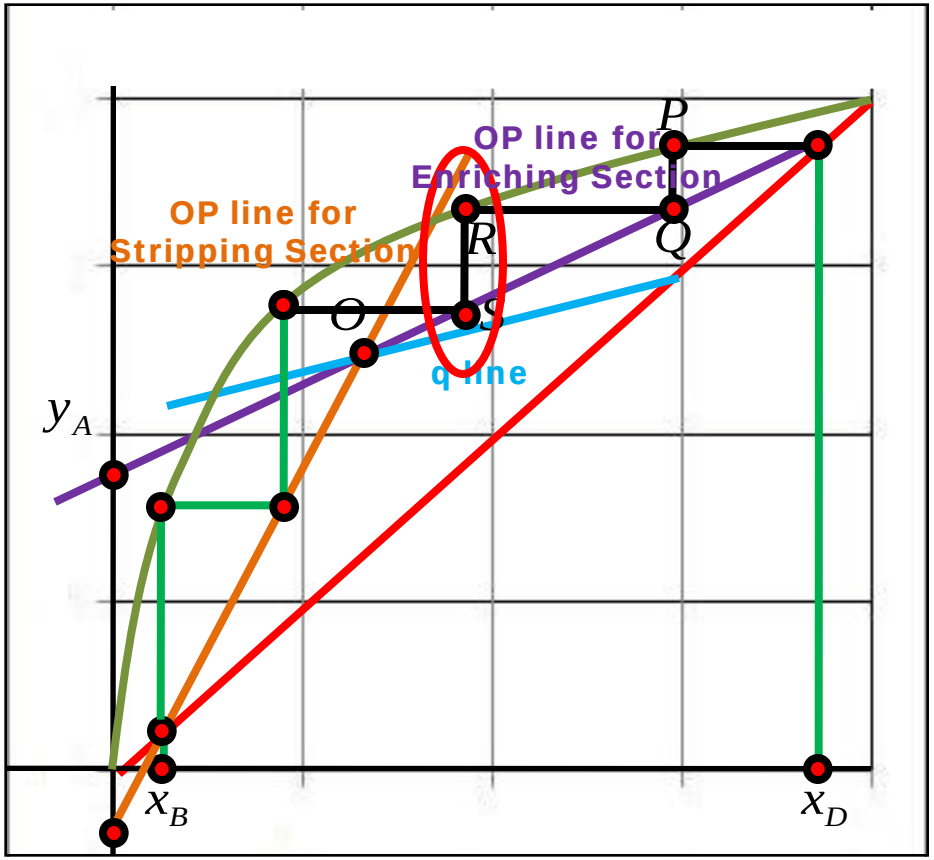
- The stair casing should be continued till the stairs cross the point **O** (the intersection of OP lines) as shown in the figure.

McCabe – Thiele Method



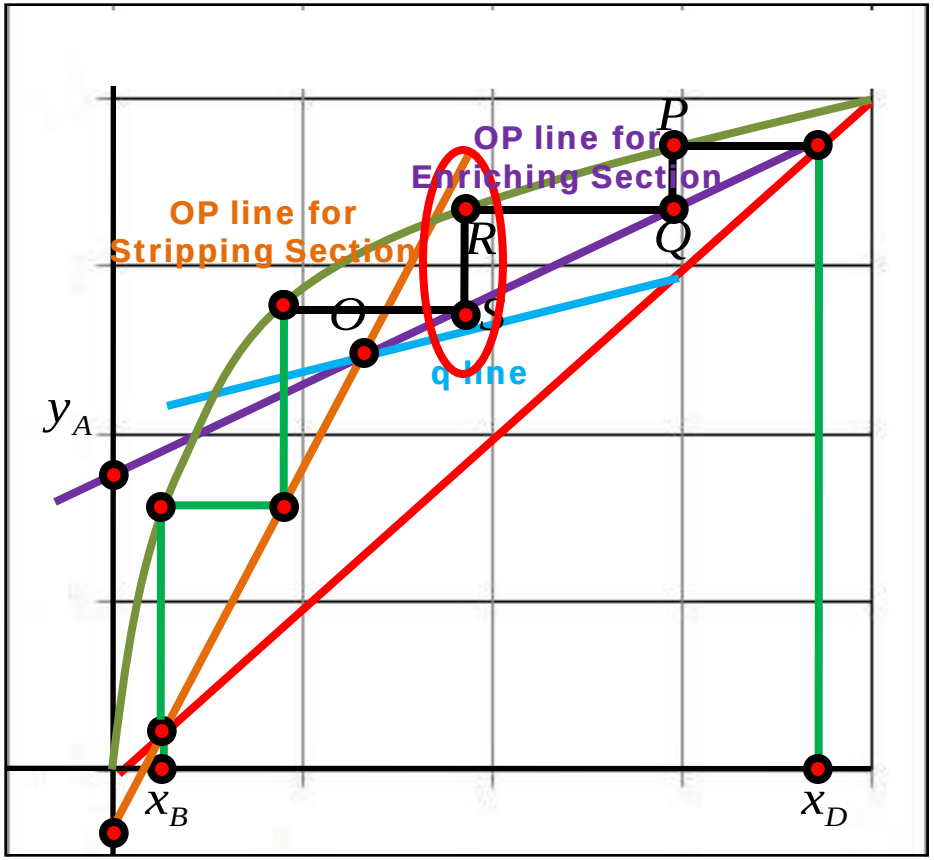
- It is clear that
  - Every horizontal line gives the condition of liquid – vapor on the same plate which are in thermal equilibrium.
  - Every vertical line gives the vapor condition for the plate below the earlier plate.

McCabe – Thiele Method



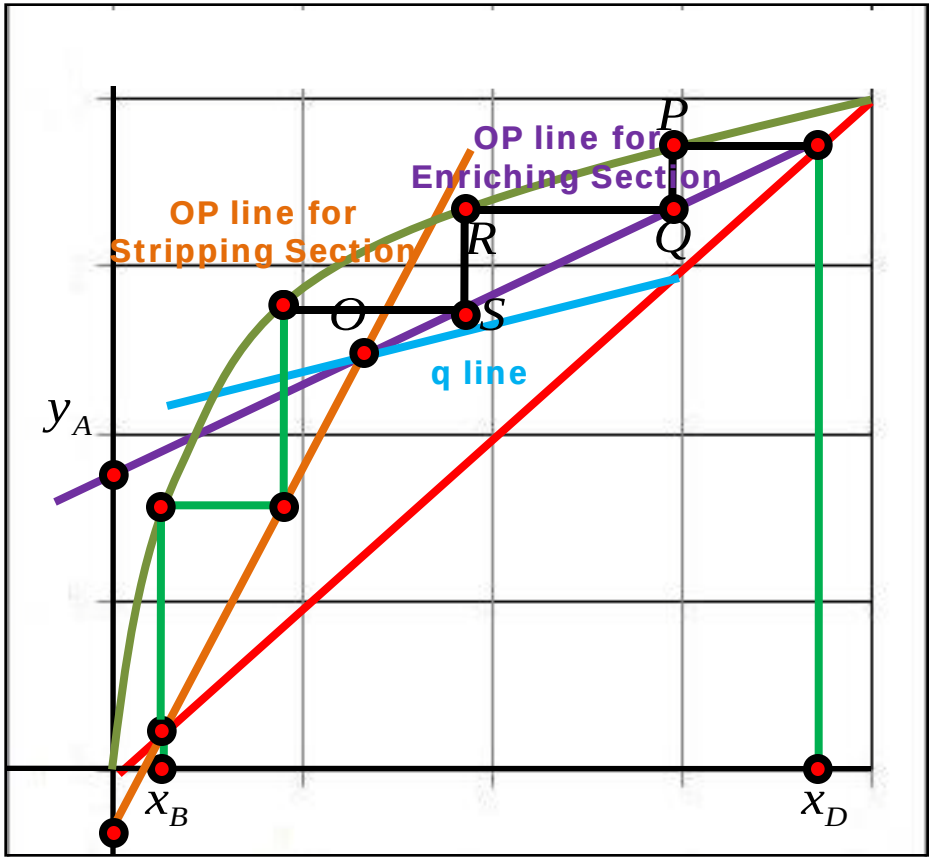
- It means that, every vertical line indicates the need for the plate in the enriching section, till the stair casing crosses the point **O**.
- The same exercise could be done for the lower section, with $\mathbf{x_D}$ as the desired impurity of the component **A** in the bottom product.

McCabe – Thiele Method



- As mentioned earlier, each vertical line indicating a plate, the total number of vertical lines in top and bottom section, together with boiler and condenser surfaces gives the total number of theoretical plates required.

McCabe – Thiele Method

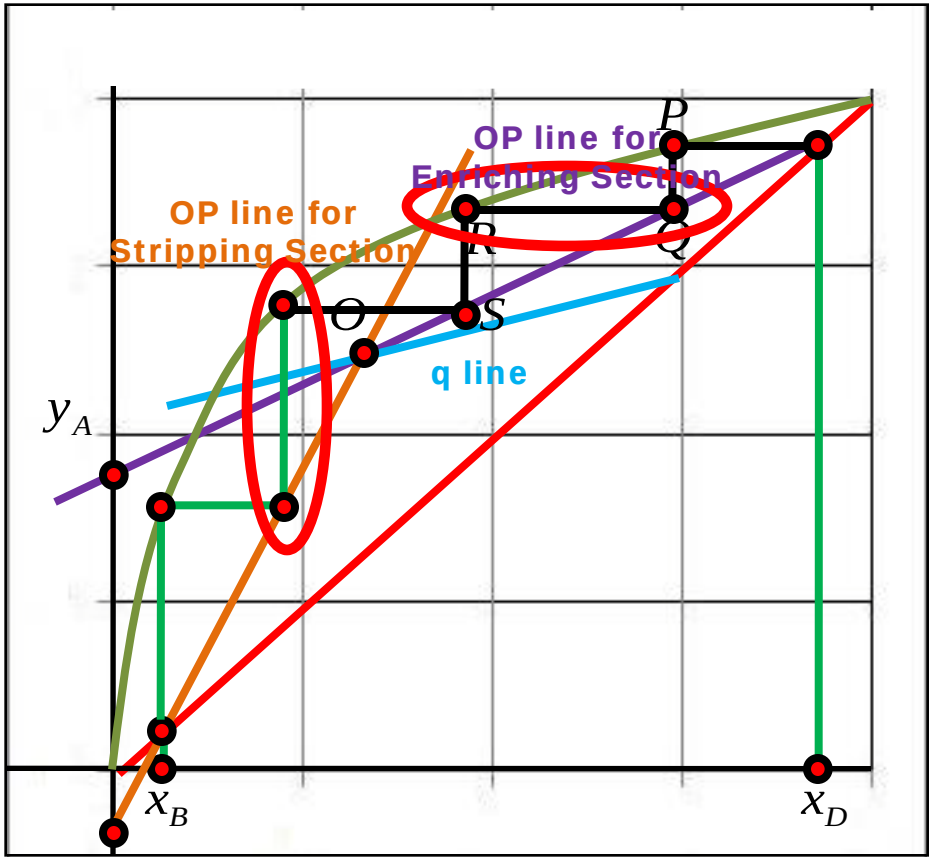


- From the adjacent hypothetical figure, the total number of vertical lines are 4.

- Hence, the total number of theoretical plates can be tabulated as shown below.

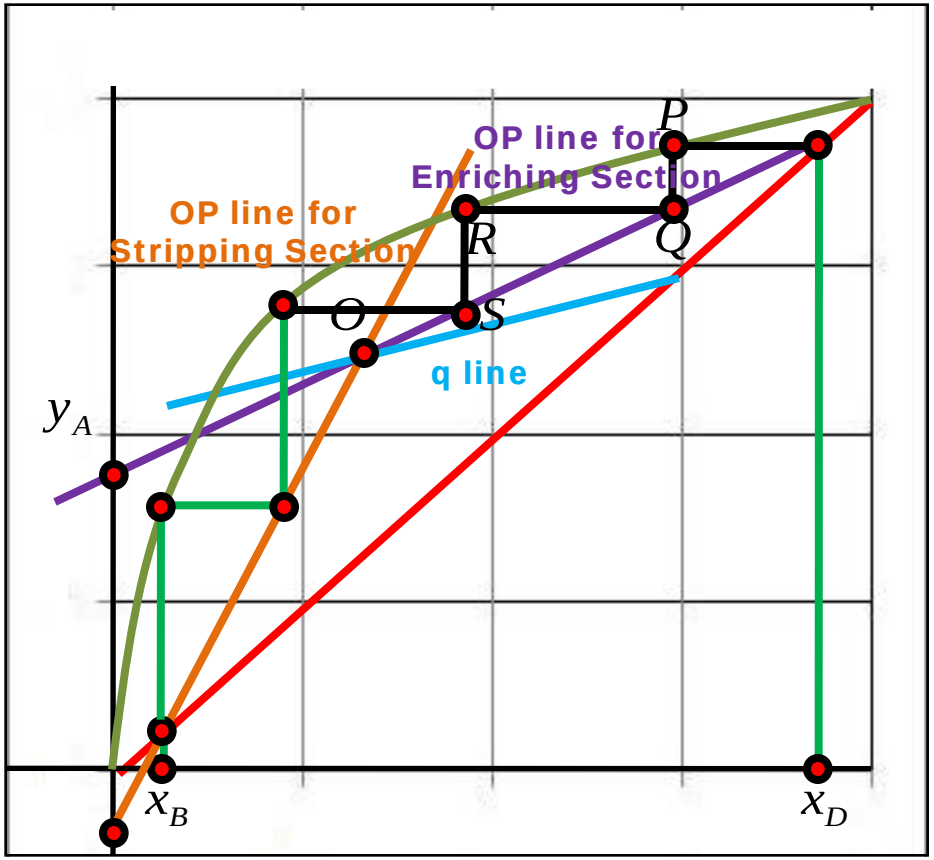
McCabe – Thiele Method	
Top	3 + 1 (Cond)
Bottom	3 + 1 (Boiler)

McCabe – Thiele Method



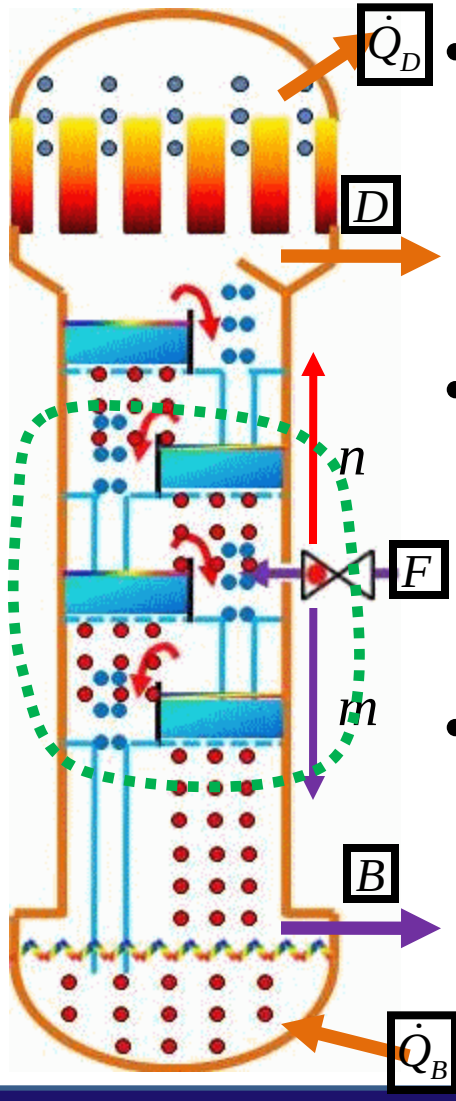
- From the adjacent figure, it is important to note that during the heat exchange process, that is along the vertical line, the liquid composition **x** is constant.
- Similarly, along the horizontal line, the vapor composition **y** remains constant.

McCabe – Thiele Method



- Also, note that in moving from top to bottom, the McCabe – Thiele diagram starts with an horizontal line and ends with a vertical line.
- This is because the liquid flows downwards and is represented by a vertical line.

McCabe – Thiele Method



- In the earlier lecture, we have balanced moles and enthalpy for top, bottom and mid-section separately.

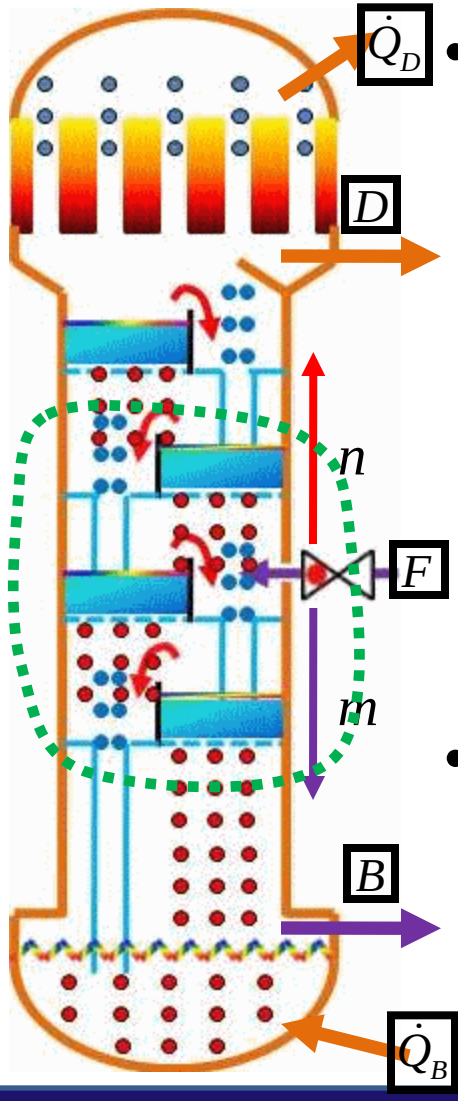
- But considering the column as a whole, the following equation hold true for mole balance.

$$F = B + D$$

- Multiplying the above equation with the mole fraction of a particular component, we get mole balance for that component.

$$x_F F = x_B B + x_D D$$

McCabe – Thiele Method



- Similarly, for the entire column, the enthalpy or the energy balance can be written as

IN	OUT
Q_B	Q_D
$h_F F$	$h_D D$
	$h_B B$

$$\dot{Q}_B + h_F F = \dot{Q}_D + h_D D + h_B B$$

- All the arguments regarding the plate calculations would be clear in tutorial as explained in next slide.

Tutorial

- Consider a rectification column for **N_2** and **O_2** separation operating at 1 atm. Determine the number of theoretical plates required to yield **97% N_2** at top and **95% O_2** at bottom. Feed stream is **50% N_2** and **50% O_2**. Molar fraction of liquid in feed stream is **0.7** mole liquid/mole mixture. The desired flow rate at the bottom product is **20** mole/sec and the heat removed in the condenser at top of the column is **500** kW.

Tutorial

Given
Working Pressure : 1 atm
Mixture : N_2 + O_2
Feed stream : 50% N_2 + 50% O_2
Bottom flow rate : 20 mole/sec = **B**
Feed liq. : 0.7 = **q**

For above mixture
Reqt. of N_2 (top) : 97% = $\mathbf{x_D}$
Reqt. of O_2 (bottom) : 95% = $\mathbf{x_B = 0.05}$
Total number of theoretical plates

Tutorial

$$F = B + D$$

- Mole balance

$$x_F F = x_B B + x_D D$$

- Mole balance for $\mathbf{N_2}$

Data	
x_F	0.5
x_B	0.05
x_D	0.97
B	20

$$F = 20 + D$$

$$0.5F = (0.05)(20) + (0.97)D$$

- Solving, we have
- F=39.14 mole/sec, D=19.14 mole/sec.

Tutorial

- Enthalpy balance

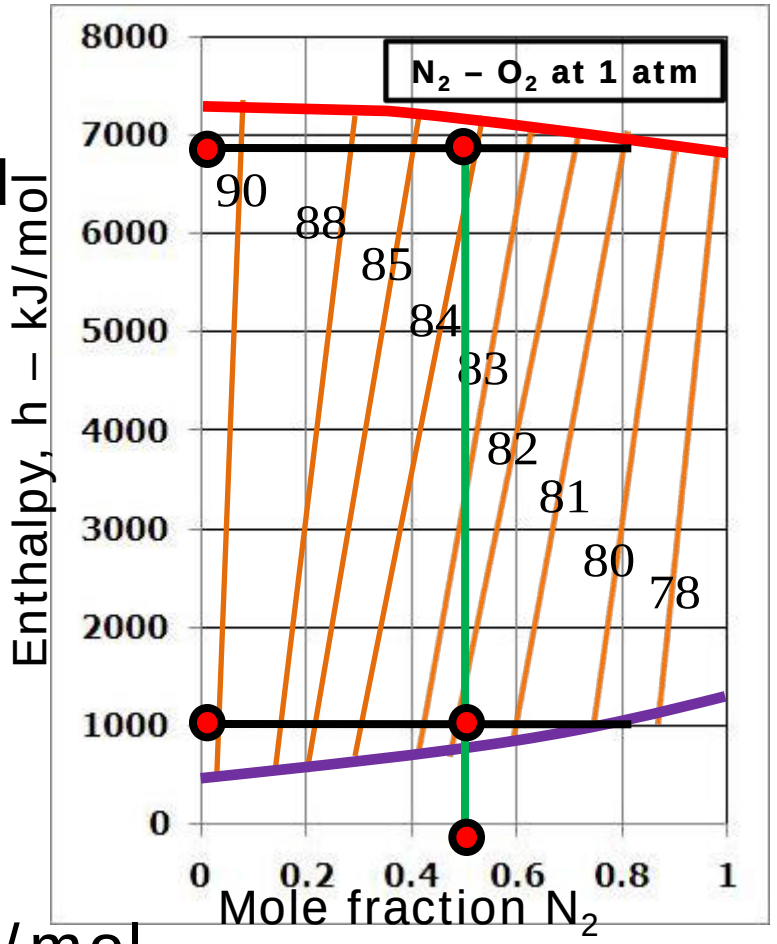
$$\dot{Q}_B = \dot{Q}_D + h_D D + h_B B - h_F F$$

- Fraction of stream in feed

$$q = \frac{H - h_F}{H - h}$$

- Rearranging, we have

$$h_F = qh + (1-q)H$$



- For 50% $\mathbf{N_2}$ + 50% $\mathbf{O_2}$
- h=1084 J/mol, H=6992 J/mol

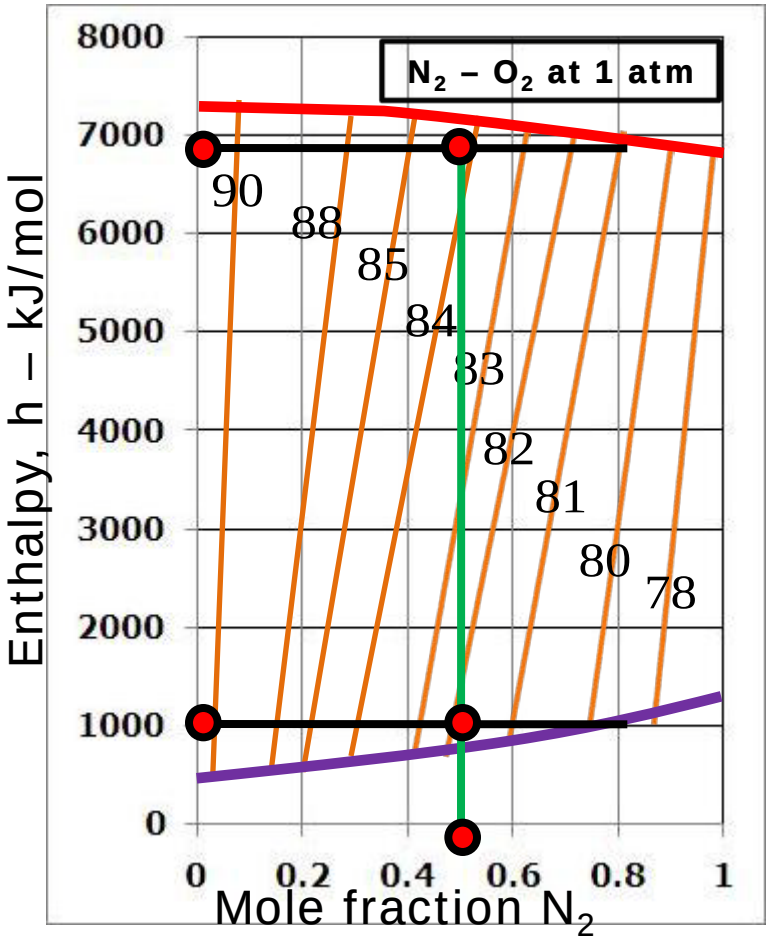
Tutorial

$$h_F = qh + (1-q)H$$

Data	
h	1084 J/m
H	6992 J/m
q	0.7

$$h_F = (0.7)1084 + (1-0.7)6992$$

$$h_F = 2856.4\ J/mol$$

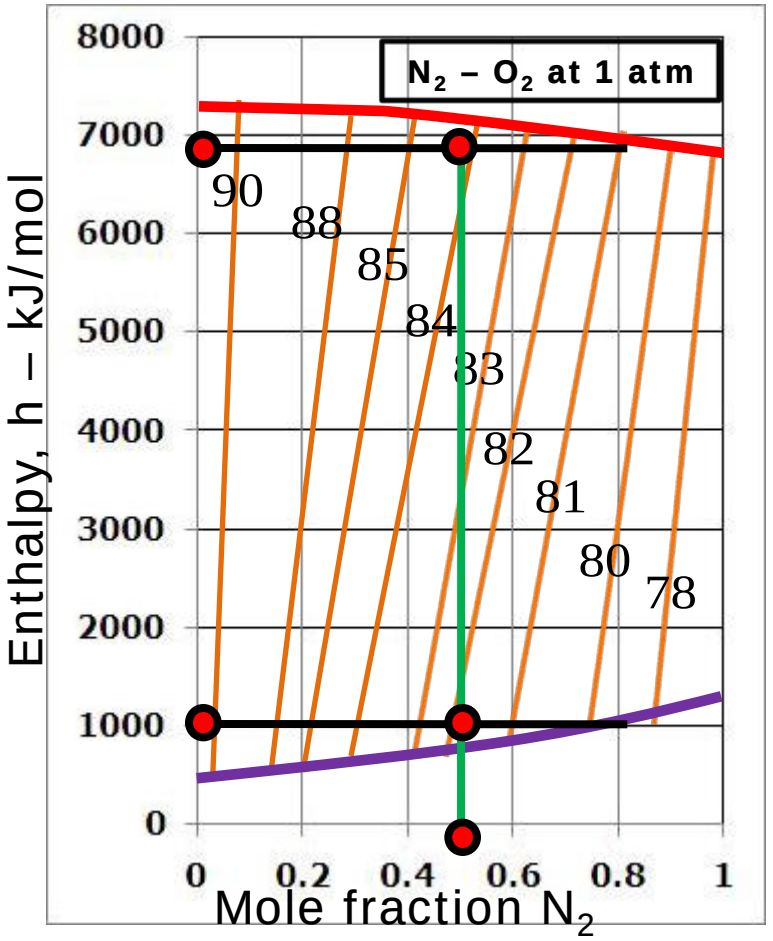


Tutorial

$$\dot{Q}_B = \dot{Q}_D + h_D D + h_B B - h_F F$$

Data	
Q_D	500 kW
h_D	1084 J/m
h_B	1084 J/m
h_F	2856.4 J/m

Data	
F	39.14
B	20
D	19.14



$$\dot{Q}_B = (500)10^3 + (1084)(19.14) + (1084)(20) - (2856.4)(39.14)$$

$$\dot{Q}_B = 430.6kW$$

Tutorial

- Operating line for **Enriching Section**

$$\frac{D}{V_n} = \frac{H_n - h_{n+1}}{\frac{\dot{Q}_D}{D} + h_D - h_{n+1}}$$

Data	
Q_D	500kW
H_n	6992 J/m
h_{n+1}	1084 J/m
h_D	1084 J/m
D	19.14 mol/s

$$\frac{D}{V_n} = \frac{6992 - 1084}{\frac{500000}{19.14} + 1084 - 1084}$$

$$\frac{D}{V_n} = 0.226$$

$$\frac{L_{n+1}}{V_n} = 1 - \frac{D}{V_n}$$

$$\frac{L_{n+1}}{V_n} = 1 - 0.226$$

$$\frac{L_{n+1}}{V_n} = 0.773$$

Tutorial

- Operating line for **Enriching Section**

$$y_n = \left(\frac{L_{n+1}}{V_n}\right) x_{n+1} + \left(\frac{D}{V_n}\right) x_D$$

$$\frac{D}{V_n} = 0.226$$

$$\frac{L_{n+1}}{V_n} = 0.773$$

$$y_n = (0.773)\, x_{n+1} + (0.226)(0.97)$$

Data	
x_D	0.97

$$y_n = 0.773 x_{n+1} + 0.22$$

Tutorial

- Operating line for **Stripping Section**

$$\frac{B}{V_m} = \frac{H_m - h_{m+1}}{\frac{\dot{Q}_B}{B} - h_B + h_{m+1}}$$

Data	
Q_B	430.6 kW
H_m	6992 J/m
h_{m+1}	1084 J/m
h_B	1084 J/m
B	20 mol/s

$$\frac{B}{V_m} = \frac{6992 - 1084}{\frac{(430.6)10^3}{20} - 1084 + 1084}$$

$$\frac{B}{V_m} = 0.274$$

$$\frac{L_{m+1}}{V_m} = 1 + \frac{B}{V_m}$$

$$\frac{L_{m+1}}{V_m} = 1 + 0.274$$

$$\frac{L_{m+1}}{V_m} = 1.274$$

Tutorial

- Operating line for **Stripping Section**

$$y_m = \left(\frac{L_{m+1}}{V_m}\right) x_{m+1} - \left(\frac{B}{V_m}\right) x_B$$

$$\frac{B}{V_m} = 0.274$$

$$\frac{L_{m+1}}{V_m} = 1.274$$

$$y_m = (1.274)\, x_{m+1} - (0.274)(0.05)$$

Data	
x_B	0.05

$$y_m = 1.274 x_{m+1} - 0.013$$

Tutorial

- Equation of **Feed Line**

$$q = \frac{H - h_f}{H - h}$$

$$y = \frac{q}{q-1}x + \frac{x_F}{1-q}$$

$$y = \frac{0.7}{0.7-1}x + \frac{0.5}{1-0.7}$$

$$y = -2.34x + 1.67$$

Data	
H	6992 J/m
h_f	2856.4 J/m
h	1084 J/m
x_F	0.5

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- Summarizing, we have the following.
- OP line for enriching section : $y_n = 0.773x_{n+1} + 0.22$
- OP line for stripping section : $y_m = 1.274x_{m+1} - 0.013$
- q line : $y = -2.34x + 1.67$
- The stair casing procedure is shown on an excel sheet to have a better understanding of the method.

Tutorial

- From the excel sheet, it is clear that the total number of vertical lines are **9**.

- Therefore, the total number of theoretical plates for this column can be tabulated as shown below.

McCabe – Thiele Method	
Enriching Section	3 + 1 (Condenser)
Stripping Section	6 + 1 (Boiler)

Summary

- Equilibrium curve gives the relation between liquid composition **(x_n)** and vapor composition **(y_n)** on the same plate.

- The OP lines relate the variation of liquid (x_{n+1}) and vapor (y_n) mole fractions of a particular component across the length of the column.

- The plate calculation is a stair casing method which involves locating equilibrium conditions on equilibrium line and OP line.

Summary

- In a McCabe – Thiele diagram
  - Each horizontal line gives the condition of liquid – vapor on the same plate which are in thermal equilibrium.
  - Each vertical line gives the vapor condition for the plate with respect to liquid that leaves the earlier plate on the top.
- The total number of vertical lines in top and bottom section, together with boiler and condenser surfaces is the total number of theoretical plates required.

Thank You!