
CRYOGENIC ENGINEERING
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Lecture No - 25

Earlier Lecture

- In the earlier lecture, we have seen the importance of equilibrium curve, OP lines and **q** line in calculation of number of plates using graphical **McCabe – Thiele** method.

- A tutorial problem was solved using an excel sheet to understand the stair casing method and the concept of equilibrium of liquid – vapor mixture.

Earlier Lecture

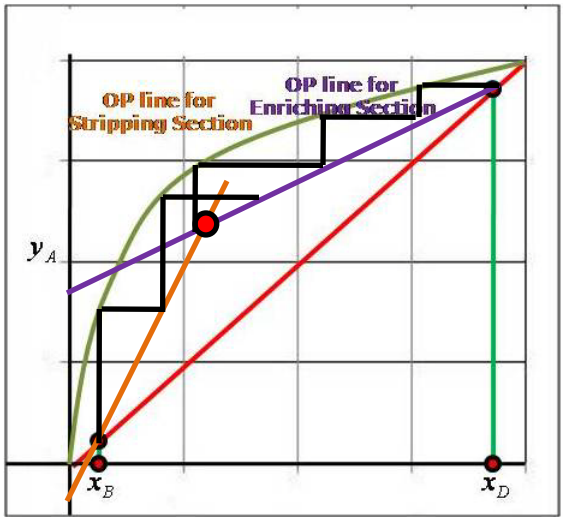
- The total number of theoretical plates is given by the number of vertical lines between the equilibrium and the operating lines.

Outline of the Lecture

Topic : Gas Separation (contd)

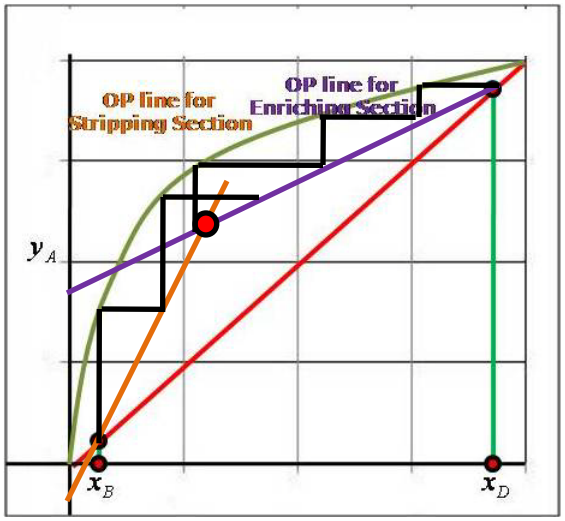
- Concept of Reflux
- Parametric study – Tutorial
- Summary of the topic
- Assignments

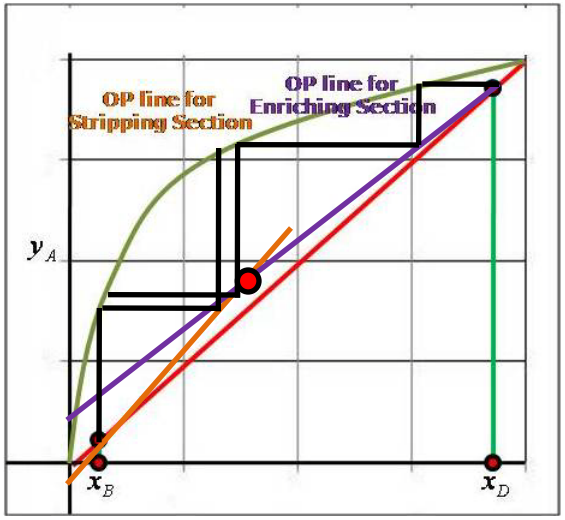
McCabe – Thiele Method



- Consider the OP lines for a column as shown in the figure.
- The number of theoretical plates in this case are **5**.

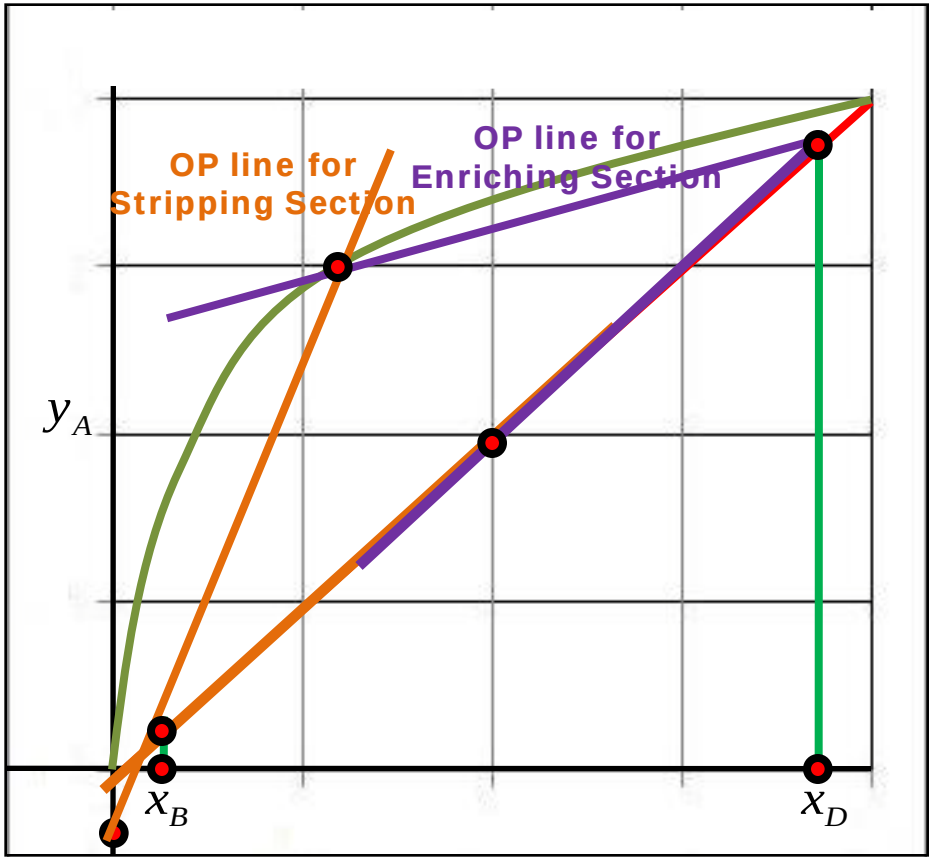
McCabe – Thiele Method





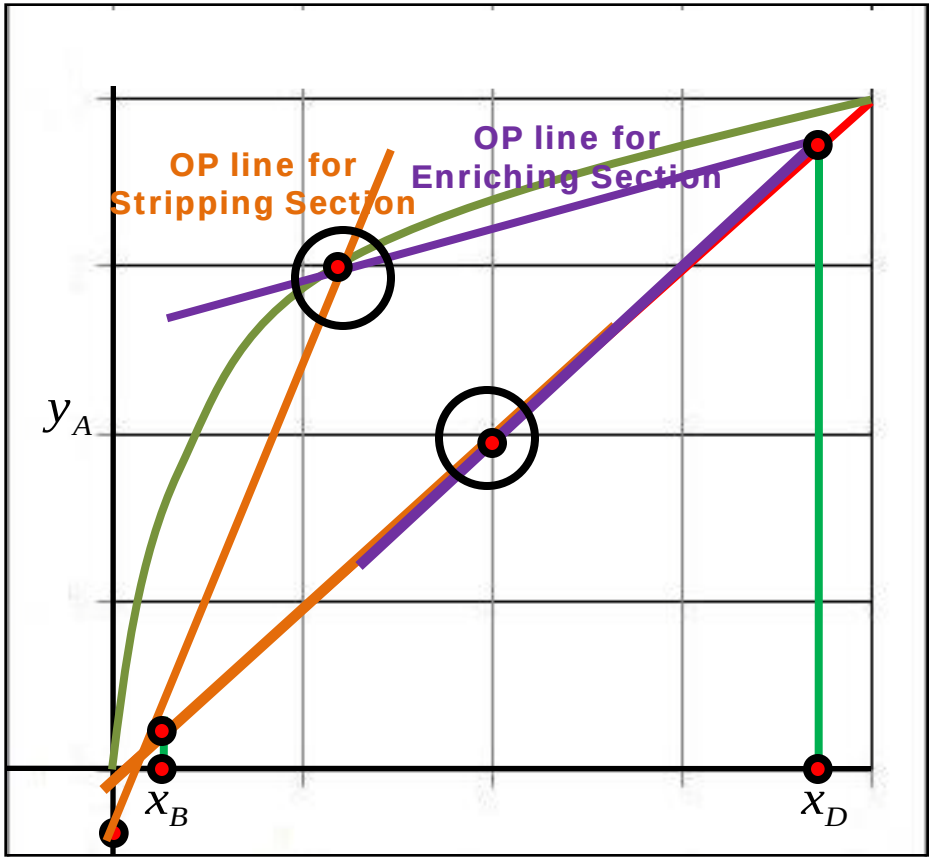
- If the OP lines approach closer to the diagonal, as shown in the figure,
- The number of theoretical plates are reduced to **4**.

McCabe – Thiele Method



- The extreme cases of point of intersection of OP lines are on
  - Equilibrium curve
  - **45º** diagonal
- The point of intersection cannot be above equilibrium and below diagonal lines because, the law of conservation of mass gets violated.

McCabe – Thiele Method



- Therefore, it is clear that the number of theoretical plates are
  - Maximum, when the point of intersection is on equilibrium line.
  - Minimum, when the point of intersection is on diagonal line.

Reflux Ratio

- In the earlier lecture, we have seen the equation for an operating line (for enriching section) as

$$y_n = \left(\frac{L_{n+1}}{V_n}\right) x_{n+1} + \left(\frac{D}{V_n}\right) x_D$$

- In the above equation,
  - $\mathbf{L_{n+1}/V_n}$ is called as internal reflux ratio **(IR)**.
  - $\mathbf{D/V_n}$ is called as external reflux ratio **(ER)**.

- From the earlier lecture, we have

$$\frac{L_{n+1}}{V_n} = 1 - \frac{D}{V_n}$$

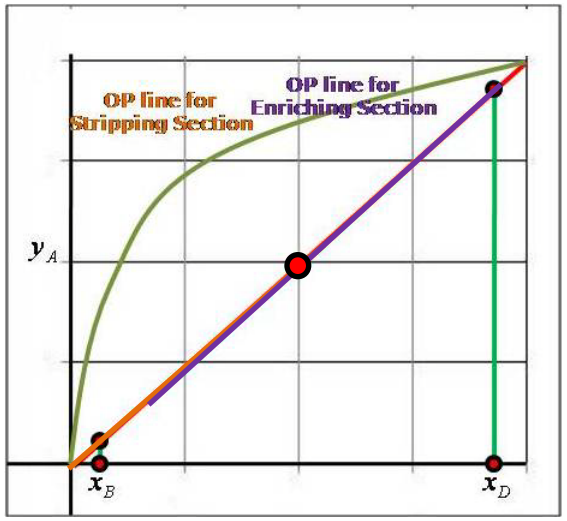
- Therefore, **IR = 1 - ER**.

Reflux Ratio

- The column has two limiting reflux conditions. They are

- **Condition of Total Reflux**
  - In this condition, all the vapor of top plate is condensed and is returned as liquid to column.

  - Hence, **L / V = 1** (for enriching section) making the OP lines to match with diagonal. The column has minimum number of plates.



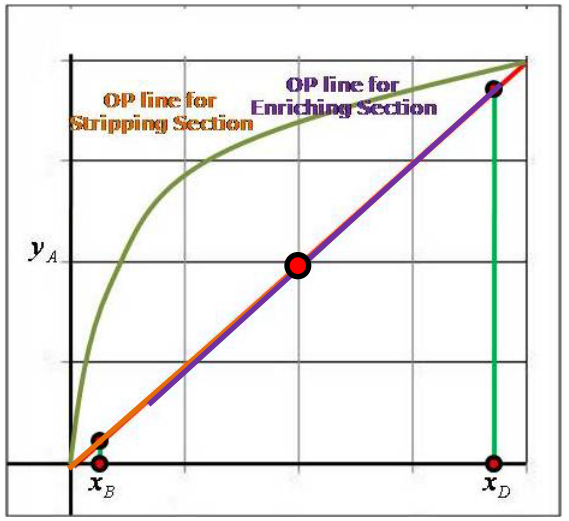
Reflux Ratio

- For **IR=1**, from the equation **IR=1 – ER**, we get **ER=0**.

$$ER = \frac{D}{V_n} = 0$$

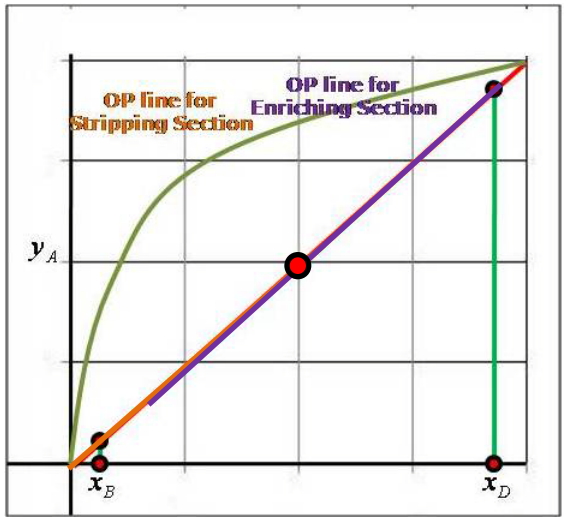
- Hence, in this condition, the product output of column is **zero**.

- Continuing further, it is clear that $\mathbf{L_{n+1}}$ is directly dependent on $\mathbf{Q_D}$. That is, as the value of $\mathbf{Q_D}$ increases, **IR** approaches **1**.



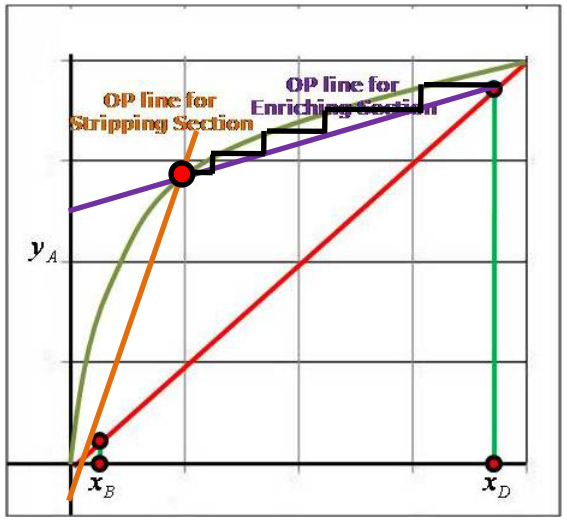
Reflux Ratio

- Summarizing, at high **IR** values (approaching **1**), we have
  - Reduced number of plates.
  - For a given **D**, high condenser loads.
  - If **IR=1**, the product output of the column is **zero** (condition of total reflux).



Reflux Ratio

- **Condition of Minimum Reflux**
  - In this condition, the amount of liquid to the column is decreased with a decrease in vapor condensed and returned to the top of the column.
  - Here, **L / V < 1** (for enriching section) makes the OP lines to move closer to the equilibrium curve. The column has maximum number of plates.



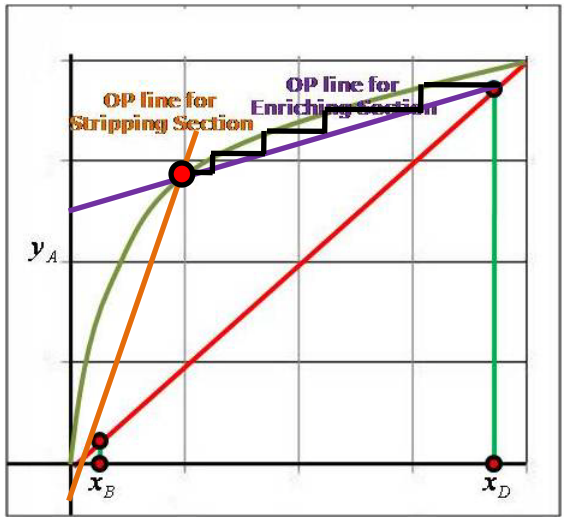
Reflux Ratio

- Again, for **IR < 1**, from the equation **IR = 1 – ER**, we get **ER > 0**.

$$ER = \frac{D}{V_n} > 0$$

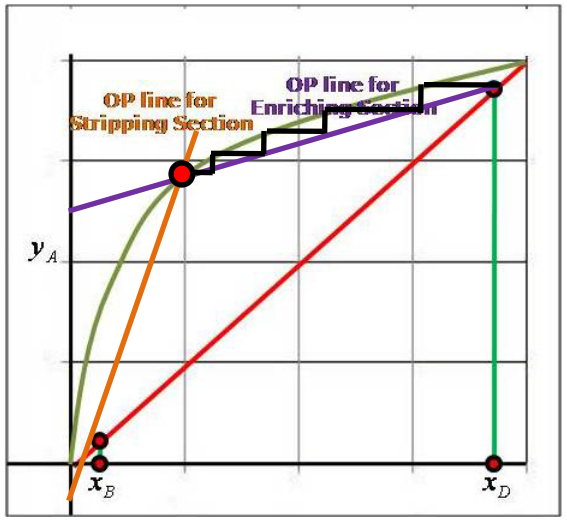
- Hence, in this condition, unlike the earlier case, there is a finite quantity of product output.

- We know that L_{n+1} is directly dependent on Q_D, therefore as Q_D decreases, **IR** also decreases.



Reflux Ratio

- Summarizing, at low **IR** values (less than **1**), we have
  - Increased number of plates.
  - Low condenser loads.
  - A finite quantity of product output from the column.



Reflux Ratio

- For the extreme cases of high and low reflux ratios,
  - Plates – minimum, high refrigeration costs.
  - Plates – maximum, low refrigeration costs.

- But in practical cases, the design of column is a compromise between the number of plates and refrigeration costs.

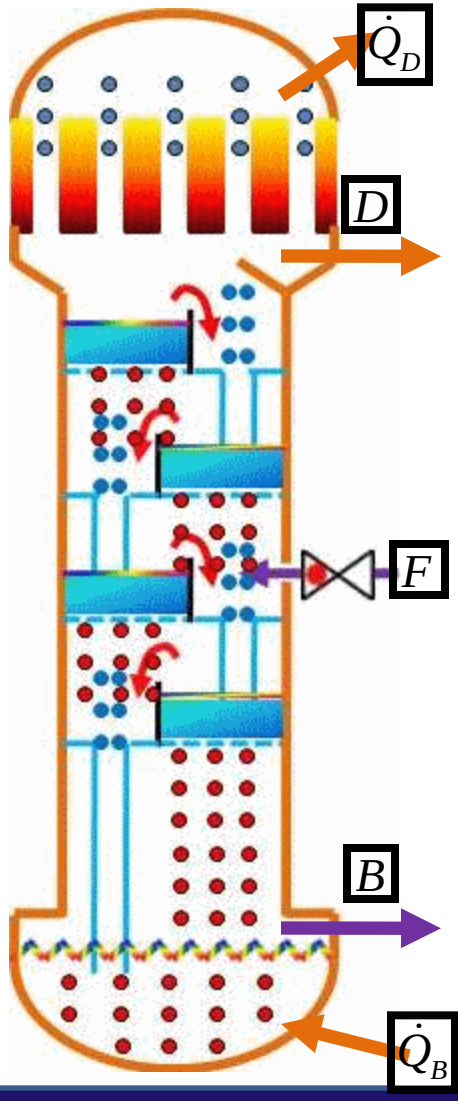
Reflux Ratio

- The typical values of Reflux ratio for a distillation column are **1.15** to **1.35** times the minimum reflux ratio.

- Similarly, for a cryogenic column, we have typical values as **1.05** to **1.15** times the minimum reflux ratio.

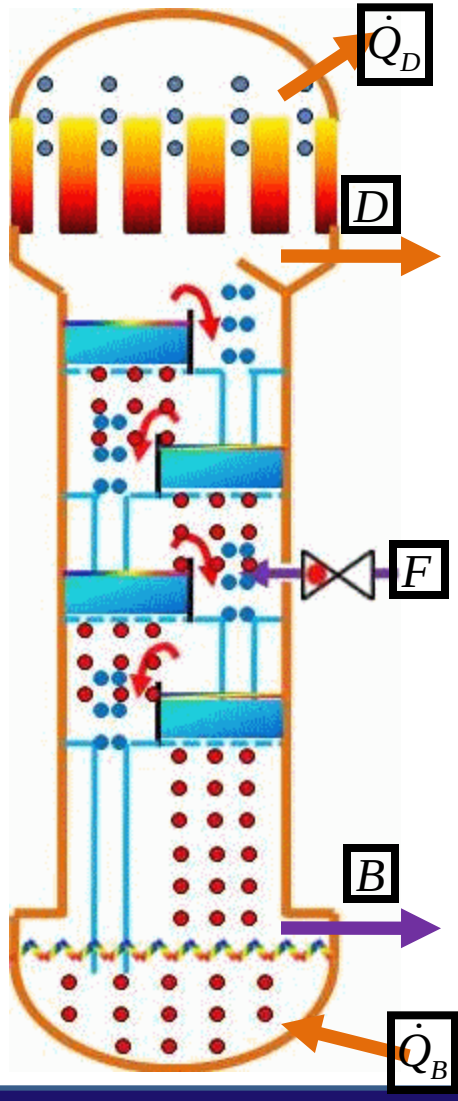
- It is important to note that the minimum reflux ratio is dependent on equilibrium curve.

Parametric Study



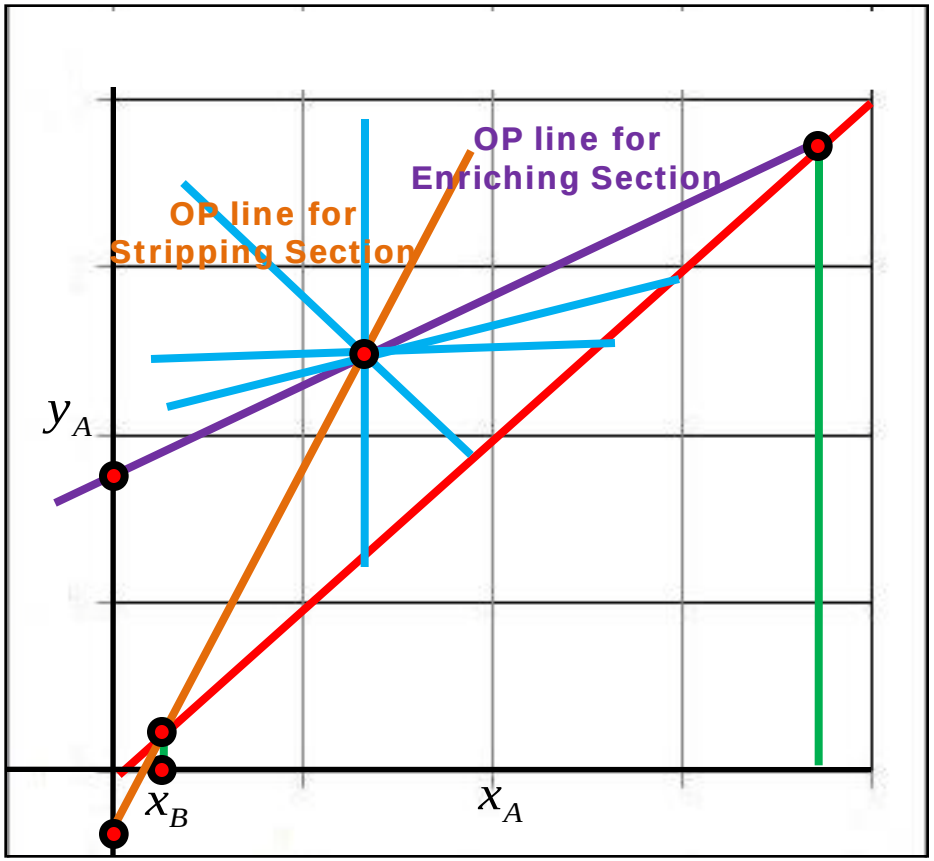
- The slope of an OP line is a function of both **L** and **V**, which are in turn dependent on **D**, $\mathbf{Q_D}$, **B**, $\mathbf{Q_B}$.
- It is important to note that, any change in **D**, alters other parameters as well and vice versa.
- This is to conform with the laws of conservation of mass and energy for the column as a whole.

Parametric Study



- We know that the impurity of component **A** in the bottom product is given by x_B.
- Reducing x_B implies more purity of the bottom product, hence, more number of plates in lower section.
- Similarly, the purity of component **A** in the top product is given by x_D.
- Again, increasing x_D would increase the number of plates in top section.

Parametric Study



- The parameter **q** is very important in determining
  - The slope of **q** line.
  - An approximate feed inlet position.
  - The theoretical number of plates.

McCabe – Thiele Method

- In the earlier lecture, we have solved the following tutorial.

- Consider a rectification column for **N_2** and **O_2** separation operating at 1 atm. Determine the number of theoretical plates required to yield **97% N_2** at top and **95% O_2** at bottom. Feed stream is **50% N_2** and **50% O_2**. Molar fraction of liquid in feed stream is **0.7** mole liquid/mole mixture. The desired flow rate at the bottom product is **20** mole/sec and the heat removed in the condenser at top of the column is **500** kW.

McCabe – Thiele Method

Given
Working Pressure : 1 atm
Mixture : N_2 + O_2
Feed stream : 50% N_2 + 50% O_2
Bottom flow rate : 20 mole/sec = **B**
Feed liq. : 0.7 = **q**

For above mixture
Reqt. of N_2 (top) : 97% = $\mathbf{x_D}$
Reqt. of O_2 (bottom) : 95% = $\mathbf{x_B = 0.05}$

- For the above problem, calculate the number of theoretical plates for changes in $\mathbf{Q_D}$, **q**, $\mathbf{x_B}$ and $\mathbf{x_D}$.

McCabe – Thiele Method

- From the earlier lecture, we have the following.
- OP line for enriching section : $y_n = 0.773x_{n+1} + 0.22$
- OP line for stripping section : $y_m = 1.274x_{m+1} - 0.013$
- **q** line : $y = -2.34x + 1.67$
- In the earlier lecture, we have seen the stair casing procedure on an excel sheet.

McCabe – Thiele Method

- From the excel sheet, it was clear that the total number of vertical lines are **9**.

- Therefore, the total number of theoretical plates for the column are tabulated as shown below.

McCabe – Thiele Method	
Enriching Section	3 + 1 (Condenser)
Stripping Section	6 + 1 (Boiler)

Parametric Study – Tutorial

- Let us see the effect of Q_D, holding all other parameters as constant.

	I	II	III	IV	V
Q_D(kW)	500	350	900	9000	300
Q_B(kW)	430.6	280.6	830.6	8930	930.6
$(L/V)_{ES}$	0.77	0.68	0.87	0.99	0.89
$(L/V)_{SS}$	1.27	1.42	1.14	1.01	1.13
Plates	3 + 6	4 + 8	3 + 5	3 + 5	4 + 9

Parametric Study – Tutorial

- Let us see the effect of x_D and x_B, holding all other parameters as constant.

x_D	Plates
0.97	3 + 6
0.99	4 + 6

x_B	Plates
0.05	3 + 6
0.02	3 + 8

Parametric Study – Tutorial

- Let us see the effect of **q**, holding all other parameters as constant.

q	Slope	Plates
q=0.7	-2.3	3 + 6
q=1	∞	2 + 6
q=0	0	4 + 6

Parametric Study – Summary

- Summarizing the parametric study, we have the following inferences.

- The number of theoretical plates decrease, as the OP lines approach close to the diagonal.

- The slope of the OP line of enriching section decreases by increasing **D** or reducing $\mathbf{Q_D}$.

Parametric Study – Summary

- Decreasing x_B (more purity of bottom product) or increasing x_D (more purity of top product), increases the number of plates in the column.

- The value of **q** is vital in determining the **q** line, an approximate feed inlet position and number of theoretical plates.

Summary

- In an Ideal Gas Separation system, all the processes are reversible and the work requirement is called as an Ideal Work.

- The equation for work requirement per mole of mixture to separate a mixture with **N** constituents is

$$\frac{-W_i}{n_m} = \Re T_m \sum_{j=1}^{N} y_j \ln\left(\frac{1}{y_j}\right)$$

- The work/mole of mixture is always less than work/mole of its constituents for any mixture. Also, $\mathbf{W_{i,m} / n_m}$ is maximum when the percentage compositions of all its ingredients are equal.

Summary

- If number of components, number of phases and degrees of freedom for a mixture in thermal equilibrium are denoted by C , P and F respectively, then the Gibbs Phase Rule

$$F = C - P + 2$$

- Dalton's Law relates partial pressures of non – reacting ideal gases.

- Raoult's Law relates the vapor pressure with the liquid mole fraction of a component in a mixture.

- Gibbs – Dalton's Law is an application of Dalton's law to the vapor above the liquid phase.

Summary

- **Murphree efficiency** of a plate is defined as the ratio of actual change in mole fraction to the maximum possible change that can occur.

- **McCabe – Thiele** method is less general and is widely used for binary mixtures at cryogenic temperatures.

- The major assumption in this method is that the liquid and vapor enthalpies are independent of mole fraction.

Summary

- The equations of operating lines for enriching and stripping sections are

$$y_n = \left(\frac{L_{n+1}}{V_n}\right) x_{n+1} + \left(\frac{D}{V_n}\right) x_D$$

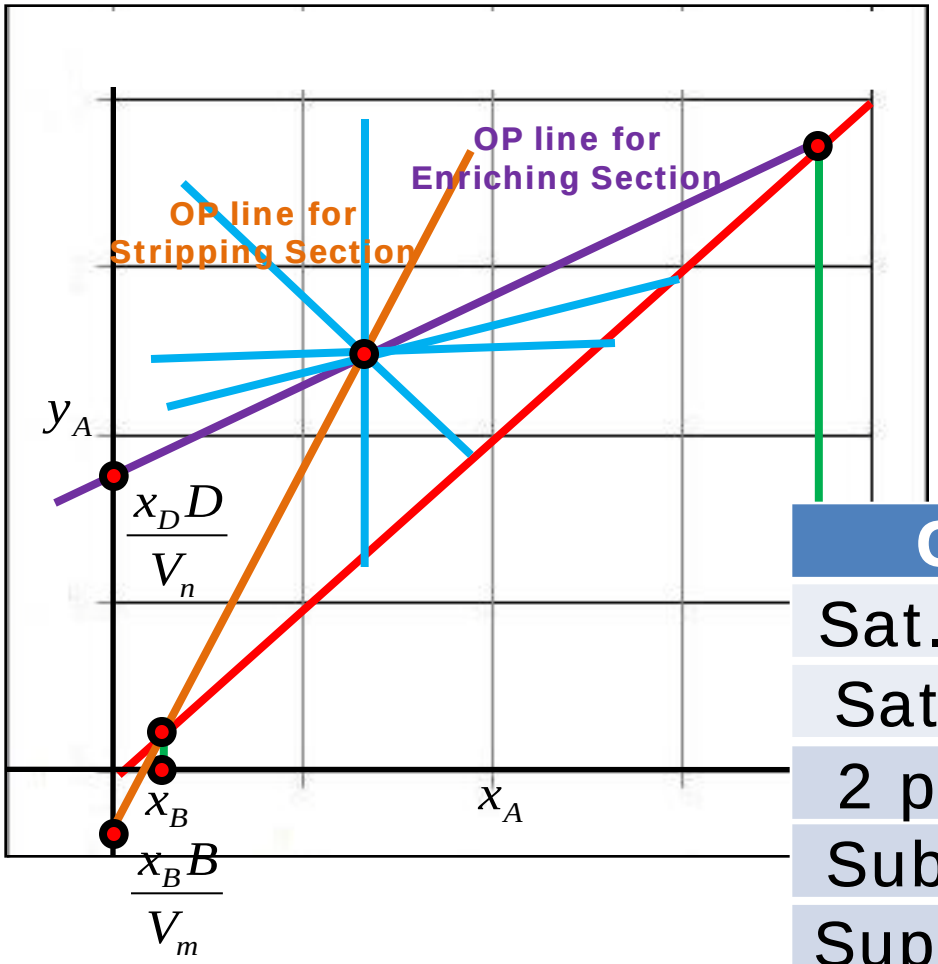
$$y_m = \left(\frac{L_{m+1}}{V_m}\right) x_{m+1} - \left(\frac{B}{V_m}\right) x_B$$

- The locus of intersection of these operating lines denotes the feed condition. It is given as

$$y = \left(\frac{q}{q-1}\right) x + \frac{x_F}{1-q}$$

- The point of intersection of feed line or **q** line and **y = x** gives the content of a component in the feed, $\mathbf{x_F}$.

McCabe – Thiele Method



- The equation of feed line is

$$y = \left(\frac{q}{q-1}\right)x + \frac{x_F}{1-q}$$

Condition	q	Slp
Sat. Vap. ($h_F = H$)	$q = 0$	0
Sat. Liq. ($h_F = h$)	$q = 1$	∞
2 ph. ($H < h_F < h$)	$0 < q < 1$	-ve
Sub. Liq. ($h_F < h$)	$q > 1$	+ve
Sup. Vap. ($h_F > h$)	$q < 0$	+ve

Summary

- Equilibrium curve gives the relation between liquid composition **(x_n)** and vapor composition **(y_n)** on the same plate.

- The OP lines relate the variation of liquid (x_{n+1}) and vapor (y_n) mole fractions of a particular component across the length of the column.

- The plate calculation is a stair casing method which involves locating equilibrium conditions on equilibrium line and OP line.

Summary

- In a McCabe – Thiele diagram
  - Each horizontal line gives the condition of liquid – vapor on the same plate which are in thermal equilibrium.
  - Each vertical line gives the vapor condition for the plate with respect to liquid that leaves the earlier plate on the top.

- The total number of vertical lines in top and bottom section, together with boiler and condenser surfaces is the total number of theoretical plates required.

Summary

- At high **IR** values, we have
  - Reduced number of plates.
  - High condenser loads.
  - Product output is zero (**IR=1**).

- At low **IR** values, we have
  - Increased number of plates.
  - Low condenser loads.
  - Finite quantity of product output.

- The number of theoretical plates decrease, as the OP lines approach close to the diagonal.

Summary

- Decreasing x_B (more purity of bottom product) or increasing x_D (more purity of top product), increases the number of plates in the column.

Assignment - 1

- Determine the number of theoretical plates required to yield **95% N_2** at top and **96% O_2** at bottom. Feed stream is **70% N_2** and **30% O_2**. the operating pressure is 1 atm. Molar fraction of liquid in feed stream is **0.5** mole liquid/mole mixture. The desired flow rate at the bottom product is **30** mole/sec and the heat removed in the condenser at top of the column is **2000** kW.

- Also, calculate the maximum and minimum number of plates for the extreme cases of Q_D.

Answers

- OP line for enriching section : $y_n = 0.77 x_{n+1} + 0.22$
- OP line for stripping section : $y_m = 1.11 x_{m+1} - 0.001$
- **q** line : $y = -1.0x + 1.4$
- The total number of vertical lines are **8**.

McCabe – Thiele Method	
Enriching Section	2 + 1 (Condenser)
Stripping Section	6 + 1 (Boiler)

Thank You!