CRYOGENIC ENGINEERING

Prof. Milind D. Atrey

Department of Mechanical Engineering,
IIT Bombay

Lecture No - 27

Earlier Lecture

- A Cryocooler is a mechanical device operating in a closed cycle, which generates low temperature.
- It eliminates cryogen requirement, offers reliable operation and is also cost effective.
- Heat exchangers can either be regenerative or recuperative type depending upon heat exchange.
- **Recuperative Type:** J – T, Brayton, Claude.
- **Regenerative Type:** Stirling, GM, Pulse Tube.

Outline of the Lecture

Topic : Cryocoolers

- Ideal Stirling cycle
- Working of Stirling Cryocooler
- Schmidt's Analysis
- Conclusions

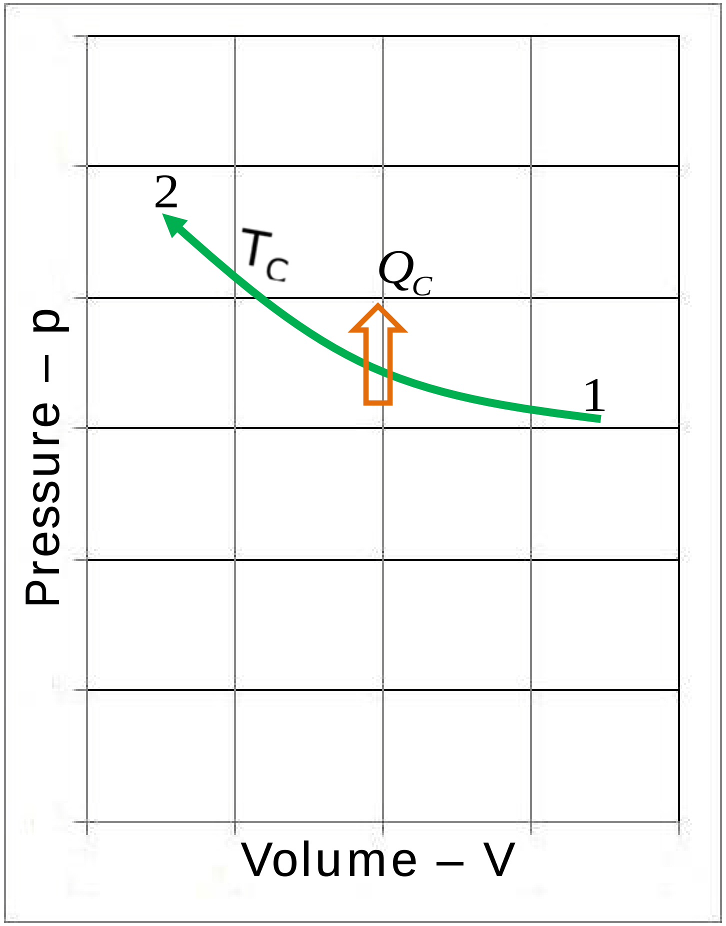
History

- A well developed and a most commonly used Cryocooler is the Stirling Cycle Cryocooler.
- This cycle was first conceived by Robert Stirling in the year 1815. It was an engine cycle and was aimed to produce work (engine).
- The important events that occurred in the history of cryocoolers are as given in the next slide.

The Chronology

Year	Event
1815	Robert Stirling – Stirling Engine
1834	John Herschel – concept of using as a cooler
1861	Alexander Kirk – The concept into practice
1873	Davy Postle – Free Piston system
1956	Jan Koehler – First commercial machine for air liquefaction
1965	Jan Koehler – Nitrogen Liquefaction

An Ideal Stirling Cycle



- Consider a p – V chart as shown in the figure.

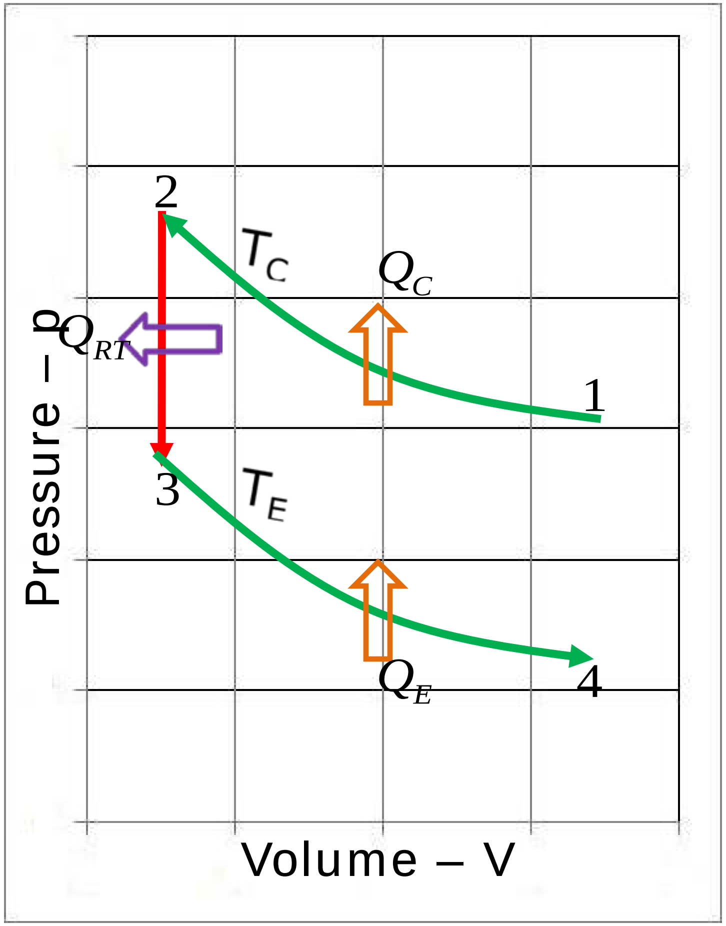
- **1→2:** Isothermal compression at **T_C**.

$$p_1V_1 = p_2V_2$$

$$T_1 = T_2 = T_C$$

$$dQ = dW = -\Re T_C \ln\left[\frac{V_2}{V_1}\right]$$

An Ideal Stirling Cycle



- **2→3:** Constant volume heat rejection.

$$V_2 = V_3$$

$$dQ = +C_V\left(T_E - T_C\right)$$

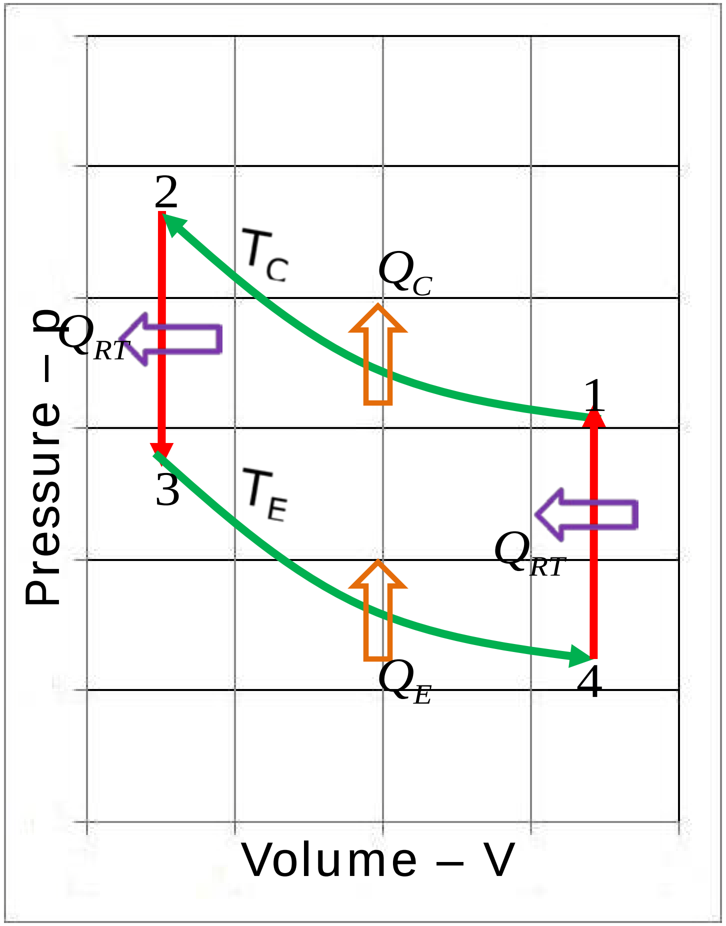
- **3→4:** Isothermal expansion.

$$p_3V_3 = p_4V_4$$

$$T_3 = T_4 = T_E$$

$$dQ = dW = -\Re T_C \ln\left[\frac{V_4}{V_3}\right]$$

An Ideal Stirling Cycle



- **4→1:** Constant volume heat absorption.

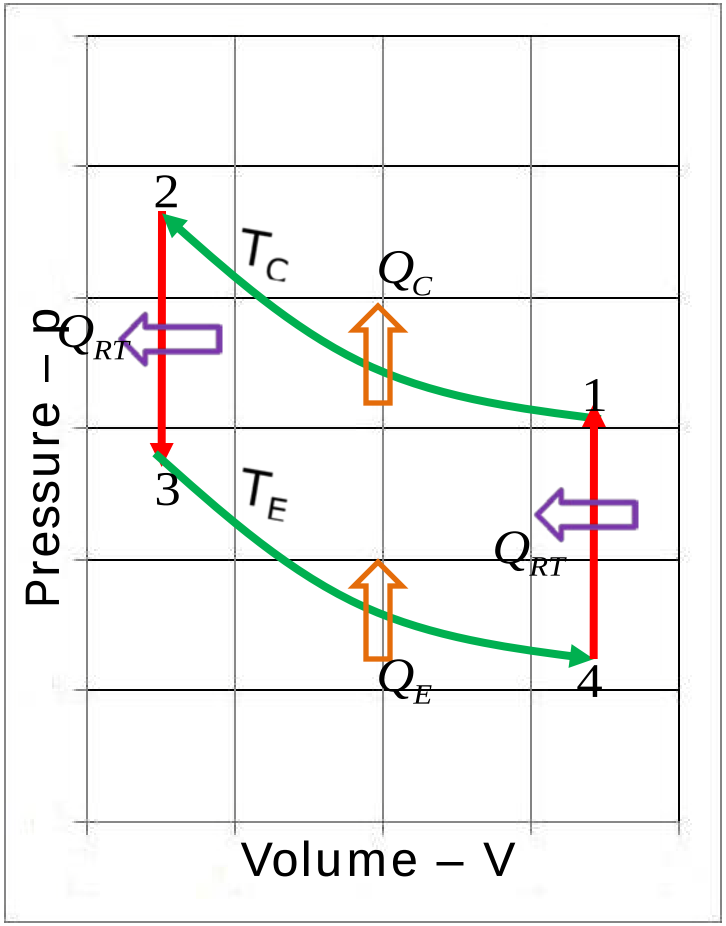
$$V_4 = V_1$$

$$dQ = -C_V\left(T_C - T_E\right)$$

$$COP = \frac{Q_E}{Q_C - Q_E}$$

$$= \frac{+\Re T_E \ln\left[\frac{V_4}{V_3}\right]}{-\Re T_C \ln\left[\frac{V_2}{V_1}\right] - \Re T_E \ln\left[\frac{V_4}{V_3}\right]}$$

An Ideal Stirling Cycle



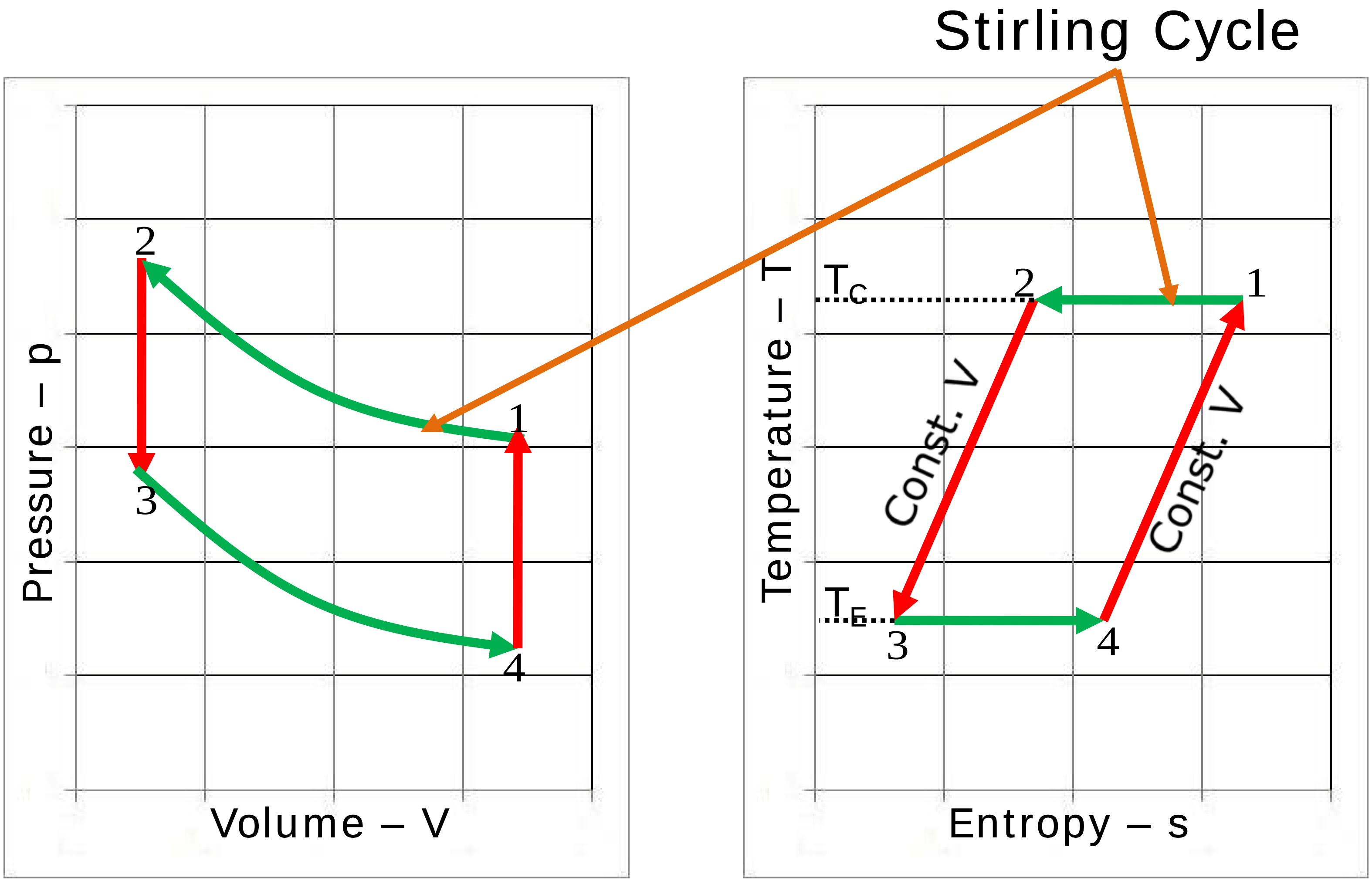
$$\boxed{\frac{V_2}{V_1}} = \frac{V_3}{V_4}$$

$$= \frac{+\Re T_E \ln\left[\frac{V_4}{V_3}\right]}{-\Re T_C \ln\boxed{\left[\frac{V_2}{V_1}\right]} - \Re T_E \ln\left[\frac{V_4}{V_3}\right]}$$

$$COP = \frac{T_E}{T_C - T_E}$$

$\text{COP}_{(\text{Stirling})} = \text{COP}_{(\text{Carnot})}$

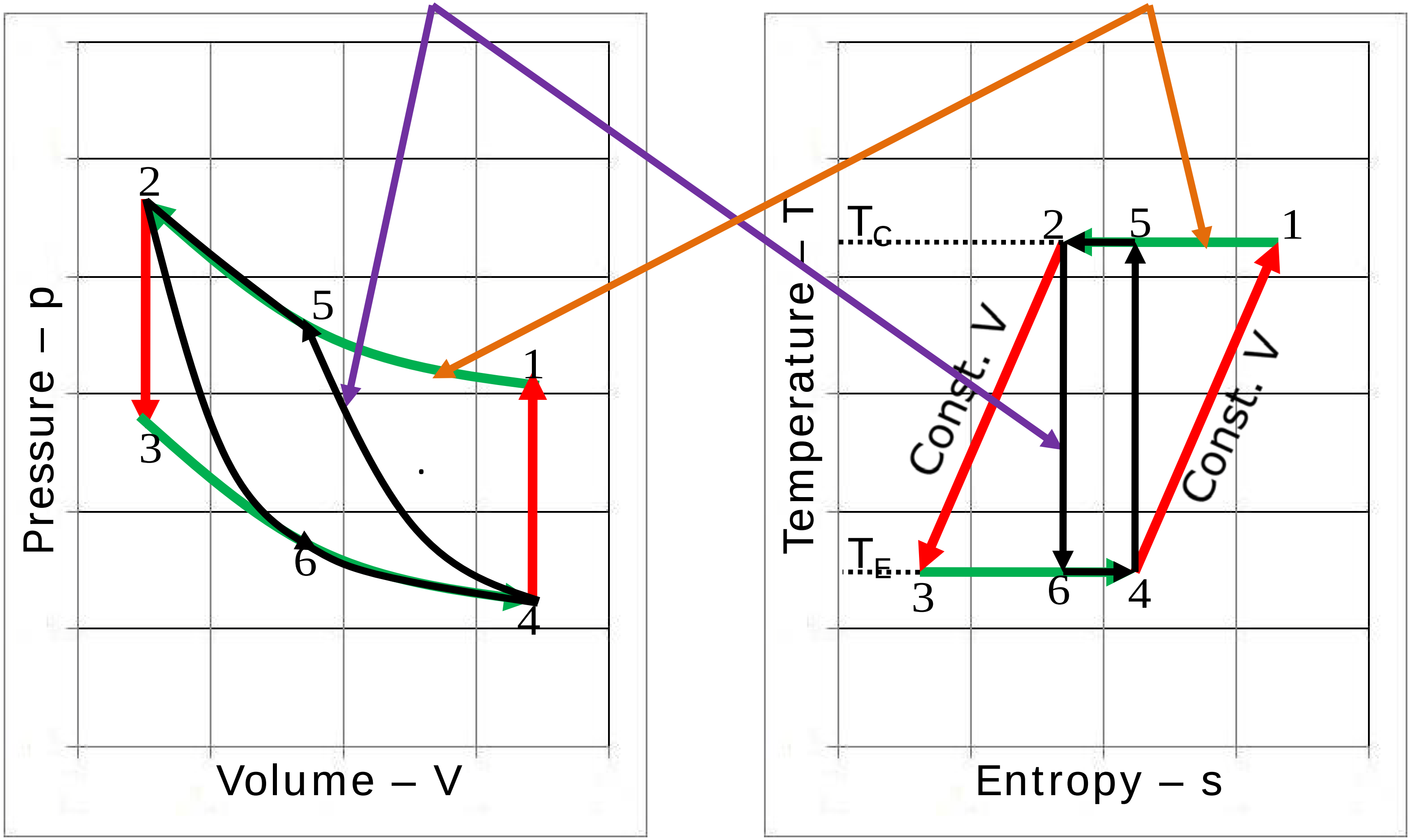
Stirling & Carnot Cycles


Stirling Cycle

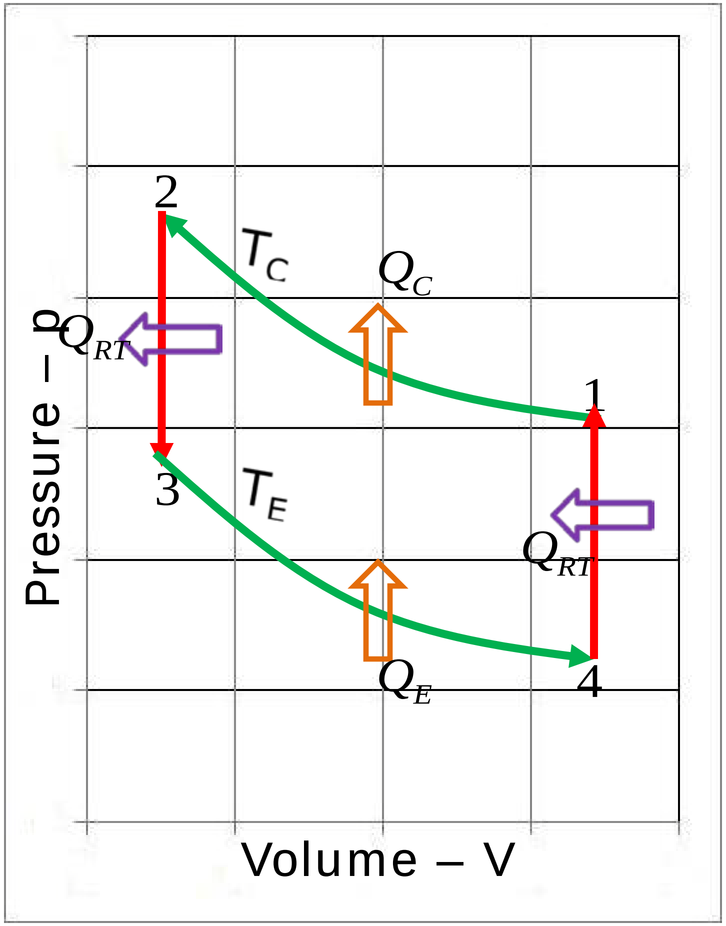
Stirling & Carnot Cycles

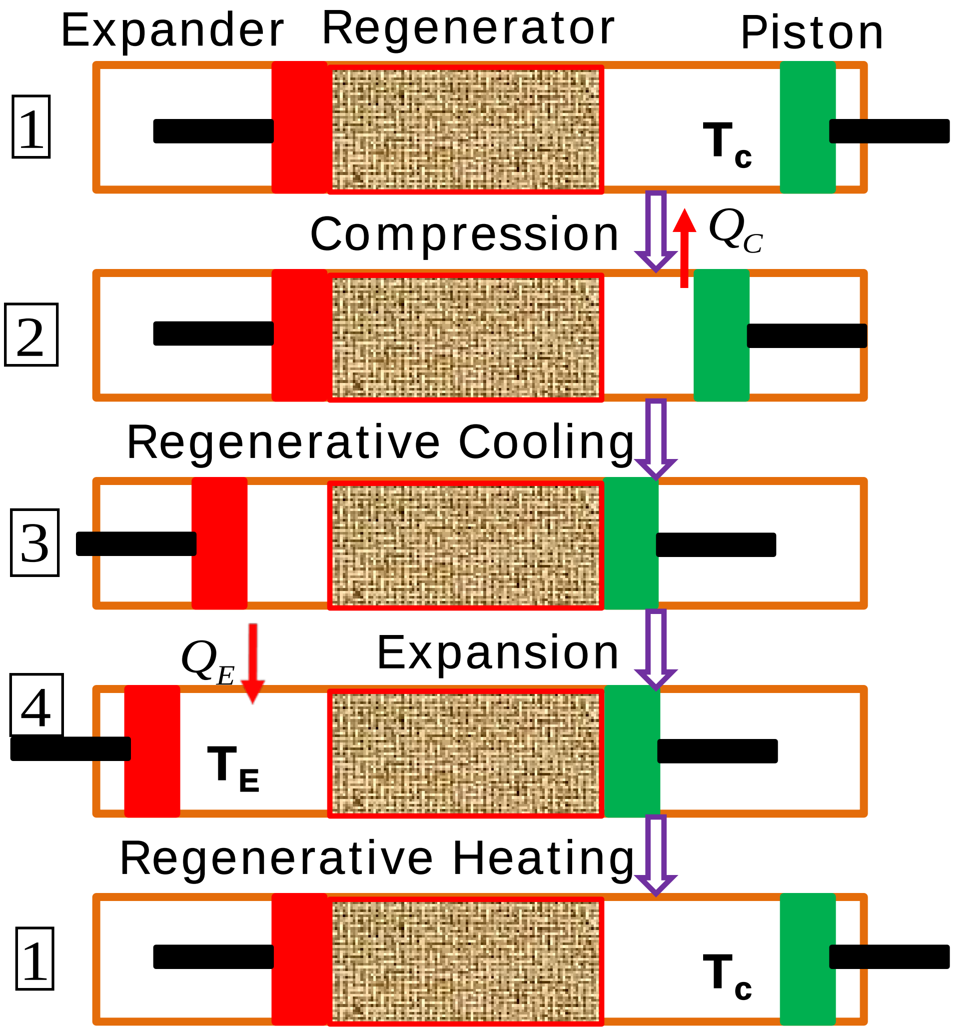
Carnot Cycle

Stirling Cycle

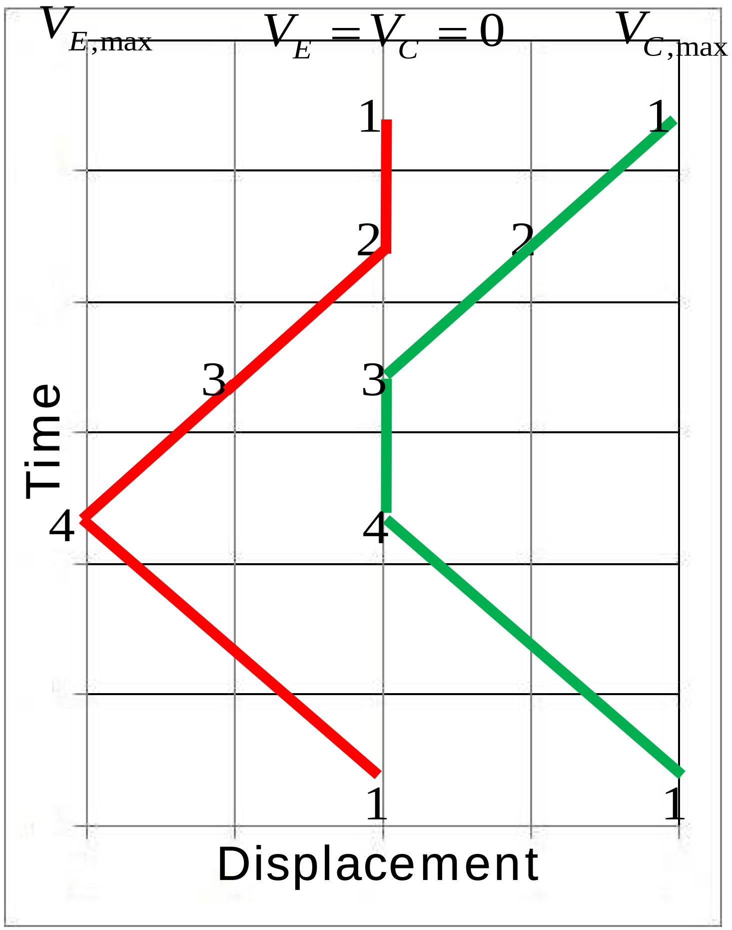


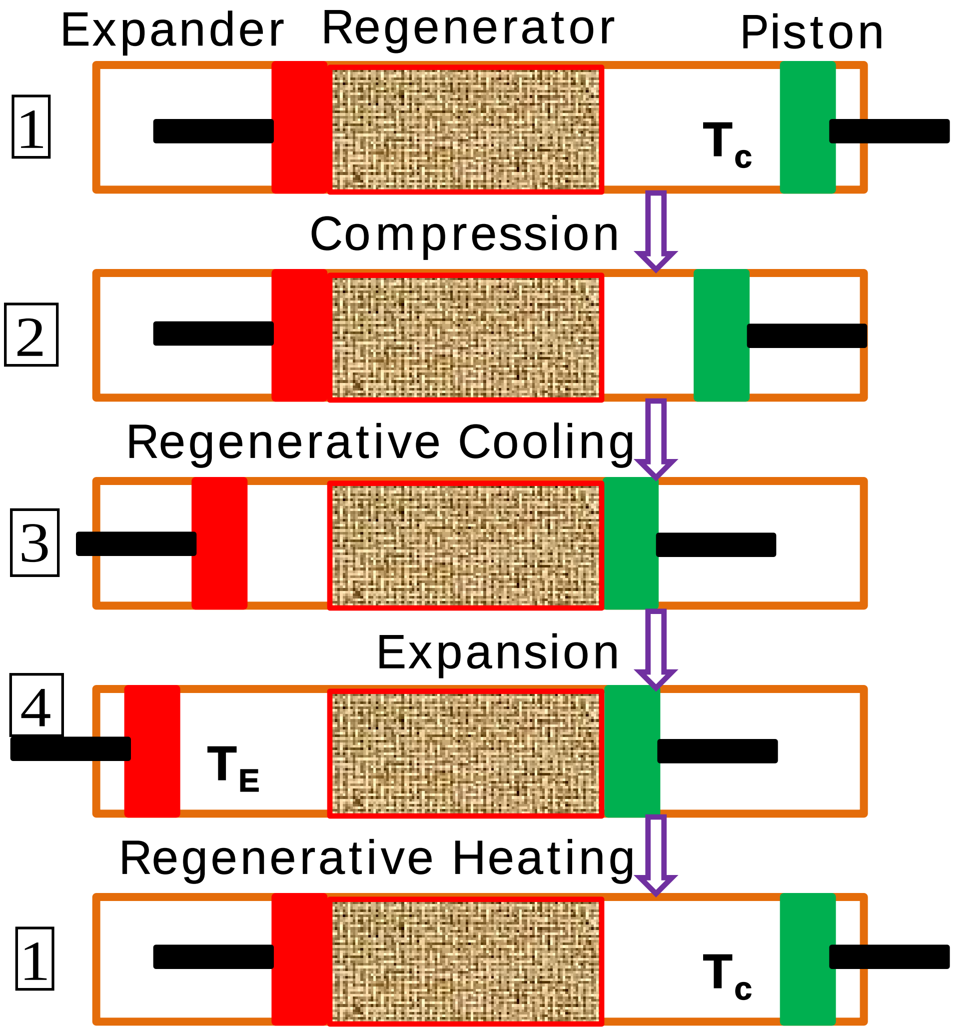
Ideal Stirling Cycle



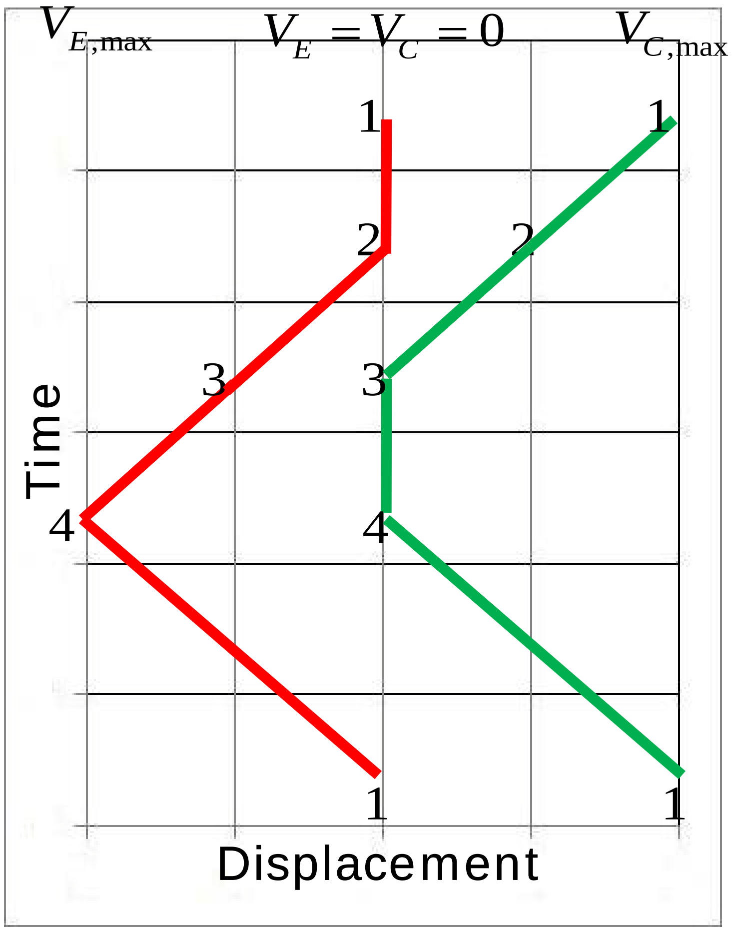


Ideal Stirling Cycle



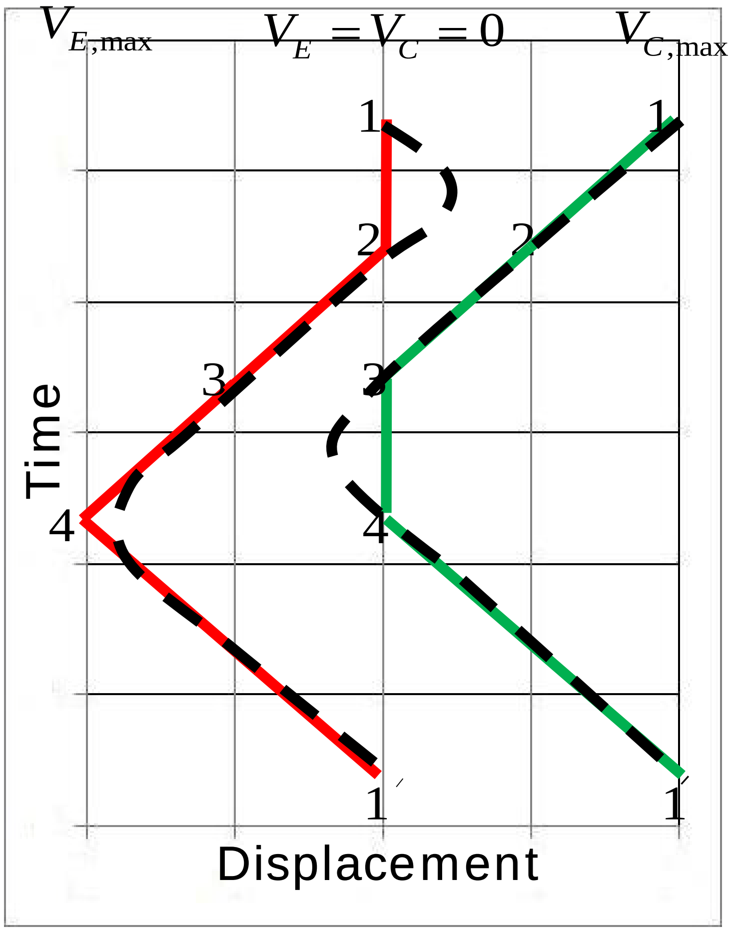


Ideal Stirling Cycle



- As mentioned in the earlier lecture, the characteristics of a Stirling cycle are
  - High frequency.
  - Regenerative heat exchanger.
  - Phase difference between the piston and the displacer motions.

Actual Stirling Cycle



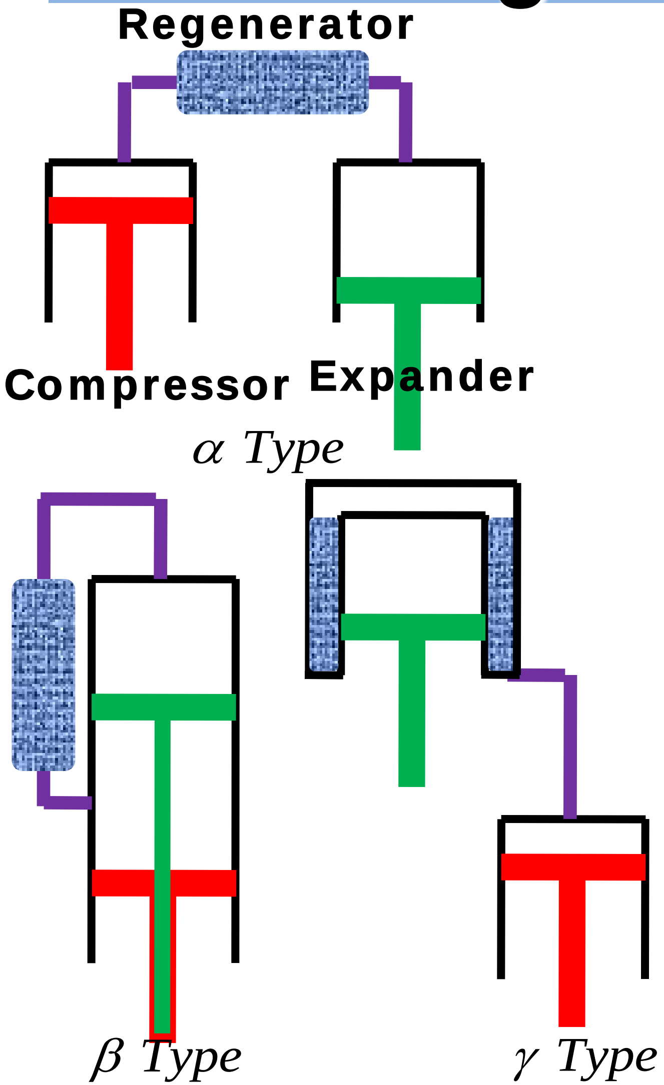
- In actual Stirling cycle the discontinuous motion can not be achieved. In view of this sinusoidal motion may be implemented.

- This motion is realistic and can be achieved using a Crank or gas spring mechanism.

Actual Stirling Cycle

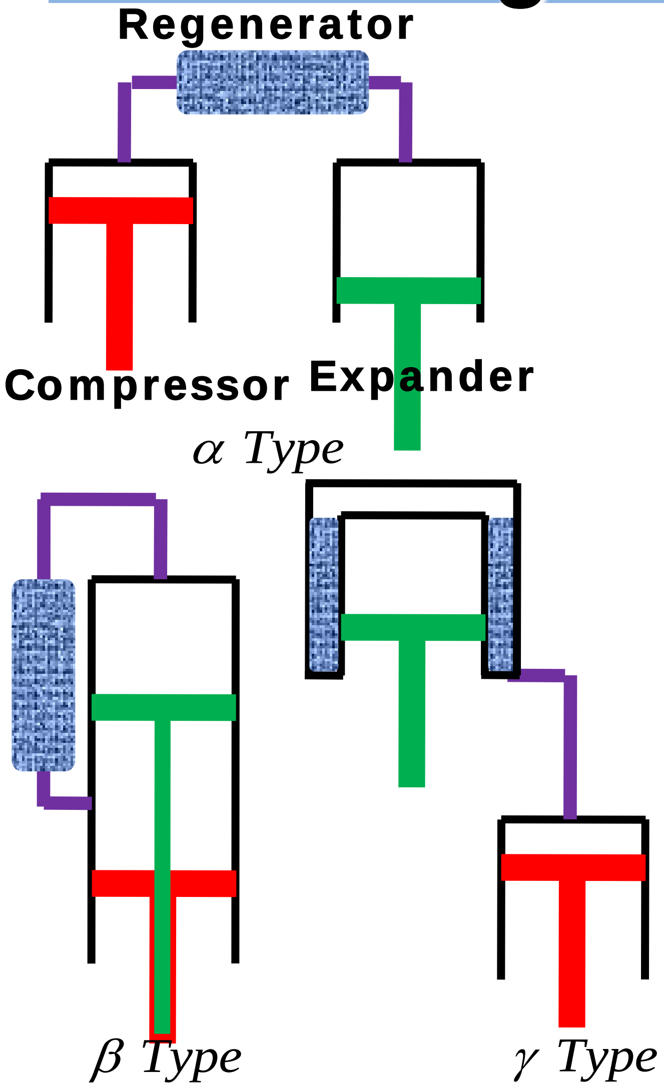
- In reality, the actual working cycle will be different from Ideal Stirling Cycle in following ways.
  - Discontinuous motion, difficult to realize in practice.
  - Presence of void volume or dead space (not swept by piston or displacer), pressure drop.
  - Ineffectiveness in heat transfer or regeneration.
  - Non isothermal compression and expansion.

Stirling Cryocooler – Types



- Depending upon the relative arrangements of piston and displacer/piston, various types of Stirling Cryocoolers are possible, namely
    - **α** type Stirling Cryocooler.
    - **β** type Stirling Cryocooler.
    - **γ** type Stirling Cryocooler.

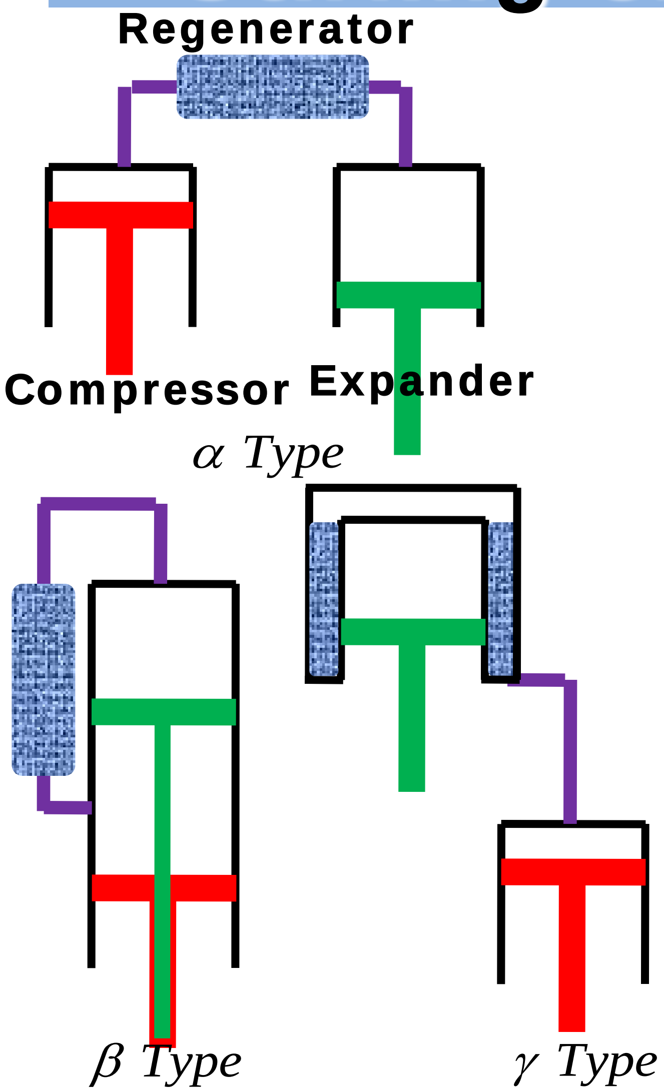
Stirling Cryocooler – Types



- **Two Piston arrangement (α type)**
- whose drive mechanisms may be mounted on same crank shaft.

- **Integral Piston & Displacer arrangement (β type)**
- The piston and displacer are housed inside same cylinder.

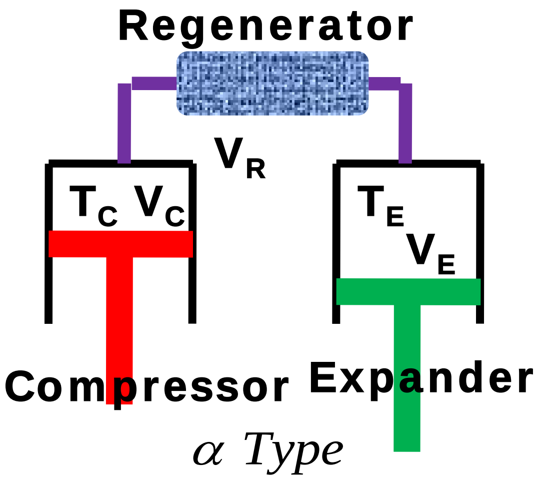
Stirling Cryocooler – Types



- **Split Piston & Displacer arrangement (γ type)**
- The compression space is divided.
- These systems have variable dead volume in compression space due to the movement of displacer.

Design Parameters

- The various design parameters of a Stirling Cryocooler are as follows.



- Evaporator temperature **(T_E)**
- Condenser temperature **(T_C)**
- Compression Volume **(V_C)**
- Expansion Volume **(V_E)**
- Regenerator Volume **(V_R)**
- **P_{max}, P_{min}, P_{avg}.**
- Phase angle **(α)**
- Crank angle **(ø)**

Schmidt's Analysis

- In the year 1861, Gustav Schmidt, a German scientist, presented a Stirling Cryocooler analysis.

- This analysis is based on a realistic cycle and is assumed to provide a first guess of dimensions. The following are the assumptions.

  - Perfect isothermal compression, expansion.

  - Harmonic motion of piston and displacer.

  - Perfect regeneration.

Schmidt’s Analysis

- The non – dimensional parameters in the Schmidt’s analysis are
- Swept volume ratio : $k = \frac{V_C}{V_E}$
- Temperature ratio : $\tau = \frac{T_C}{T_E}$
- Dead volume ratio : $X = \frac{V_D}{V_E}$

Schmidt's Analysis

- Expansion volume variation :

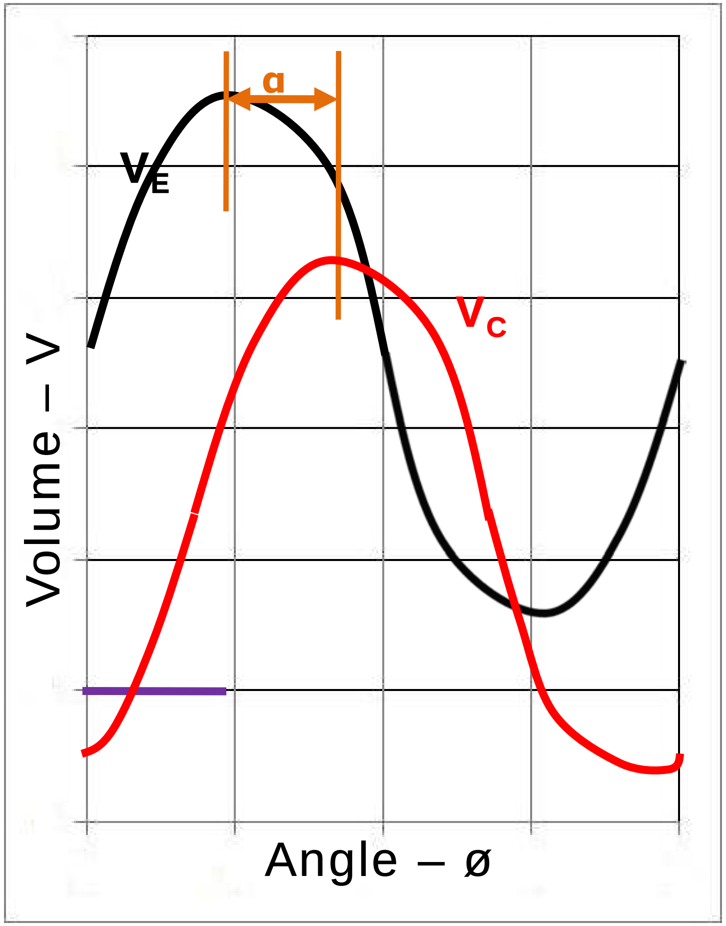
$$V_e = \frac{1}{2}V_E\left(1+\cos\phi\right)$$

- Compression volume variation

$$V_c = \frac{1}{2}\boxed{V_C}\left(1+\cos(\phi-\alpha)\right)$$

$$k = \frac{\boxed{V_C}}{V_E}$$

$$V_c = \frac{1}{2}kV_E\left(1+\cos(\phi-\alpha)\right)$$



Schmidt's Analysis

$$M_T = \frac{p_e V_e}{R\boxed{T_e}} + \frac{p_c V_c}{RT_c} + \frac{p_d V_d}{R\boxed{T_d}} \qquad = \frac{KV_E}{2RT_C}$$

- Let the instantaneous pressure in the system be same throughout the system, **p**.
- Also, **T_e** and **T_c** are assumed to be constants as **T_E** and **T_C** respectively.
- Let **M_T** be given as shown.

$$M_T = \frac{KV_E}{2RT_C}$$

Schmidt's Analysis

$$\frac{pV_E}{RT_C}\left[\frac{(1+\cos\phi)T_C}{2\boxed{T_E}}+\frac{k\left(1+\cos(\phi-\alpha)\right)}{2}+\frac{\boxed{V_D}T_E}{V_E\boxed{T_D}}\right]=\frac{KV_E}{2RT_C}$$

$$\tau=\frac{T_C}{\boxed{T_E}}$$

$$X=\frac{\boxed{V_D}}{V_E}$$

$$\boxed{T_d}=\frac{T_E+T_C}{2}$$

$$S=\frac{2X\tau}{\tau+1}$$

$$\frac{K}{p}=\left[\tau(1+\cos\phi)+k\left(1+\cos(\phi-\alpha)\right)+2S\right]$$

$$A=\boxed{\sqrt{(\tau+k\cos\alpha)^2+(k\sin\alpha)^2}}$$

$$B=\boxed{\tau+k+2S}$$

$$\delta=\boxed{\frac{A}{B}}$$

$$\tan\theta=\frac{k\sin\alpha}{\tau+k\cos\alpha}$$

Schmidt’s Analysis

- Substituting, **A**, **B**, **θ** and **δ** in the mass equation and rearranging, we get

$$p = \frac{K}{B\left[\delta\cos(\theta-\phi)+1\right]}$$

$$p_{\min} = \frac{K}{B\left[1+\delta\right]}$$

$$@ \quad \phi = \theta$$

$$p_{\max} = \frac{K}{B\left[1-\delta\right]}$$

$$@ \quad \phi = \theta - \pi$$

$$p_{ratio} = \frac{\left[1+\delta\right]}{\left[1-\delta\right]}$$

Schmidt's Analysis

- Mean pressure

$$p_m = \frac{1}{2\pi}\int_{0}^{2\pi} p\,d(\theta-\phi)$$

$$p_m = p_{\max}\sqrt{\frac{1-\delta}{1+\delta}}$$

$$Q_E = \int p\,dV_e = \frac{\pi p_m \delta \sin\theta V_E}{1+\left[1-\delta^2\right]^{0.5}}$$

$$Q_C = \int p\,dV_c = \frac{\pi p_m V_E \delta \sin(\theta-\alpha)k}{1+\left[1-\delta^2\right]^{0.5}}$$

$$COP = \frac{Q_E}{W_T}$$

$$COP = \frac{Q_E}{Q_C - Q_E}$$

$$= \frac{T_E}{T_C - T_E}$$

Losses

- In the earlier slide, we saw the cooling effect based on Schmidt's analysis.

- But, in an actual system, there are many losses. Few of them are as listed below.

  - Ineffectiveness of regenerator.
  - Pressure drop in system.
  - Solid conduction losses.
  - Shuttle conduction losses.
  - Losses in power input.

Losses

- Considering the above mentioned losses, the net cooling effect and gross power required is given by the following correlations.

  - **$Q_{net} = Q_E - \Sigma(\text{losses})$.**

  - **$W_{total} = W_T + \Sigma(\text{losses})$.**

- In general, **Q_E** calculated from Schmidt's analysis, in which 60 – 70% are considered as losses, while losses in power input is due to mechanical efficiency.

Summary

- A Stirling Cycle was first conceived by Robert Stirling in the year 1815.

- $\text{COP}_{(\text{Stirling})} = \text{COP}_{(\text{Carnot})}$.

- In reality, the actual working cycle has discontinuous motion, pressure drop, ineffectiveness and non isothermal processes.

- Depending upon the relative arrangements of piston and displacer/piston, **α**, **β**, **γ** are the different types of Stirling cryocooler.

Summary

- Gustav Schmidt presented a Stirling Cryocooler analysis in the year 1861, it is assumed to provide a first guess of dimensions.

- The net cooling effect and gross power required is given by the following correlations.

  - $\mathbf{Q_{net} = Q_E - \Sigma(losses).}$

  - $\mathbf{W_{total} = W_T + \Sigma(losses).}$

- A self assessment exercise is given after this slide.

- Kindly asses yourself for this lecture.

Self Assessment

1. A Stirling cycle consist of two ________ processes.
2. In an isothermal process, dQ is given by ________.
3. In a constant volume process, dU is given by ____.
4. COP_{Carnot} and $COP_{Stirling}$ are ______.
5. COP of Stirling cycle is ________.
6. In an actual Stirling cycle, the discontinuous motion is approximated to ________ motion.
7. The volume not swept by piston/displacer is ____.
8. In a ______ type unit, the piston and displacer are housed inside same cylinder.
9. In Schmidt's analysis, instantaneous pressure is assumed to be _____.

Answers

1. Isothermal and Constant volume
2. $dQ = dW = -\Re T_C \ln\left[V_2 / V_1\right]$
3. $dU = +C_V \left(T_E - T_C\right)$
4. Equal.
5. $T_E / \left(T_C - T_E\right)$
6. Sinusoidal
7. Void volume
8. Beta
9. Constant

Thank You!