CRYOGENIC ENGINEERING

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Lecture No -   28

Earlier Lecture

- In the earlier lecture, we have studied the Stirling Cycle and proved that

$$COP_{Stirling} = COP_{Carnot}$$

- **α**, **β**, **γ** are the different arrangements of Stirling cryocoolers. Each has its own advantages and disadvantages.

- The actual working cycle has a continuous motion, non isothermal processes, pressure losses and pressure drops.

Earlier Lecture

- Schmidt's analysis (1861) serves as a first guess of dimensions. The cooling effect and work input are denoted by Q_E and W_T respectively.

- The net cooling effect and the total work input required, considering the losses, are given by

  - $Q_{net} = Q_E - \Sigma(losses)$

  - $W_{total} = W_T + \Sigma(losses)$

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Outline of the Lecture

Topic : Cryocoolers

- Design parameters (Schmidt's Analysis)

- Parametric study (Schmidt's Analysis)

- Walker's optimization charts

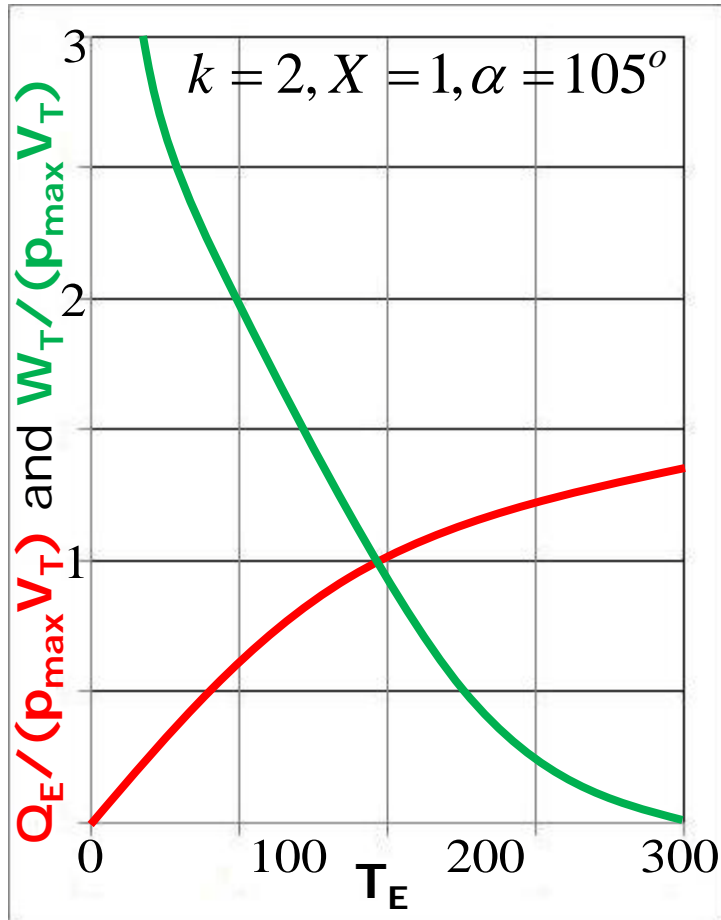- Design methodology of a Stirling cryocooler

Parametric Study

- In general, Q_E is dependent on both design and operating parameters.

  - Design : $k = \dfrac{V_C}{V_E}$ $\tau = \dfrac{T_C}{T_E}$ $X = \dfrac{V_D}{V_E}$ α, V_T

  - Operating : N, p_{max}.

- These parameters are varied to study their effect on Q_E.

- For an optimum design, a compromise between operating and design parameters may be sought.
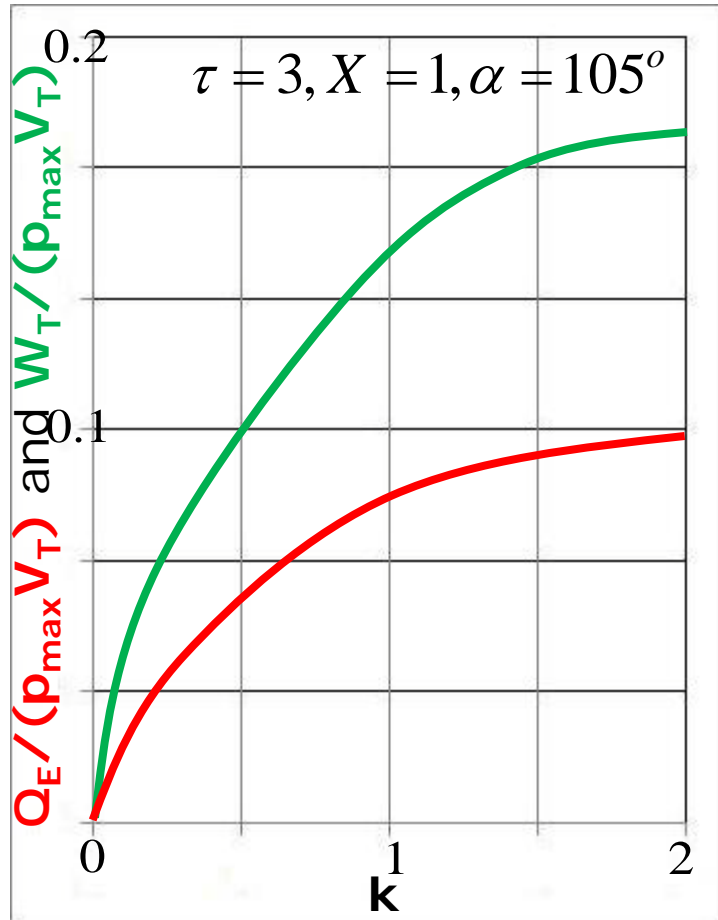
Parametric Study

- Based on Schmidt's analysis, the variation of heat lifted per unit cycle ($Q_E/(p_{max}V_T)$) and work input required per unit cycle ($W_T/(p_{max}V_T)$) for the above said non dimensional numbers is presented.

- It is important to note that Q_E and W_T are non – dimensionalized with a product of p_{max} and V_T.

- In the following study, let us call

  - $Q_E/(p_{max}V_T)$ as Q_{max}.

  - $W_T/(p_{max}V_T)$ as W_{max}.

Effect of T_E

$$k = 2, X = 1, \alpha = 105^o$$

- Consider a chart to study the variation of Q_{max} and W_{max} with variation of T_E.

- With the increase in value of T_E,
  - Q_{max} increases.
  - W_{max} decreases.

- The COP of the system increases.

Effect of k

Plot axes: $Q_E/(p_{max}V_T)$ and $W_T/(p_{max}V_T)$ vs k, with $\tau = 3, X = 1, \alpha = 105^o$

- The plot of Q_{max} and W_{max} versus **k**, for τ, **x** and **α** as **3**, **1**, **105º** respectively, is as shown in the figure.

- With the increase in the value of **k**, both Q_{max} and W_{max} increase.
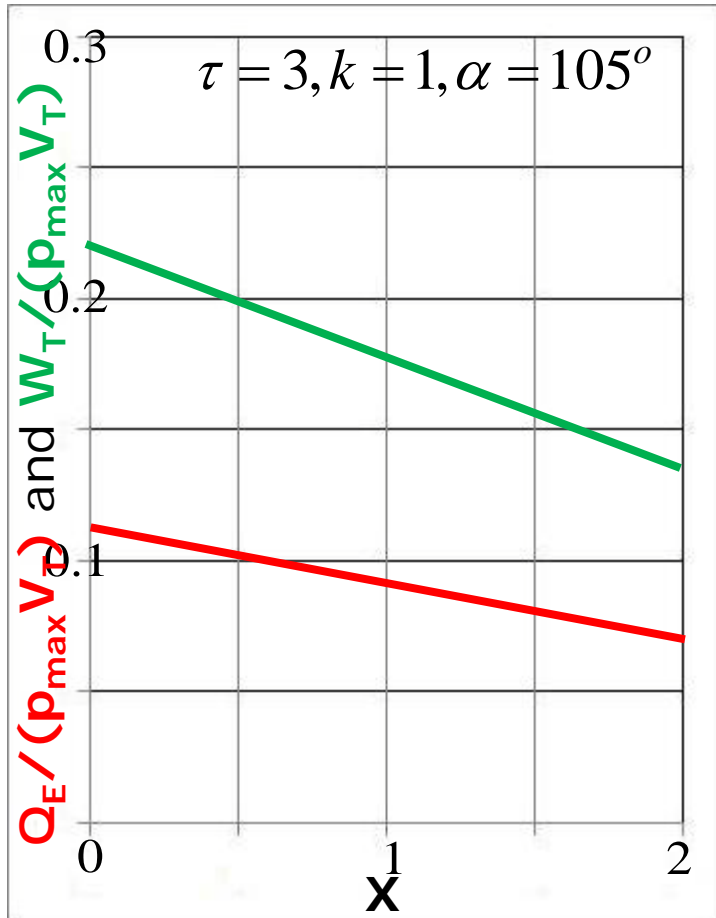
- The COP of the system remains constant.

Effect of X

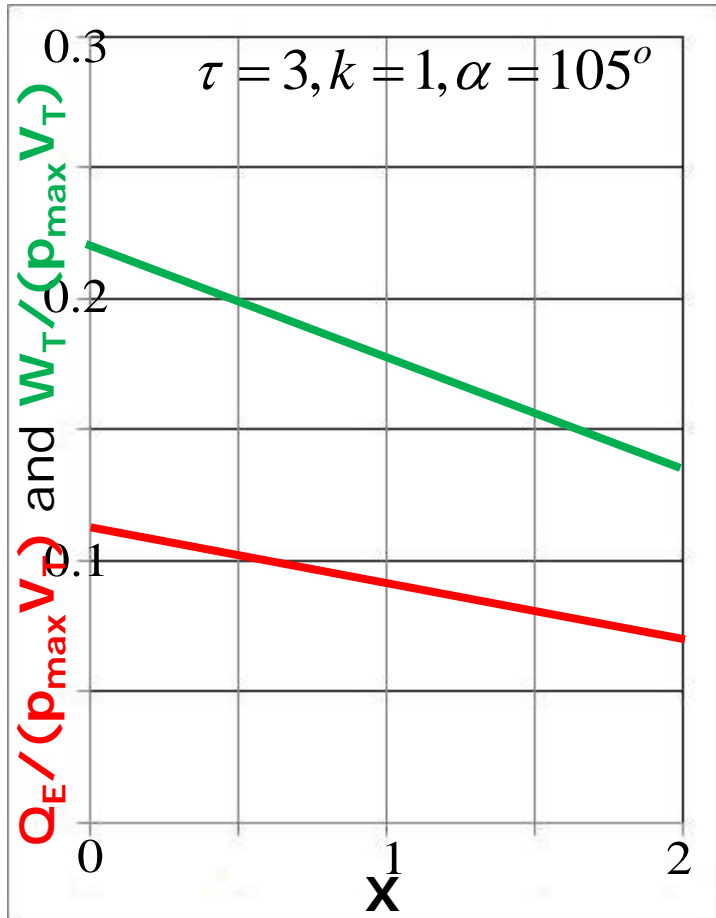$$\tau = 3, k = 1, \alpha = 105^o$$

- The dead volume in a system is that volume, which is not swept by the compressor and the expander – displacer pistons.

- With the increase in the dead volume ratio, both Q_{max} and W_{max} decrease.

Effect of X

$$\tau = 3, k = 1, \alpha = 105^o$$

- The increase in the dead volume leads to,
  - Decrease in the pressure ratio of the system.

  - Decrease in the pressure drop of the system.

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Effect of X

Plot axes: Y-axis labelled $Q_E/(p_{max}V_T)$ and $W_T/(p_{max}V_T)$; X-axis labelled **X**. Condition: $\tau = 3, k = 1, \alpha = 105^o$
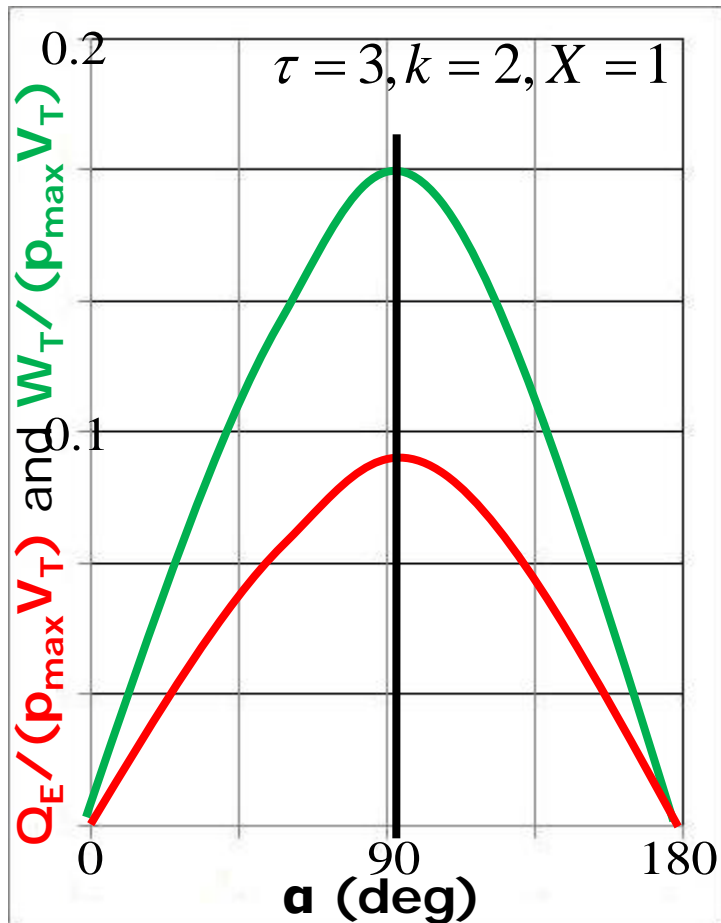
- In practice, some amount of dead volume is necessary to accommodate the after cooler and the regenerator.

- The optimum value of **X** determines the COP of the system.

Effect of Phase Angle

$$\tau = 3, k = 2, X = 1$$
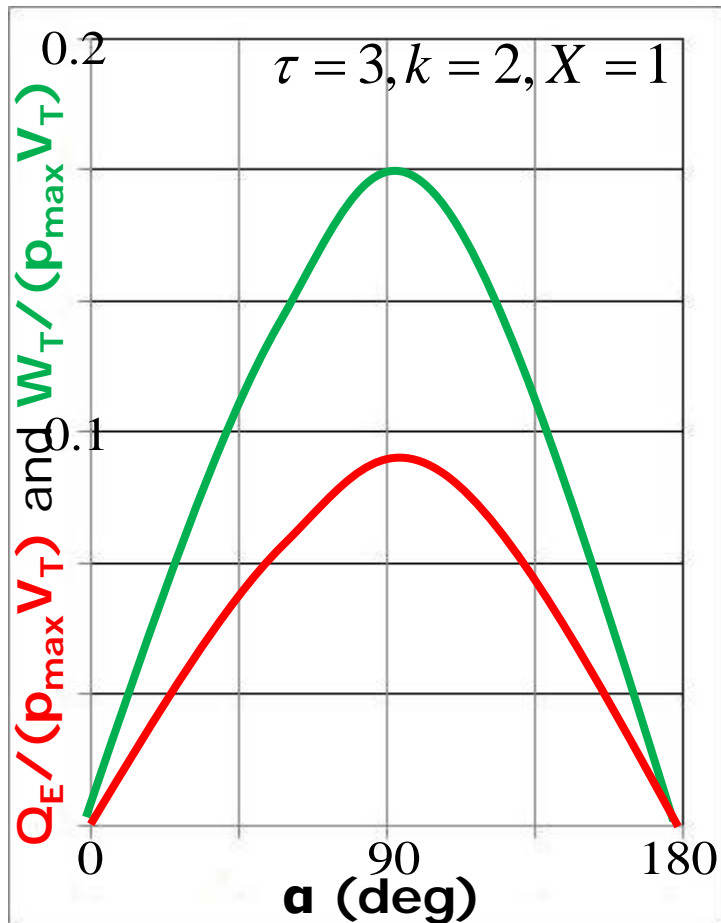
- There exists a phase difference between the expander – displacer and the compressor pistons.

- This phase angle is vital to produce cold in system. Also, there exists an optimum phase angle for a system.

- The variation plots for Q_{max} and W_{max} with α are as shown in the figure.

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Effect of Phase Angle

$$\tau = 3, k = 2, X = 1$$

Y-axis: $Q_E / (p_{max} V_T)$ and $W_T / (p_{max} V_T)$

X-axis: α (deg), values 0, 90, 180

Y-axis values: 0.1, 0.2

- Both the curves pass through a maxima with an increase in phase angle.

- This position of maxima for both the curves occurs approximately at the same phase angle.

- Also, the curves gets flattened near the maxima, in the neighborhood of **60º - 120º**.

Optimization Charts

- From the parametric study, it is clear that, each of the non – dimensional number has an impact on the performance of the system.

- However, a combined effect of these parameters on the performance of system as a whole, was first reported by **Graham Walker** in the year 1962 in the form of design charts.

- These charts were produced for both refrigerators as well as for engines and are called as **Walker's optimization charts**.

Optimization Charts

X=0.5, X=1, X=2
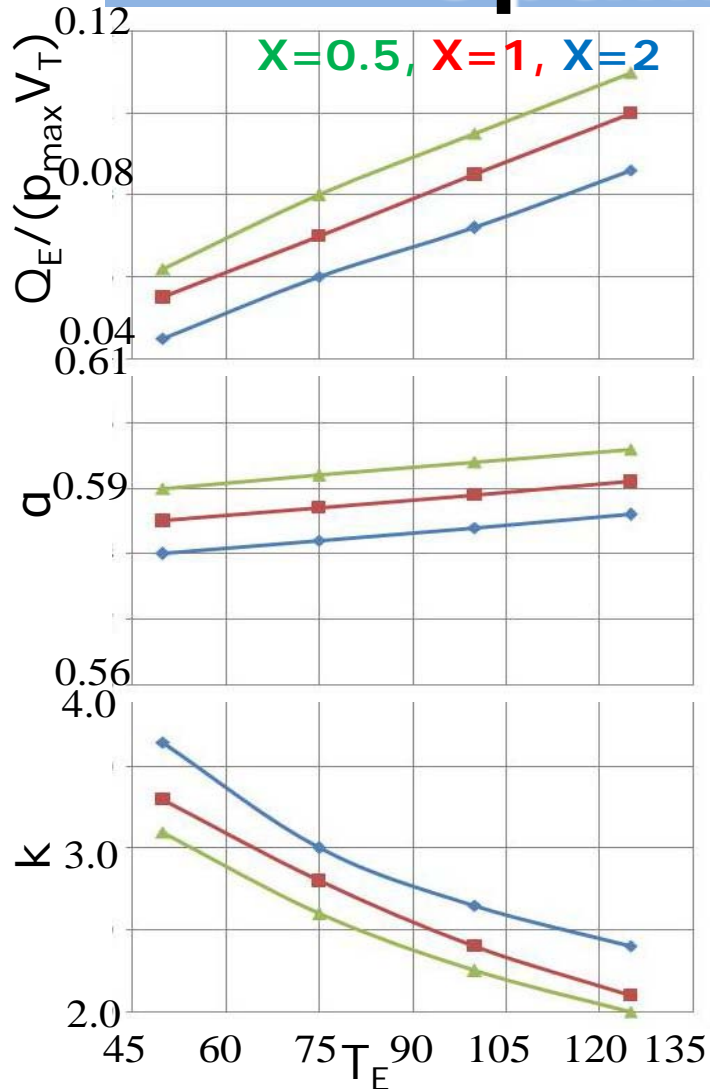
- The adjacent figure shows **Walker's Optimization Chart** for a refrigerator.

- Moving from top to bottom, this chart has three different y – axes, namely, Q_{max}, a (**radians**), k, plotted against a common x – axis representing an expander temperature (T_E).

- The value of T_C for these charts is taken as 300 K.

Optimization Charts
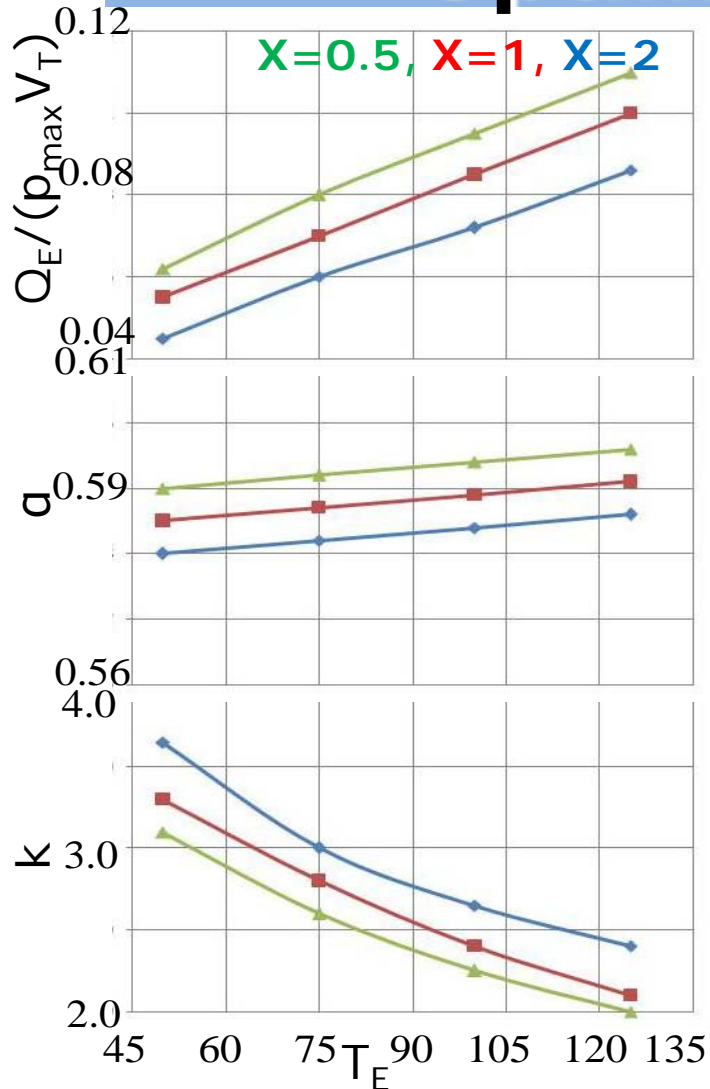
- For each of the above said parameters, this chart shows the variations for three different values of dead volume ratios, namely, **X=0.5**, **X=1**, **X=2**.

- From the figure, it is clear that the **Q_max** and **k** are more sensitive to **T_E** as compared to phase angle, **α**, in radians.

Design of a Cryocooler

- The Schmidt's analysis is a pure ideal analysis and does not take losses into account.

- As mentioned in the earlier lecture, these losses can be thermal, mechanical, shuttle conduction etc.

- In order to make the analysis more realistic, Q_E in the Walker's charts is taken as $Q_{E, Design}$, which is three times Q_E, the required cooling power, in order to account for losses.

- Therefore, $Q_{E, Design} = 3 \times Q_{E, Reqd}$.

Design Methodology
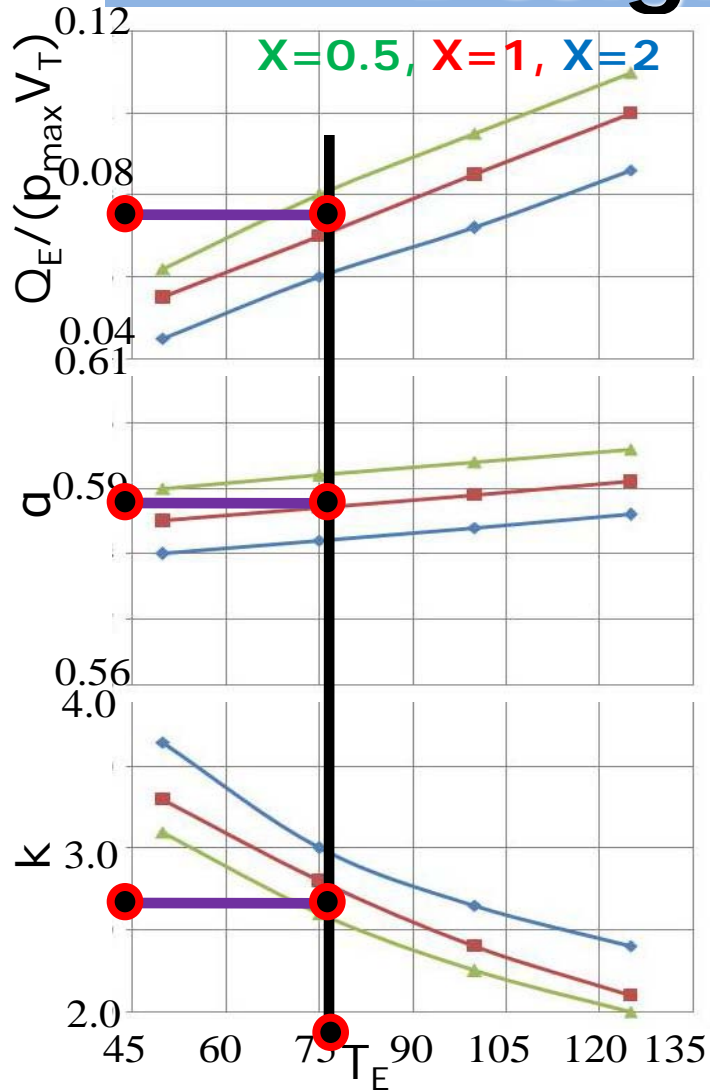
- Consider the design of a system for a given values of Q_E and T_E. The key steps are,

- An initial value of **X** is assumed, say **X = 1**.

- Assume p_{max} and locate Q_{max}.

- Get **k** and **α**.

- Get V_T, V_C, V_E and system optimization.

- Valid assumptions, wherever necessary. Iterate, if needed.

Tutorial

- Design an **α** – type Stirling Nitrogen liquefier using the Schmidt's analysis. The working gas is Helium and the capacity of the plant is 10 liter per hour of **LN_2**. The maximum allowable pressure in the system is 40 bar. The speed of the prime mover is 1440 rpm.

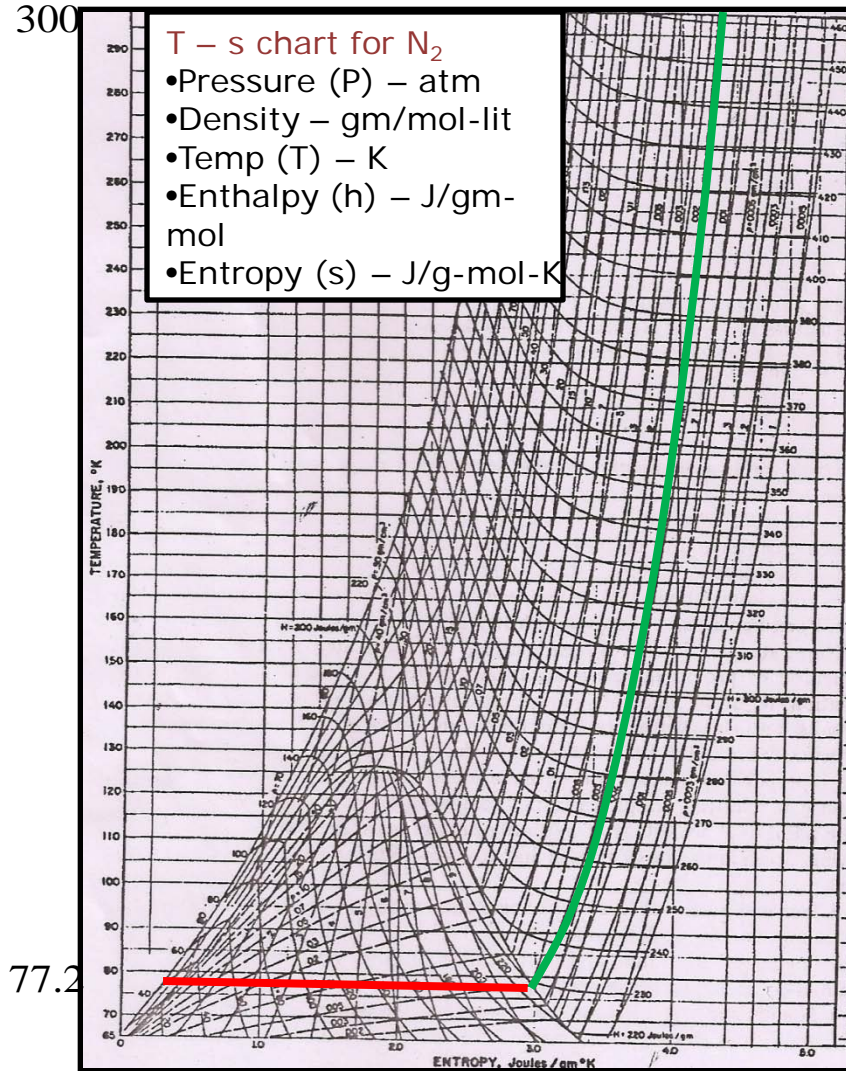- Use the standard T – s diagrams for entropy and enthalpy values.

Tutorial

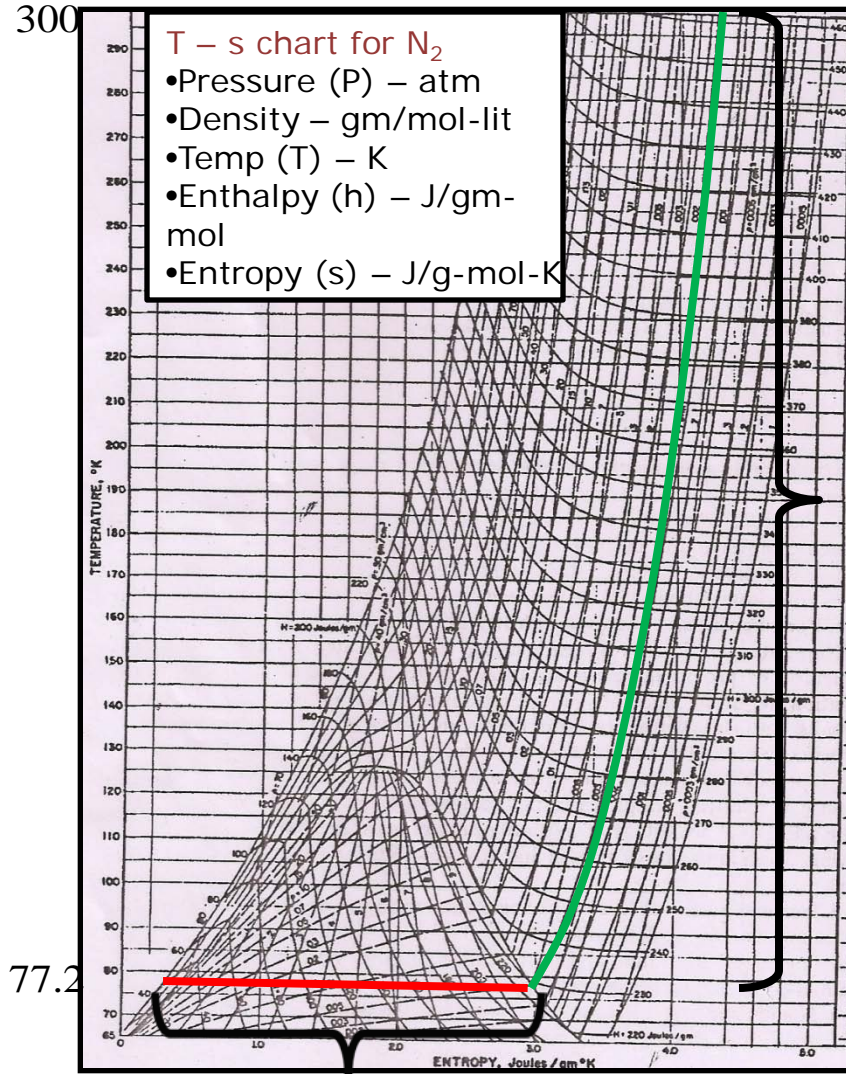- The schematic of an **α – type** Stirling cryocooler is as shown.

Regenerator

V_T

T_C V_C

T_E V_E

Compressor **Expander**

α Type
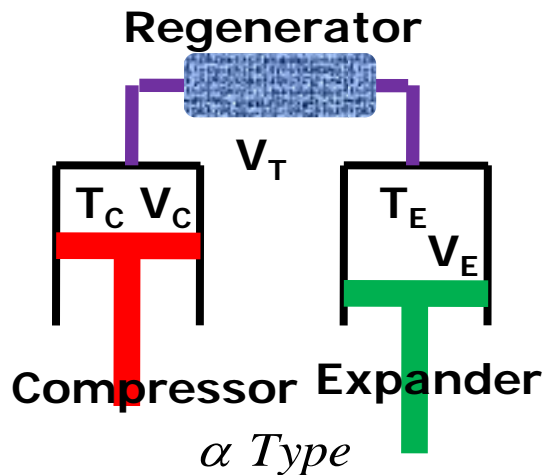
- Given parameters are
  - Evap. Temp. (T_E): 77.2 K.
  - Cond. Temp. (T_C): 300 K.
  - Max. Pressure (P_{max}): 40 bar.
  - N = 1440 rpm.

- Parameters to be calculated are
  - Volumes: (V_C), (V_E), (V_T).
  - Phase angle $(α)$

Tutorial

T – s chart for N$_2$
- Pressure (P) – atm
- Density – gm/mol-lit
- Temp (T) – K
- Enthalpy (h) – J/gm-mol
- Entropy (s) – J/g-mol-K

- Consider the T – s diagram for Nitrogen as shown in the figure.

- It is important to note that, the energy required to condense Nitrogen involves

  - Sensible heat from 300 K to 77 .2 K.

  - Latent heat of vaporization at 77.2 K.

Tutorial

T – s chart for N$_2$
- Pressure (P) – atm
- Density – gm/mol-lit
- Temp (T) – K
- Enthalpy (h) – J/gm-mol
- Entropy (s) – J/g-mol-K

- From the standard T – s diagram for Nitrogen, the change in enthalpy for these processes are as shown below.

- Sensible heat (KJ/Kg-K)
$$\Delta h_s = 231.7$$

- Latent heat (KJ/Kg-K)
$$\Delta h_l = 199.1$$

- The net change in enthalpy is $\Delta h_{net} = 430.8$

Tutorial

- The required capacity of the given liquefier is **10** liter per hour.

- The density of liquid nitrogen is 808 kg/m³. Hence, the required mass flow rate across the liquefier corresponding to 10 liter per hour is calculated as shown below.

$$\dot{m} = \frac{n(m^3/hr)\rho}{3600}$$

$$= \frac{(10)(10^{-3})(808)}{3600}$$
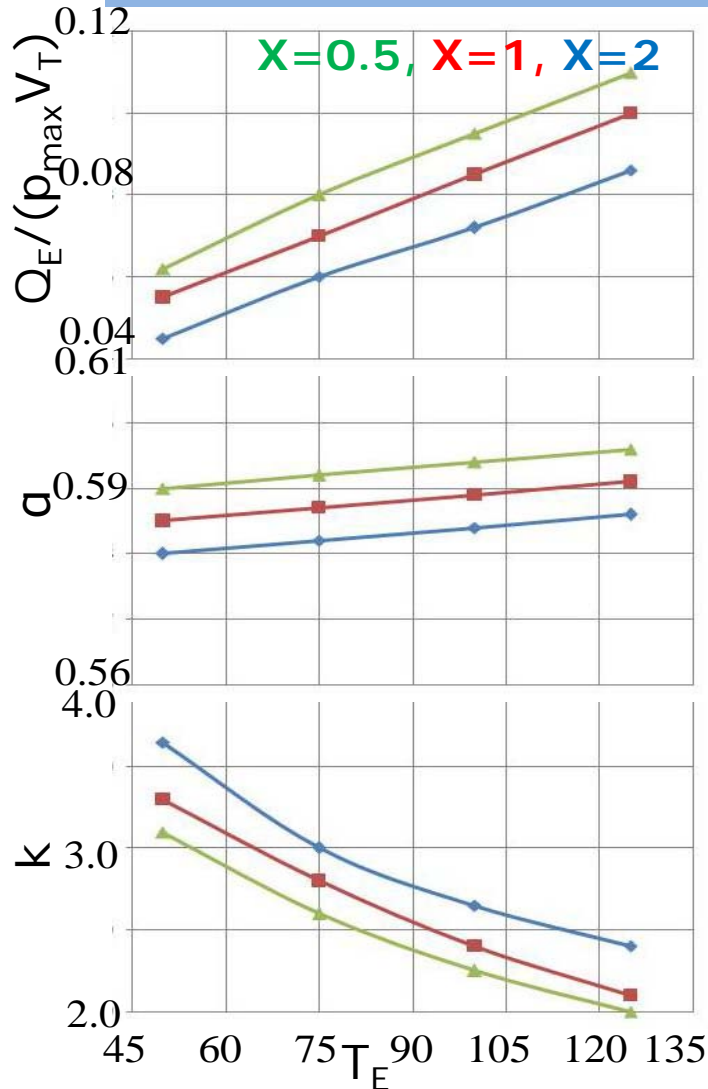
$$= 0.00224\, Kg/s$$

- The net cooling power required to produce **10** liter per hour **LN$_2$** is

$$Q_{E,reqd} = \Delta h_{net}\dot{m}$$

$$= (430.8)(0.00224)$$

$$= 965W$$

Tutorial

- Therefore, $Q_{E, Design}$ at $T_E = 77.2 \text{ K}$ is given by
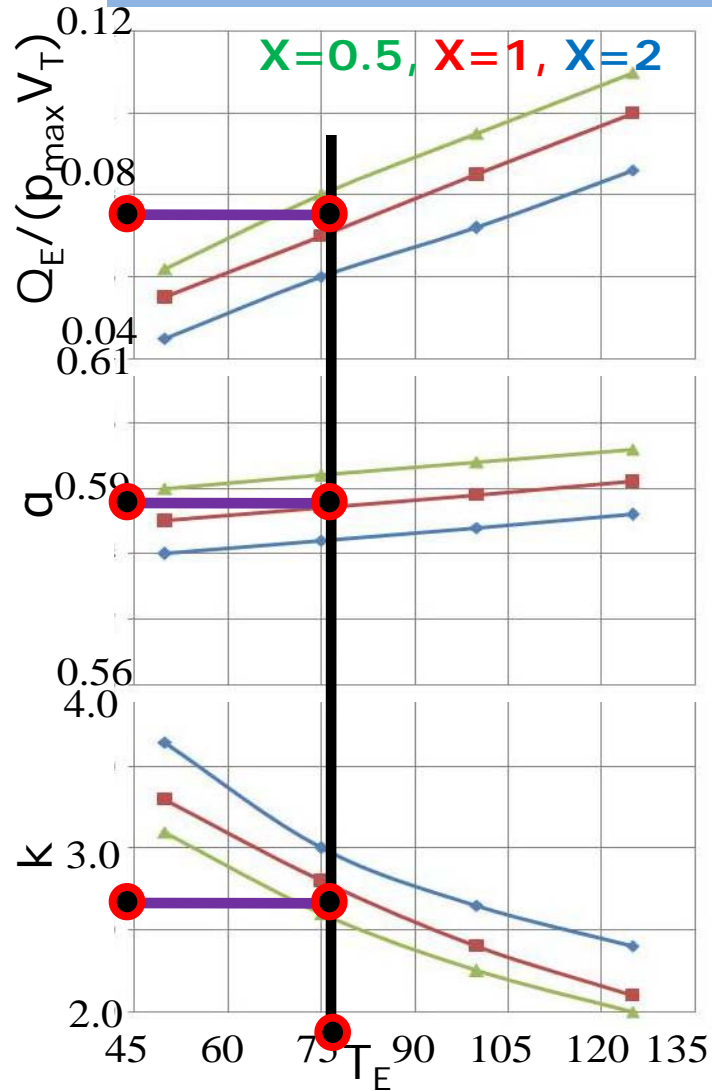
$$Q_{E,Design} = 3(965) \qquad = 2895W$$

- The RPM of the prime mover is given as 1440. Therefore, the **N(rps)** is **24**.

- The $Q_{E, Design}$ per unit cycle is calculated as shown below.

$$Q_{E,Design} = \frac{2895}{24} \qquad = 120.6$$

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Tutorial

- Choosing **X = 1** on **Walker's Optimization Chart**, we have the following values.

$$k = 2.85$$

$$\alpha = 0.575^c$$

$$= 32.9^o$$

$$Q_{max} = 0.07$$

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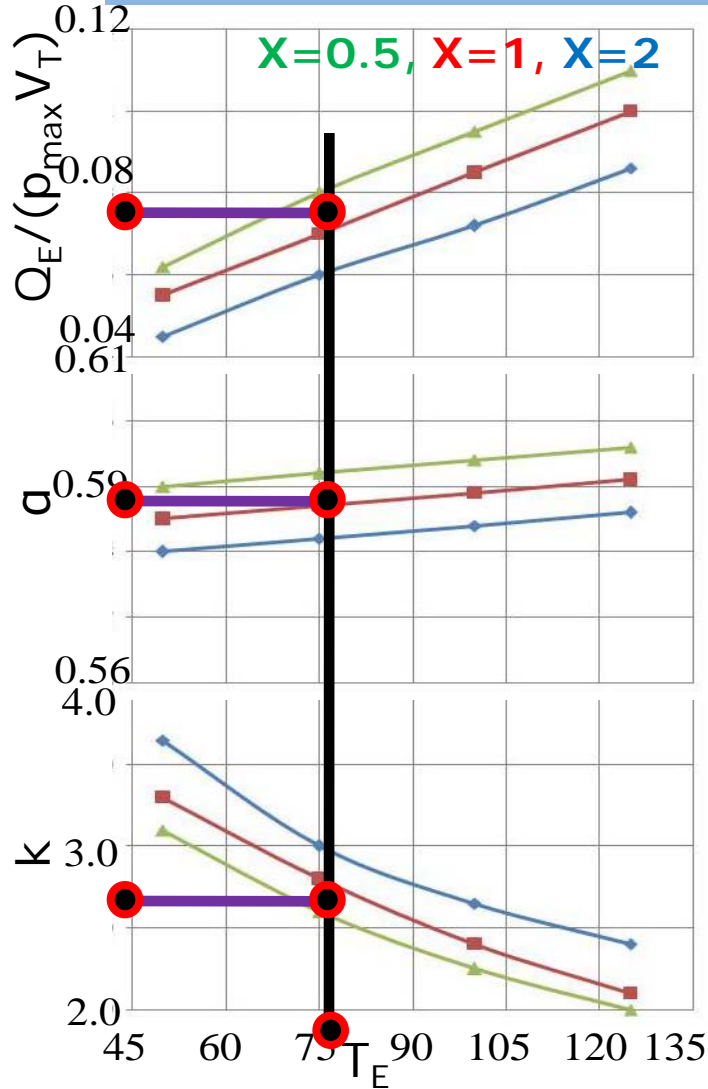
Tutorial

- From the definition of Q_{max}, we have the following.

$$Q_{max} = \frac{Q_{E,Design}}{P_{max}V_T}$$

$$p_{max} = 40\,bar$$

$$V_T = \frac{120.6}{(0.07)(40)(10^5)}$$

$$= 4.3\left(10^{-4}\right)m^3$$

$$= 430\,cc$$

$$k = \frac{V_C}{V_E} = 2.85$$

$$V_T = V_C + V_E$$

$$V_E = 1.12\left(10^{-4}\right)m^3$$

$$V_C = 3.18\left(10^{-4}\right)m^3$$

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Tutorial

- Assuming a stroke to bore ratio of **0.75**, for both compressor and expander – displacer pistons, we have the following dimensions.

$$V_C = \frac{\pi}{4} D_C^2 S_C = 3.18\left(10^{-4}\right)$$

$$\frac{S_C}{D_C} = 0.75$$
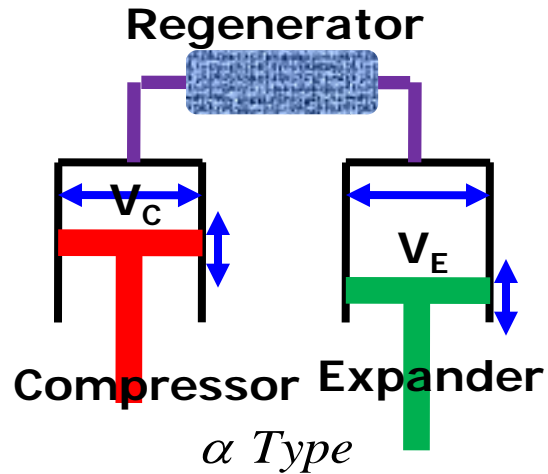
$$D_C = 81.4mm$$

$$S_C = 60.8mm$$

$$V_E = \frac{\pi}{4} D_E^2 S_E = 1.12\left(10^{-4}\right)$$

$$\frac{S_E}{D_E} = 0.75$$

$$D_E = 57.5mm$$

$$S_E = 43.1mm$$

Tutorial

Regenerator

V$_C$

V$_E$

Compressor **Expander**

α *Type*

$D_C = 81.4mm$

$D_E = 57.5mm$

$S_C = 60.8mm$

$S_E = 43.1mm$

Operating Parameters

- **T$_E$**: 77.2 K
- **T$_C$**: 300 K
- **P$_{max}$**: 40 bar
- **N**: 1440

Design Parameters

$$V_C = 3.18\left(10^{-4}\right)m^3$$

$$V_E = 1.12\left(10^{-4}\right)m^3$$

$$\alpha = 0.575^c$$

$$= 32.9^o$$

Tutorial

- For a given $Q_{E, Design}$, if the dimensions of the piston and expander – displacer are very large, say more than 150mm, the system may be designed for two cylinders or more.

- This is an iterative process until the feasible dimensions are decided.

Conclusions

- For an optimum design of a cryocooler, a compromise between the operating and the design parameters may be sought.

- With the increase in T_E,
  - Q_{max} increases, W_{max} decreases, COP increases.

- With the increase in k,
  - Q_{max} and W_{max} increase, COP remains constant.

- With the increase in X,
  - Q_{max} and W_{max} decrease.

Conclusions

- The increase in the dead volume leads to a decrease in pressure ratio and pressure drop.

- Q_{max} and W_{max} curves for phase angle variation, pass through a maxima. The curves gets flattened near the maxima, close to **60º - 120º**.

- A combined effect of parameters on performance of system as a whole, is given in **Walker's optimization charts**.

- For a realistic analysis, $Q_{E,\ Design} = 3\ X\ Q_{E,\ Reqd}$.

Assignment

- An **α** – type Stirling Nitrogen liquefier is capable of producing 15 liter per hour of **LN$_2$**. Speed of the prime mover is 1440 rpm and the maximum allowable pressure in the system is 35 bar. Use the Schmidt's analysis to design the system, if the working gas is Helium. Use **X = 0.5** for the design procedure.

- Use the standard T – s diagrams for entropy and enthalpy values.

Thank You!