CRYOGENIC ENGINEERING

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Lecture No - 30

Earlier Lecture

- We have seen the schematic and the working of a Gifford – McMahon Cryocooler (**W. E. Gifford** and **H. O. Mc Mahon**, 1950).
- It has a valve mechanism to generate the pressure pulse. The relation between the pressure pulse and the expander – displacer motion is vital.
- The basic components are Compressor, Flex lines, Regenerator(s), Displacer(s), Valve mechanism.
- A GM system can reach much lower temperatures as compared to a Stirling system.

Earlier Lecture

- Multistaging is done to reach lower temperatures (4.2 K to 10 K).

- Single stage (**~ 30 K**), **SS** mesh.
- 2 – stage (**~ 10 K**), 1st stage: **SS** mesh, 2nd stage: **Lead** balls.
- 2 – stage (**~ 4.2 K**), 1st stage: **SS** mesh **+ Lead** balls, 2nd stage: **Lead** balls **+ Er_3Ni** balls.

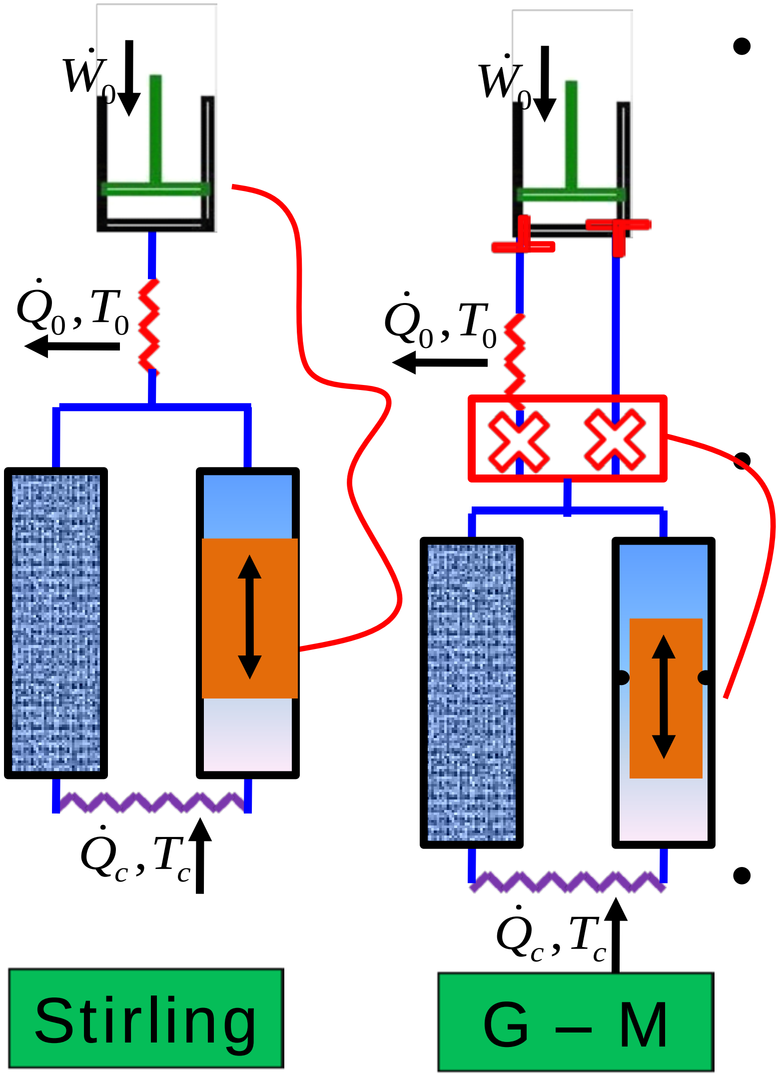
- Commercially available cryocoolers have rotary valves to control/regulate the flow of working fluid.

Outline of the Lecture

Topic : Cryocoolers

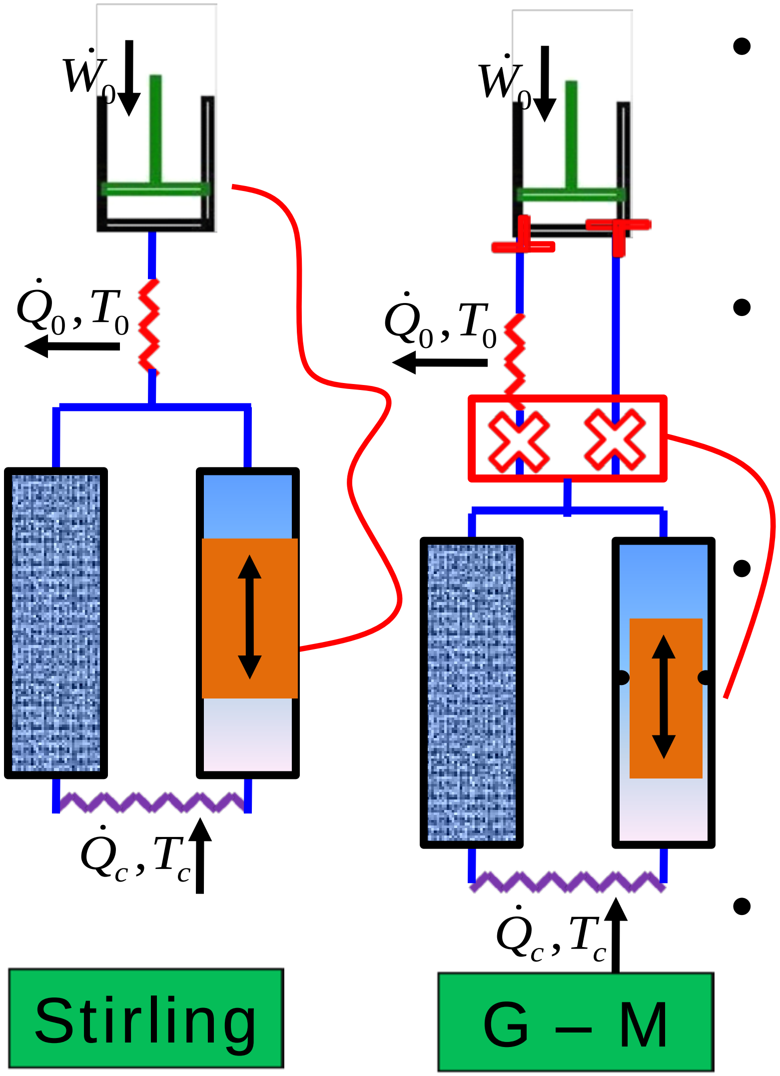
- Pulse Tube (PT) Cryocooler
- Working of a Pulse Tube Cryocooler
- PT Classification
- Stirling, GM and PT – A Comparison and Applications.
- Phasor Analysis

Introduction



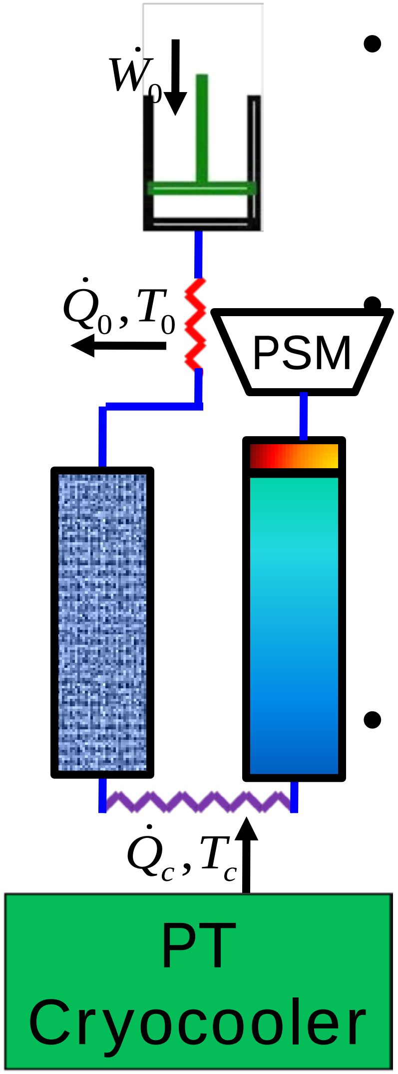
- In the earlier lectures, we have seen a regenerative type Stirling and GM systems.
- These systems have a mechanical expander – displacer to displace the working gas.
- The displacers are either free moving or driven by an external mechanism.

Introduction



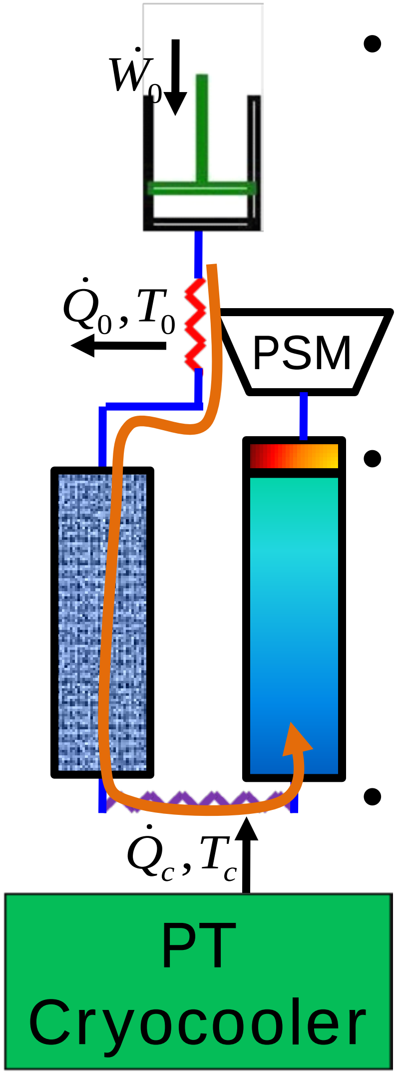
- The cold end displacers pose few problems as given below.
- The rubbing seal on displacer is difficult to maintain.
- The motion of the displacer induces unnecessary vibrations at the cold end.
- PT cooler overcomes these problems as this does not have a mechanical displacer.

Pulse Tube Cryocooler



- Consider the schematic of a Stirling system as shown in the figure.
- In a Pulse Tube cryocooler, the mechanical displacer is removed and an oscillating gas flow in the thin walled tube produces cooling.
- This gas tube is called as **Pulse Tube** and this phenomenon is called as **Pulse Tube action**.

Working of PT Cryocooler

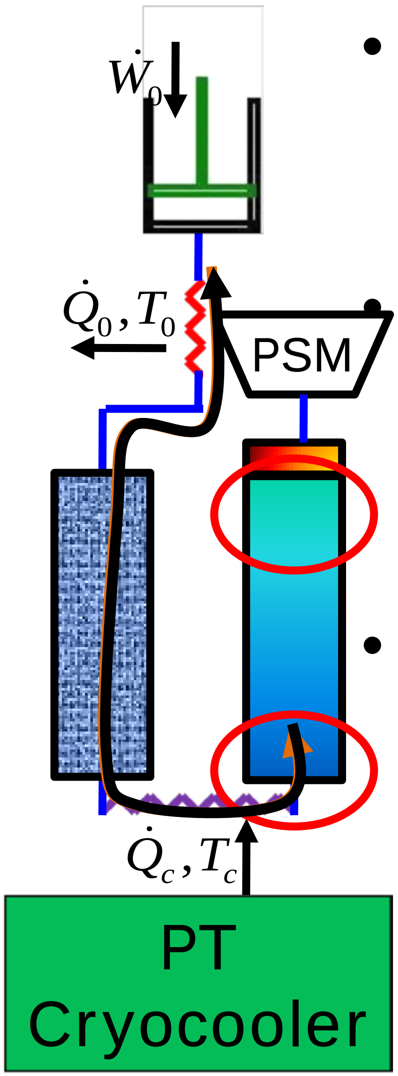


- The components of a PT system are Compressor, Heat exchangers, Regenerator, Pulse tube and Phase Shift Mechanism (**PSM**).

- The details and the requirement of the Phase Shift Mechanism (**PSM**) is explained at the later part of the lecture.

- During pressurization, the high pressure gas flows across the regenerator and into the pulse tube.

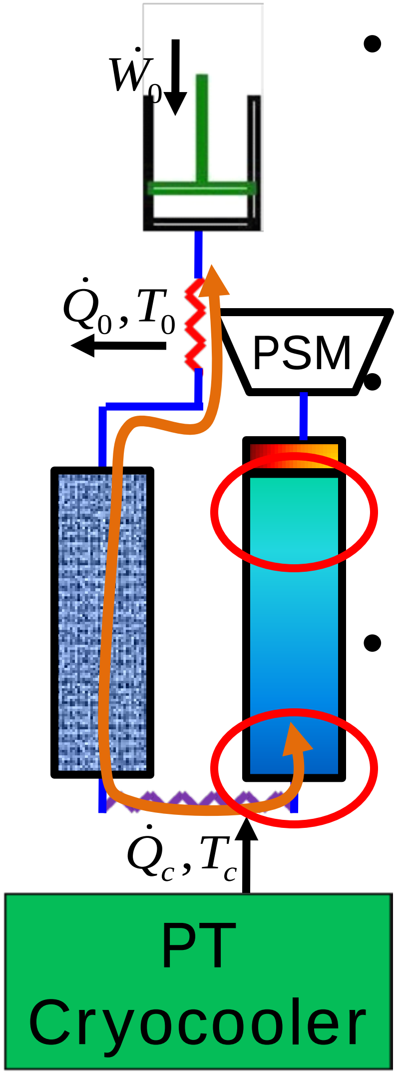
Working of PT Cryocooler



- The gas in the Pulse Tube compresses the gas present at other end (top end).
- This compression results in a rise in temperature at the top end, relative to the other end.
- During depressurization, the gas expands in the PT resulting in lowering of temperature across the Pulse Tube.

As a result, a temperature gradient is setup across the length of the PT.

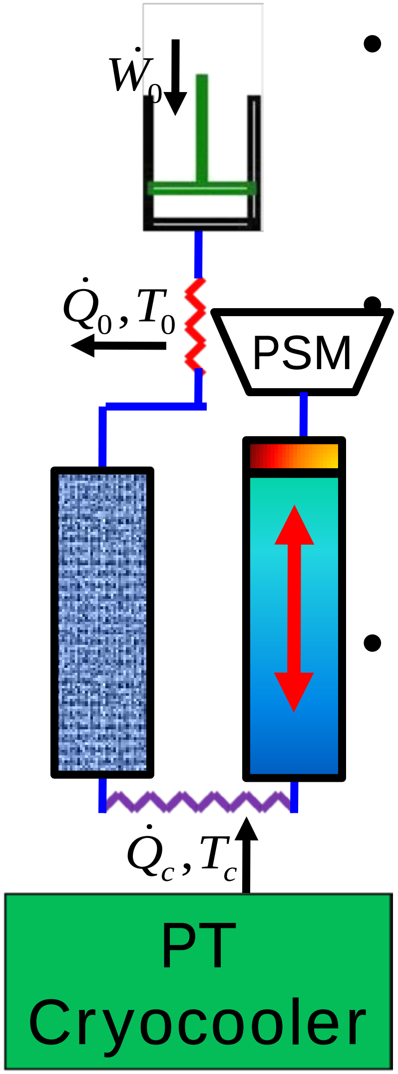
Working of PT Cryocooler



- The cold gas, during depressurization, transfers cold to the regenerator matrix, which is used to precool the gas.
- The **Hot End** temperature at the top is maintained at ambient using water.
- The cooling effect produced at the bottom end, also called as **Cold End**, is lifted by using a heat exchanger.

Also, the **After Cooler** temperature at the top is maintained at ambient.

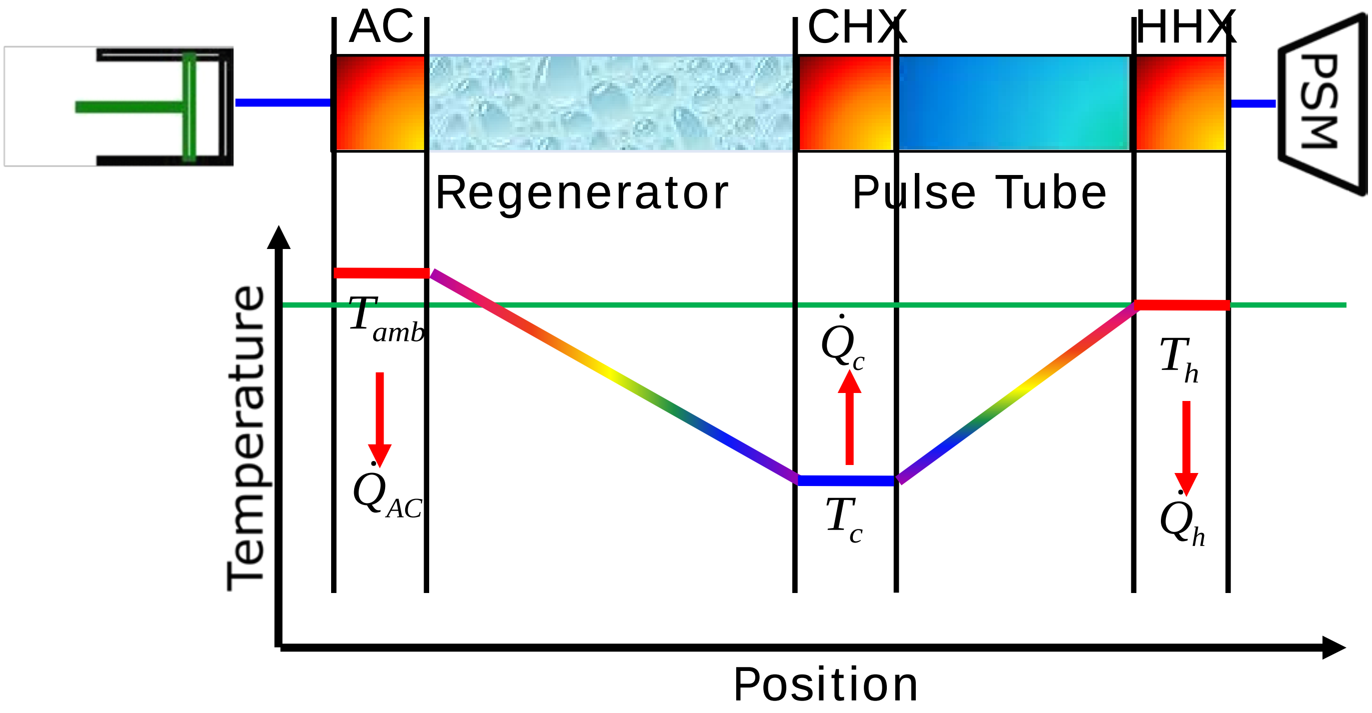
Working of PT Cryocooler



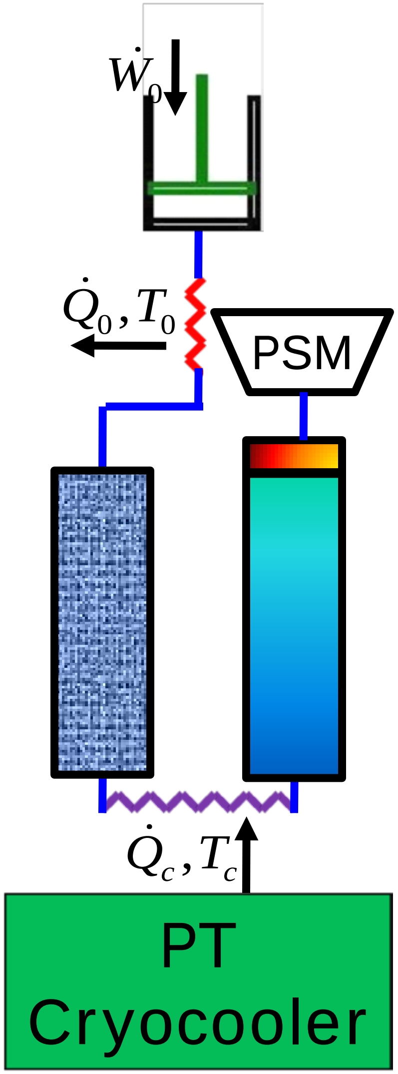
- The gas movement in the PT does not need any mechanical drive.
- Hence, the vibrations in the PT Cryocooler are less as compared to Stirling and GM Cryocoolers.
- The schematic of the PT Cryocooler, with three heat exchangers, namely, After cooler (**AC**), Cold end heat exchanger (**CHX**), Hot end heat exchanger (**HHX**) is as shown in the next slide.

Pulse Tube Cryocooler

- The temperature variation across the length of the Pulse Tube Cryocooler is as shown below.



Pulse Tube Cryocooler



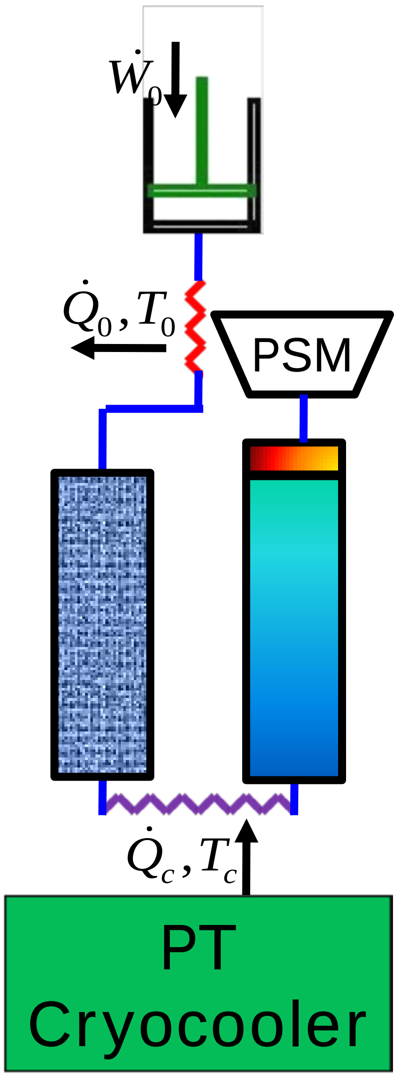
Advantages

- No moving parts in the expander, hence, less vibrations.
- No sealing requirement at low temperature.
- High reliability.

Disadvantages

- No reliability data due to less history.
- Orientation effects.

Pulse Tube Cryocooler



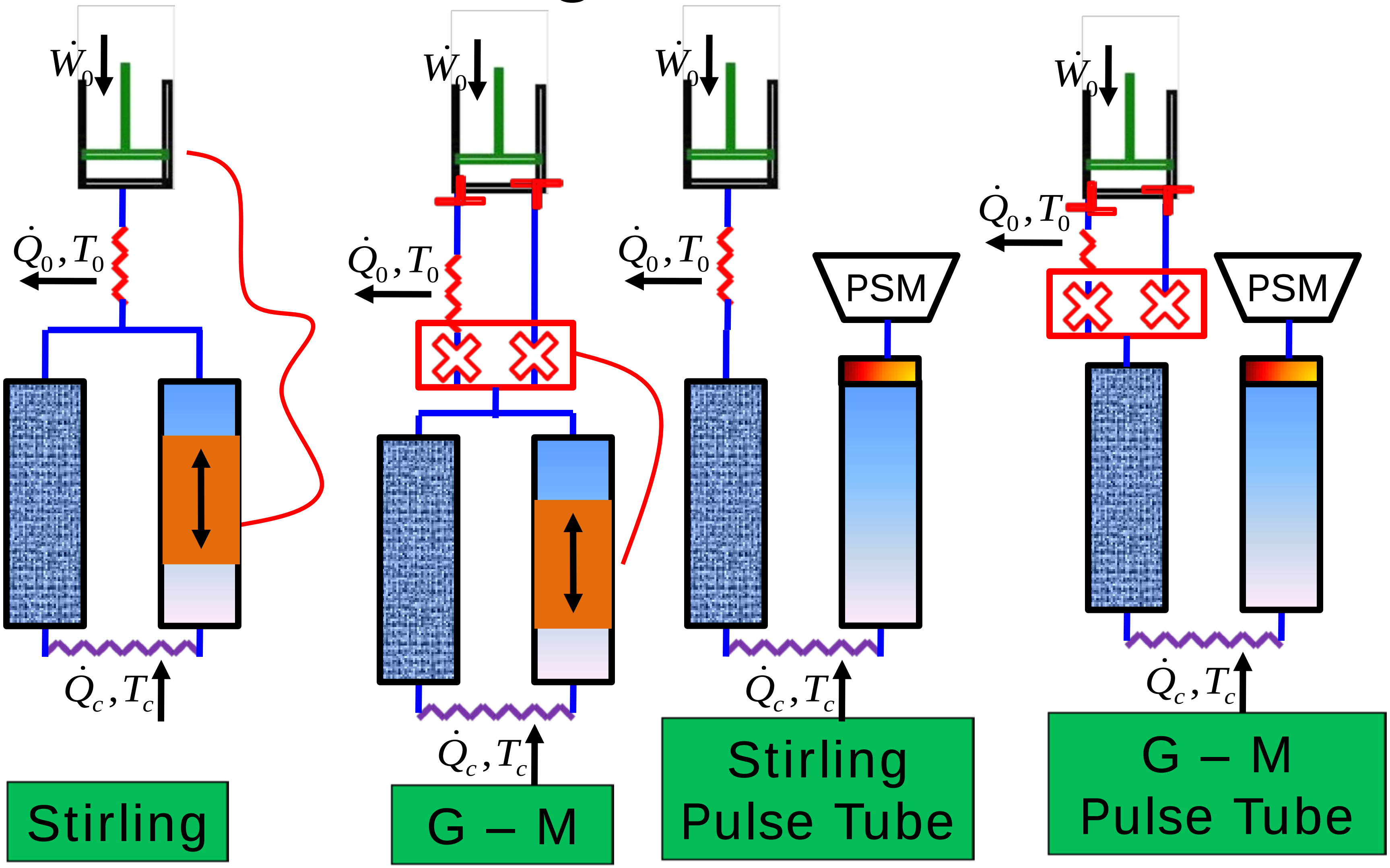
Uses

- Cooling of infrared sensors.
- Space applications.
- Re-condensing of **LHe** and **LN_2**.
- Gas liquefaction.

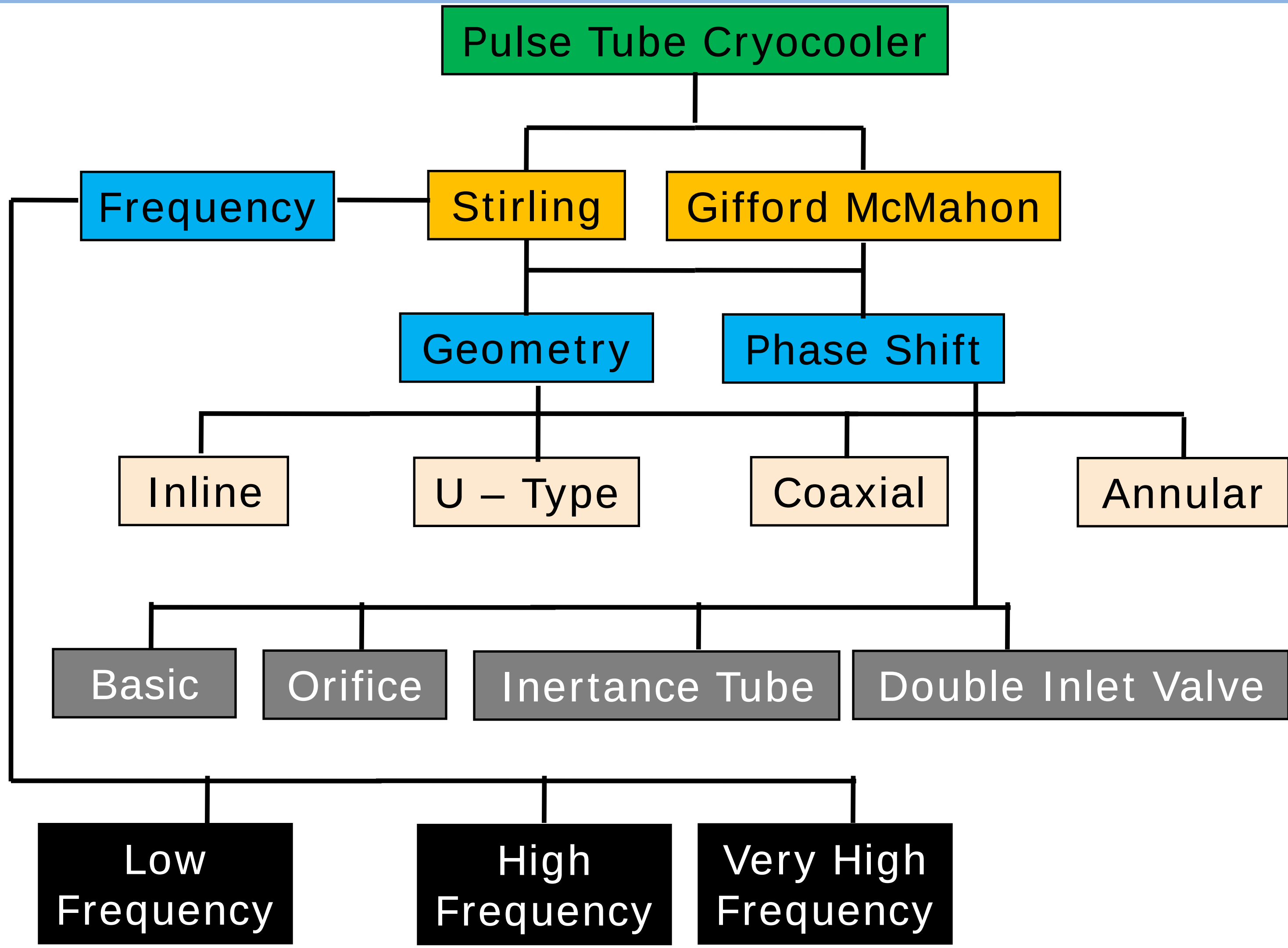
Recent Developments

- Reached below 4 K.
- Miniaturization.

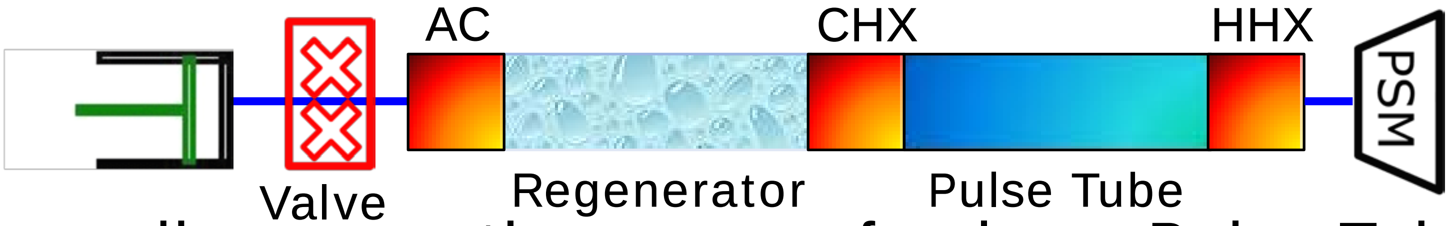
Stirling, GM and PT



PT Classification



PT Classification



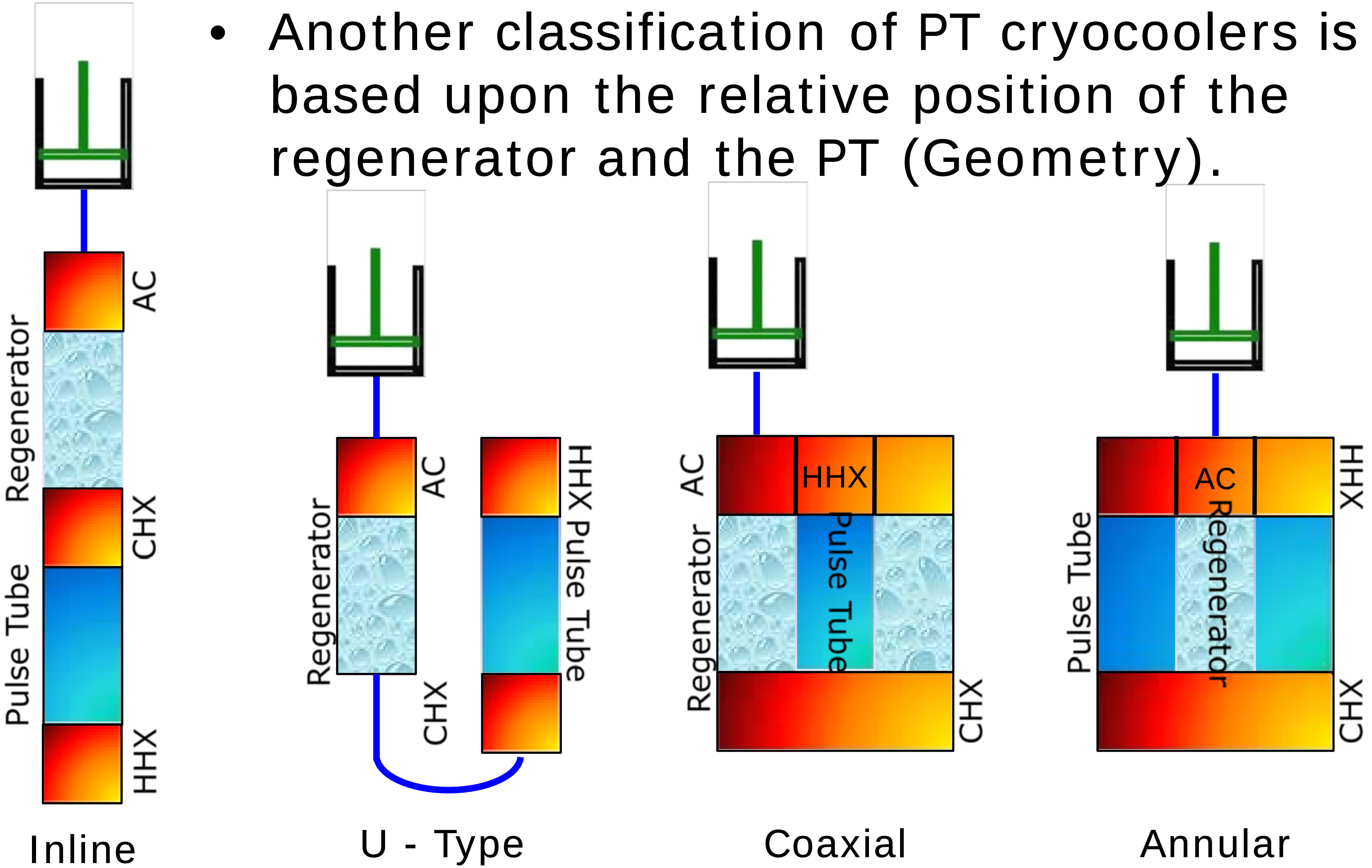
- Depending upon the usage of valves, Pulse Tube cryocooler can either be
  - Stirling type PT Cryocooler
  - GM type PT Cryocooler

- Stirling systems are high frequency machines where as, GM systems are of low frequency.

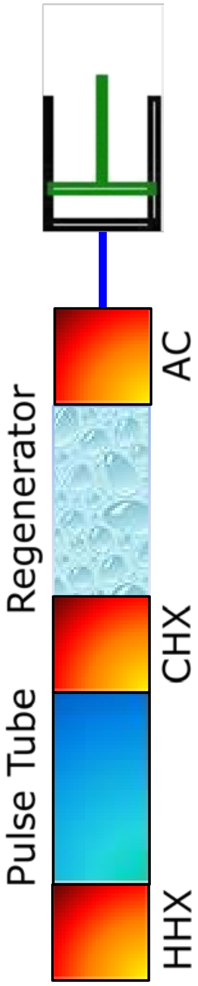
- Each of the systems has its own advantages and disadvantages.

PT Classification

- Another classification of PT cryocoolers is based upon the relative position of the regenerator and the PT (Geometry).



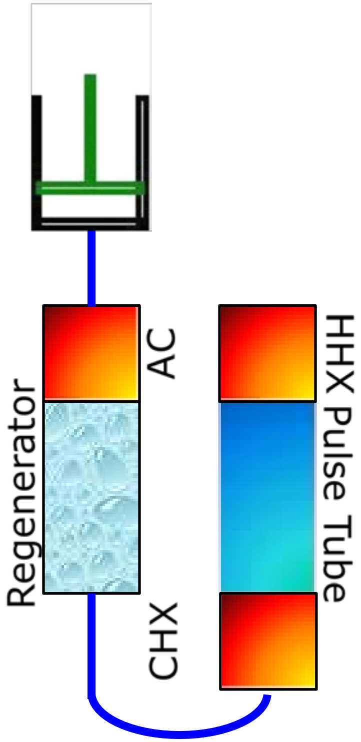
PT Classification



Inline

- The gas does not change the direction of flow. Hence, the pressure losses are minimum.
- This arrangement delivers best performance as compared to others.
- The cold end is at the center of the system.
- The system is less compact since it occupies huge space (length wise).

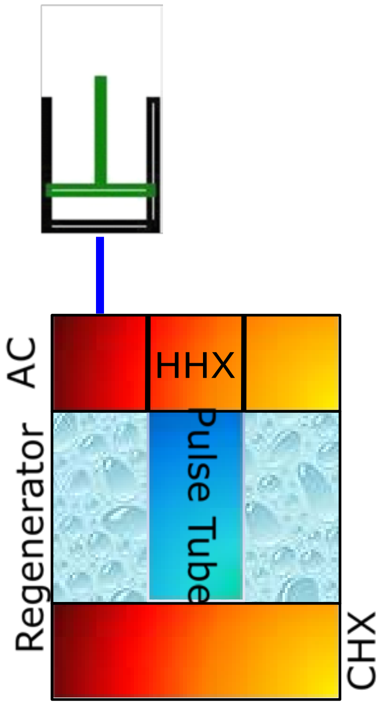
PT Classification



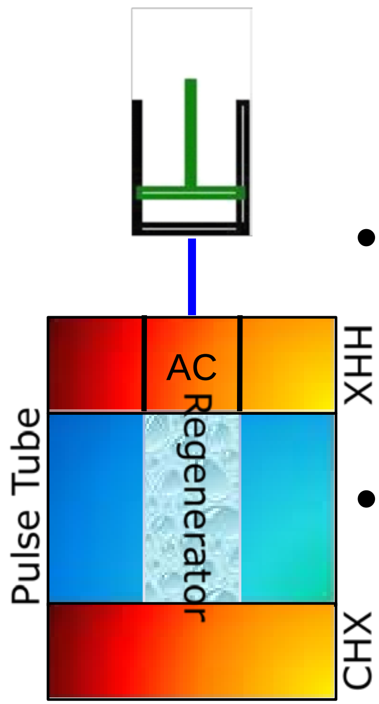
U – Type

- The gas flow undergoes a 180 degree change in flow direction. Due to which the system exhibits pressure drop.
- The cold end is exposed and it is easily accessible.
- The system is more compact and it occupies less space.
- The performance is dependent upon the sharpness of the bend.

PT Classification



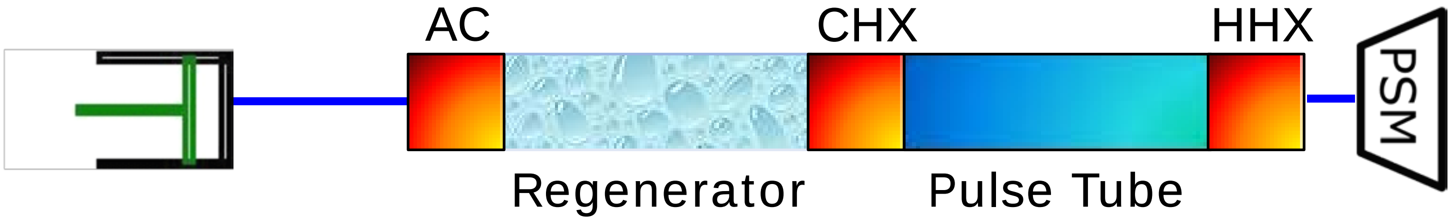
Coaxial



Annular

- The system exhibits maximum pressure drop due to change in flow direction.
- The cold end is exposed and it is easily accessible.
- The system is more compact but there is a possible heat transfer between the PT and regenerator.

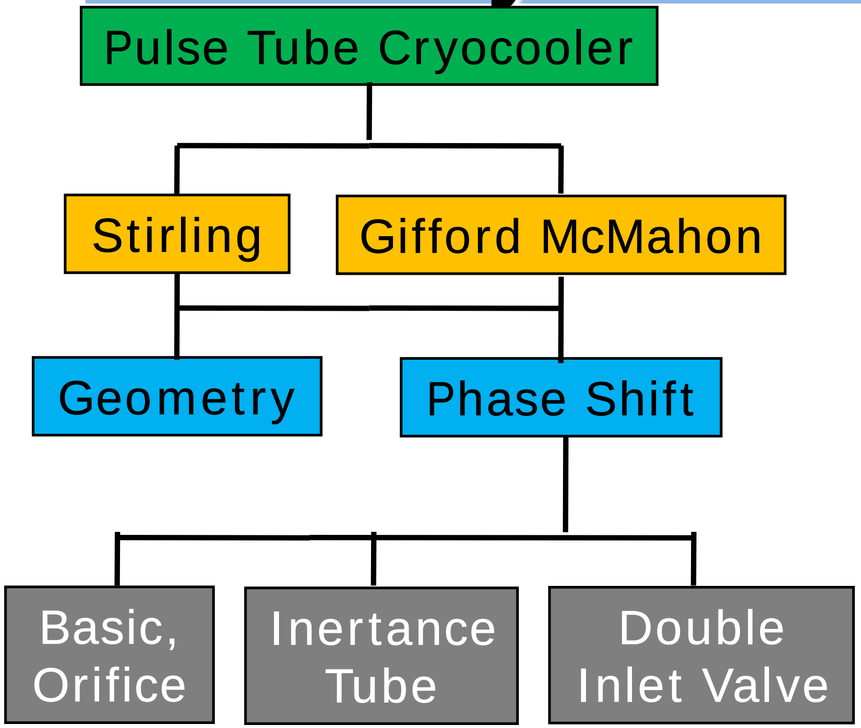
PT Classification



- Depending upon the operating frequency, the Stirling PT systems can be classified as listed below.

- Low Frequency (< 30 Hz)
- High Frequency (30 Hz – 80 Hz)
- Very High Frequency (> 80 Hz)

Analysis of PT Cryocooler



- In order to understand the need of a Phase Shift Mechanism, it is important to understand the modeling of a PT Cryocooler.
- Different analyses are published in the literature with varied difficulty and level of accuracy.

Analysis of PT Cryocooler

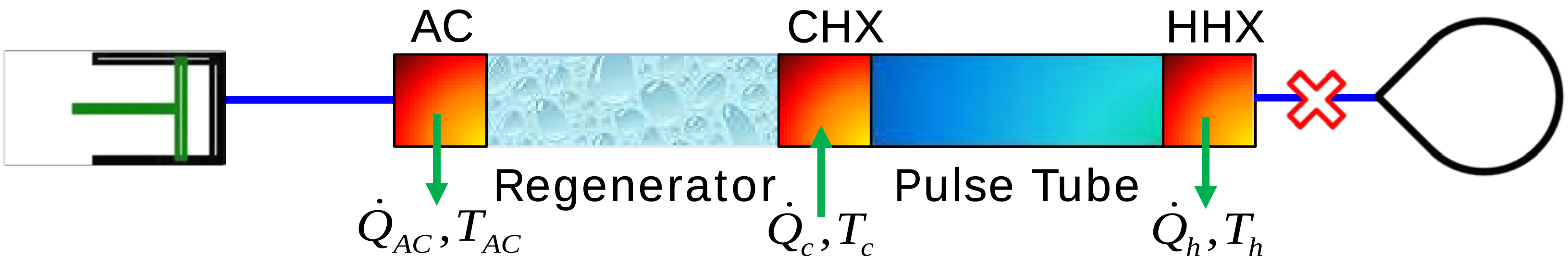
- The following are the methods used to analyze the PT Cryocooler.
- **First Order** – Phasor Analysis.
- **Second Order** – Isothermal Model, Thermodynamic Non – symmetry effect.
- **Third Order** – Numerical Methods, CFD.
- In this topic, Phasor analysis is explained and a tutorial problem is solved.

Phasor Analysis

- In the year 1990, Ray Radebaugh of NIST, proposed Phasor analysis of a PT Cryocooler.
- The theory was applied to an Orifice PT Cryocooler with a monatomic gas as a working fluid.
- The assumptions are,
  - The thermodynamic processes in the PT are adiabatic.
  - Pressure is constant through out the system.
  - Pressure (**p**) and Temperature (**T**) variations are sinusoidal.

Phasor Analysis

- Consider an Orifice Pulse Tube Cryocooler (OPTC) as shown in the figure.



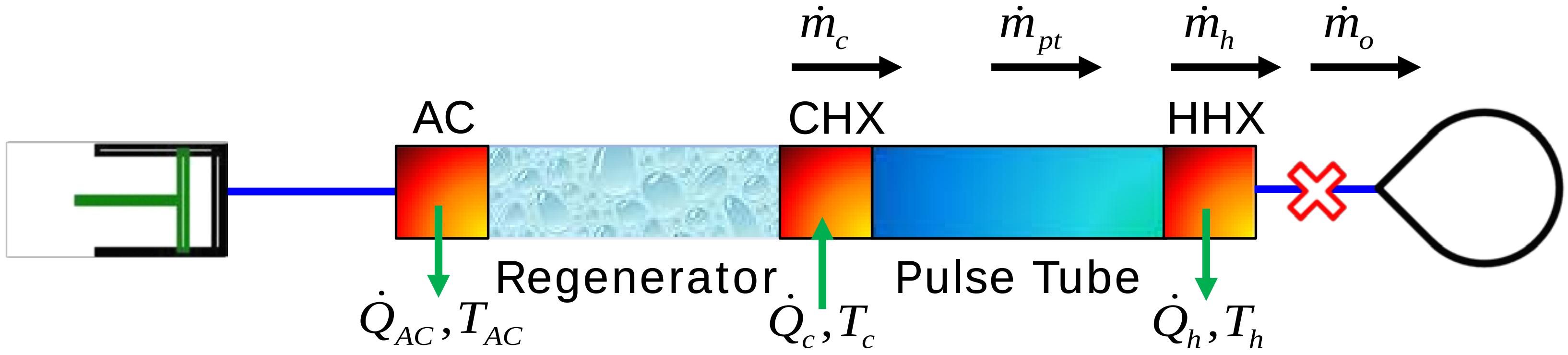
- The sinusoidal variation of Pressure (**p**) and Temperature (**T**) are as given below.

$$p = p_0 + p_1 \cos(\omega t)$$

$$T = T_0 + T_1 \cos(\omega t)$$

- In the above equation, $\mathbf{p_0}$ and $\mathbf{T_0}$ are the average pressure and ambient temperature respectively. $\mathbf{p_1}$ and $\mathbf{T_1}$ are the variations respectively.

Phasor Analysis



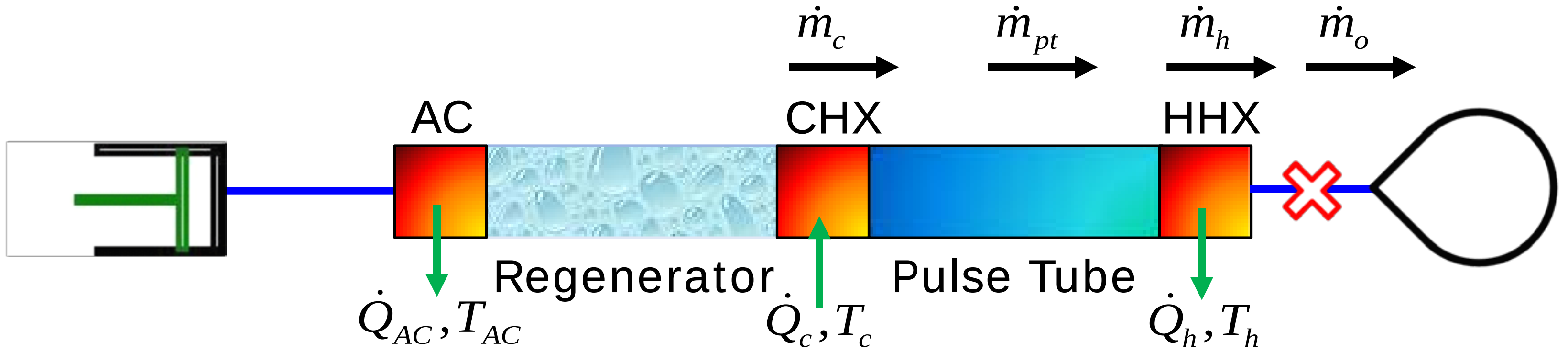
- Let $\mathbf{m_c}$, $\mathbf{m_{pt}}$, $\mathbf{m_h}$ and $\mathbf{m_o}$ be the mass flow rates in the cold end, PT, hot end and orifice respectively.

- Using the law of conservation of mass, we have

$$\dot{m}_{pt} = \dot{m}_h - \dot{m}_c$$

- In an OPTC, the following holds true. $\dot{m}_h = \dot{m}_o$

Phasor Analysis



- Rearranging the above mass equation, in terms of volume, we have

$$dV_{pt} = dV_h - dV_c$$

- Upon multiplying with pressure **p**, we get

$$pdV_{pt} = pdV_h - pdV_c$$

Phasor Analysis

- Consider an Ideal Gas equation.

$$pV = mRT$$

- Differentiating the Ideal Gas Equation, we have

$$\frac{pdV}{dt} = RT\left(\frac{dm}{dt}\right) \qquad pdV = RT\left(dm\right)$$

$$pdV_{pt} = pdV_h - pdV_c$$

- Combining the above two equations, we get

$$pdV_{pt} = RT_h dm_h - RT_c dm_c$$

Phasor Analysis

- The Pressure (**p**) and Temperature (**T**) variations are sinusoidal. They can be written as

$$p = p_0 + p_1 \cos(\omega t) \qquad T = T_0 + T_1 \cos(\omega t)$$

- At any cross section of the PT, let there be a phase **α** between pressure and temperature variations.

$$p = p_0 + p_1 \cos(\omega t) \qquad T = T_0 + T_1 \cos(\omega t + \alpha)$$

$$\frac{T}{T_0} = \left(\frac{p}{p_0}\right)^{\frac{\gamma-1}{\gamma}}$$

- For an adiabatic process, we have

$$\frac{T_0 + T_1 \cos(\omega t + \alpha)}{T_0} = \left(\frac{p_0 + p_1 \cos(\omega t)}{p_0}\right)^{\frac{2}{5}}$$

$$\frac{T_1}{T_0} = \frac{2}{5}\left(\frac{p_1}{p_0}\right)$$

Phasor Analysis

- For a Pulse Tube, an adiabatic law between the pressure and the volume is as given below.

$$pV_{pt}^{\gamma} = k$$

- Differentiating, we have

$$V_{pt}dp + \gamma\, pdV_{pt} = 0$$

- The mass equation is

$$pdV_{pt} = RT_h dm_h - RT_c dm_c$$

$$V_{pt}dp = -\gamma\left(RT_h dm_h - RT_c dm_c\right)$$

$$-\frac{V_{pt}dp}{\gamma} = -RT_c dm_c + RT_h dm_h$$

Phasor Analysis

- Rearranging the above equation, we have

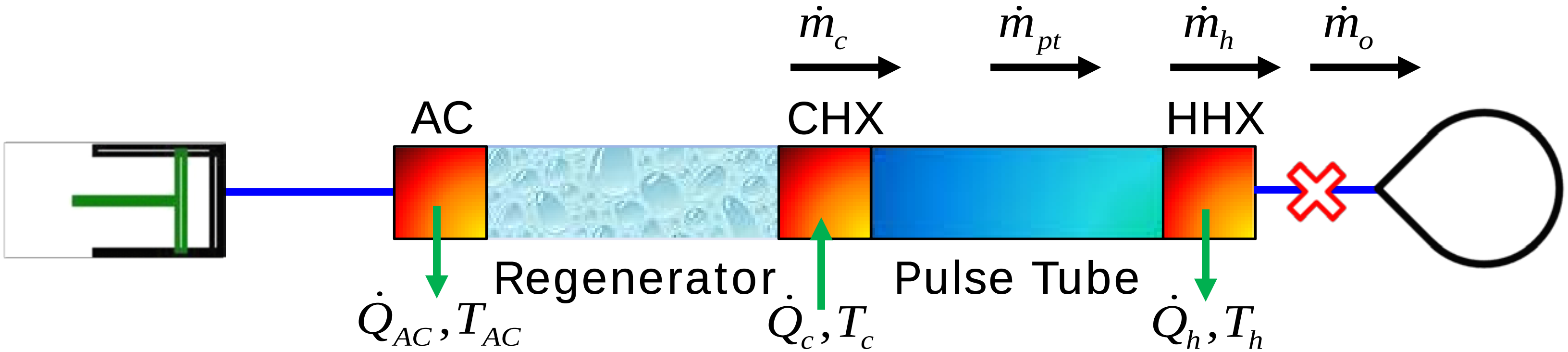
$$-\frac{V_{pt}dp}{\gamma} = -RT_c dm_c + RT_h dm_h$$

$$dm_c = \frac{V_{pt}}{\gamma RT_c} dp + \frac{T_h}{T_c} dm_h$$

- Dividing the above equation with an infinitesimal time step **dt**, we have

$$\frac{dm_c}{dt} = \frac{V_{pt}}{\gamma RT_c}\left(\frac{dp}{dt}\right) + \frac{T_h}{T_c}\left(\frac{dm_h}{dt}\right)$$

Phasor Analysis



- The mass flow rate through the orifice is directly proportional to the pressure drop.

$$\dot{m}_h \propto \Delta p$$

$$\dot{m}_h \propto p_o + p_1 \cos(\omega t) - p_o$$

$$\dot{m}_h = C_1 p_1 \cos(\omega t)$$

Phasor Analysis

- Combining the following equations, we have

$$p = p_0 + p_1\cos(\omega t)$$

$$\dot{m}_h = C_1 p_1 \cos(\omega t)$$

$$\frac{dm_c}{dt} = \frac{V_{pt}}{\gamma R T_c}\left(\frac{dp}{dt}\right) + \frac{T_h}{T_c}\left(\frac{dm_h}{dt}\right)$$

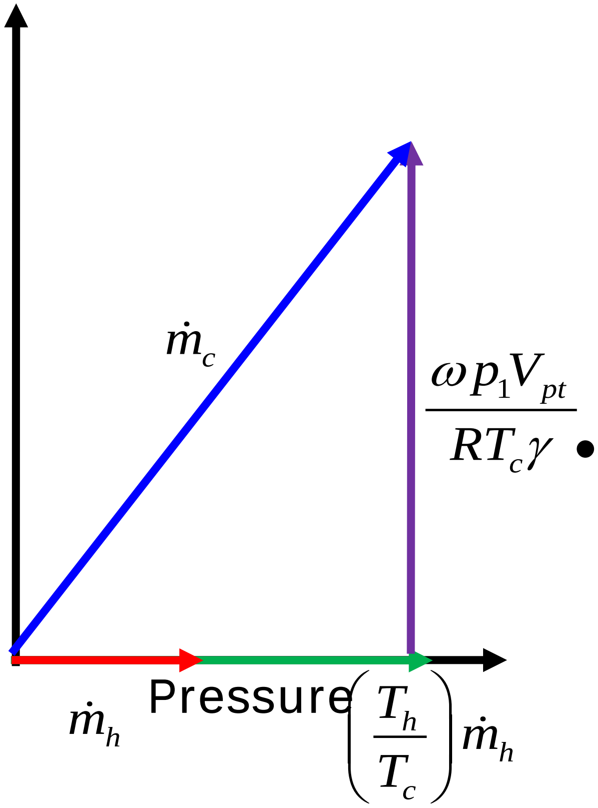
$$\dot{m}_c = -\frac{\omega p_1 V_{pt}}{\gamma R T_c}\sin(\omega t) + \frac{T_h}{T_c} C_1 p_1 \cos(\omega t)$$

$$\dot{m}_c = \frac{\omega p_1 V_{pt}}{\gamma R T_c}\cos\left(\omega t + \frac{\pi}{2}\right) + \frac{T_h}{T_c} C_1 p_1 \cos(\omega t)$$

Phasor Analysis

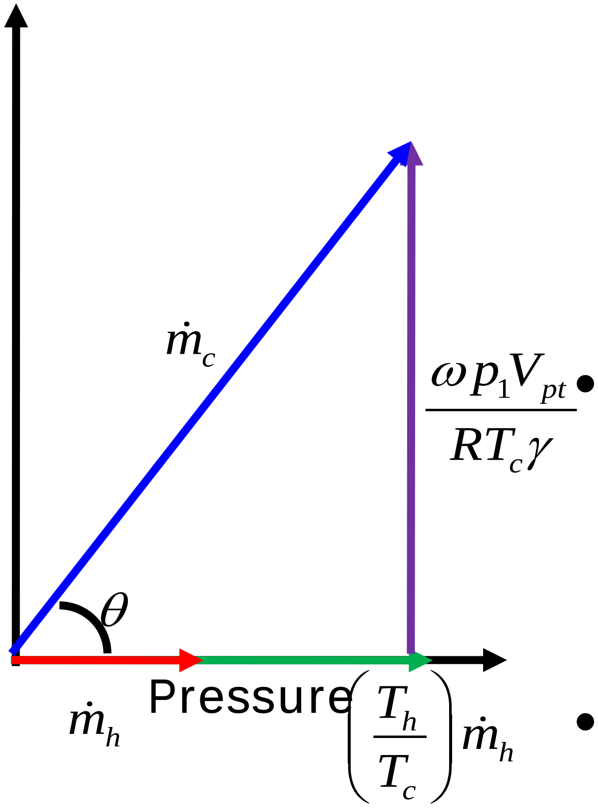
$$\dot{m}_c = \frac{\omega p_1 V_{pt}}{\gamma R T_c} \cos\left(\omega t + \frac{\pi}{2}\right) + \frac{T_h}{T_c} C_1 p_1 \cos(\omega t)$$

$$\dot{m}_c = \frac{\omega p_1 V_{pt}}{\gamma R T_c} \cos\left(\omega t + \frac{\pi}{2}\right) + \frac{T_h}{T_c} \dot{m}_h$$



- From the above equation, it is clear that vector **m_c** is a sum of two vectors which are at **90°** to each other.
- Plotting these two vectors, we have the figure as shown.

Phasor Analysis



- From the figure, it is clear that there exists a phase angle between mass flow rate at the cold end and pressure vector.
- This angle depends on the dimensions, frequency, $\mathbf{p_1}$ and other operating parameters.
- The importance of the phase angle is explained at a greater detail in the next lecture.

Summary

- In a Pulse Tube cryocooler, the mechanical displacer is removed and an oscillating gas flow in the thin walled tube produces cooling. This phenomenon is called as **Pulse Tube action**.

- PT systems can be classified based on the
  - Stirling type or GM type
  - Geometry and Operating frequency
  - Phase shift mechanism

- There exists a phase angle between mass flow rate at the cold end and pressure vector.

- A self assessment exercise is given after this slide.

- Kindly asses yourself for this lecture.

Self Assessment

1. In a PT Cryocooler, mechanical displacer is replaced with ________.
2. Stirling, GM Cryocoolers have ____ vibrations than PT Cryocoolers.
3. The accuracy of Phasor Analysis is ______.
4. In Phasor Analysis, the variation of Pressure (**p**) and Temperature (**T**) is ______.
5. A high frequency PT Cryocooler operates in ______.
6. In a PT Cryocooler, hot end and after cooler are maintained at ________.
7. In ____ system, there is a possible heat transfer between the PT and regenerator.

Self Assessment

8. Mass flow rate through the orifice is directly proportional to the ________.

Answers

1. Pulse Tube
2. More
3. First Order
4. Sinusoidal
5. 30 Hz – 80 Hz
6. Ambient
7. Co – axial
8. Pressure drop

Thank You!